UNIT – I AMPLITUDE MODULATION

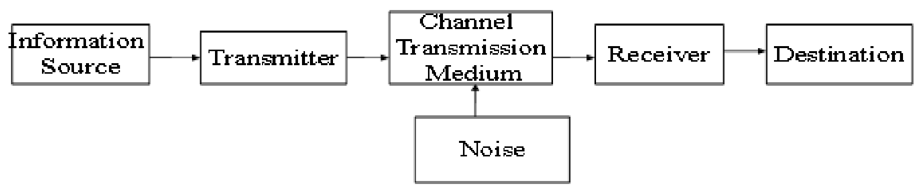
Amplitude Modulation- DSBSC, DSBFC, SSB, VSB - Modulation index, Spectra, Power relations and Bandwidth – AM Generation – Square law and Switching modulator, DSBSC Generation – Balanced and Ring Modulator, SSB Generation – Filter, Phase Shift and Third Methods, VSB Generation – Filter Method, Hilbert Transform, Pre-envelope & complex envelope –comparison of different AM techniques, Superheterodyne Receiver

Introduction

Communication involves transfer of information from source to destination via a channel or medium.

Elements of a communication system

The basic elements are Source, Transmitter, channel, Receiver and Destination

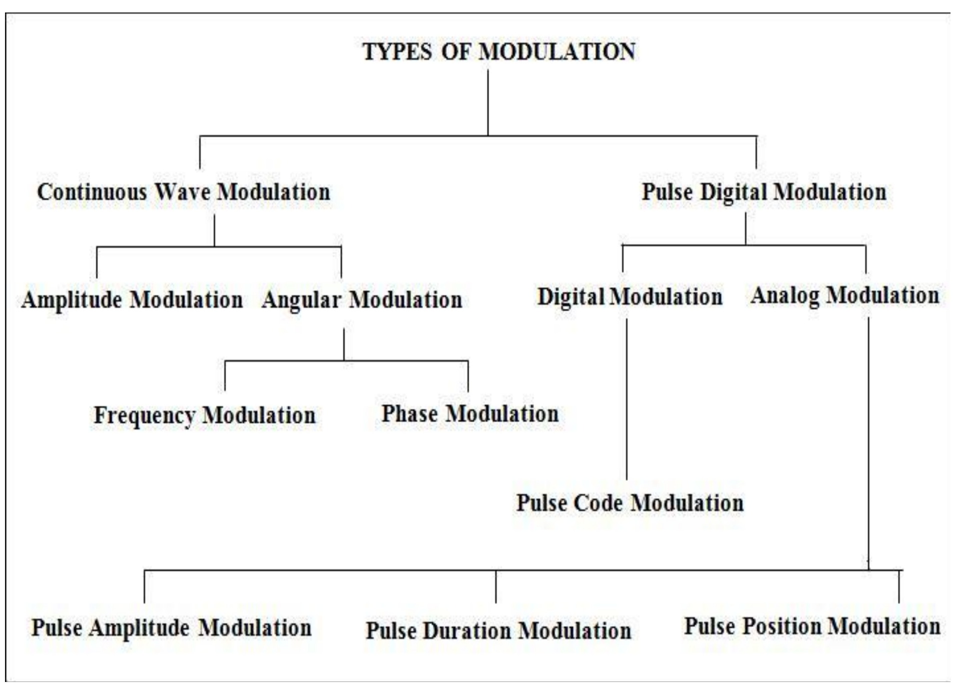


1.1 Modulation

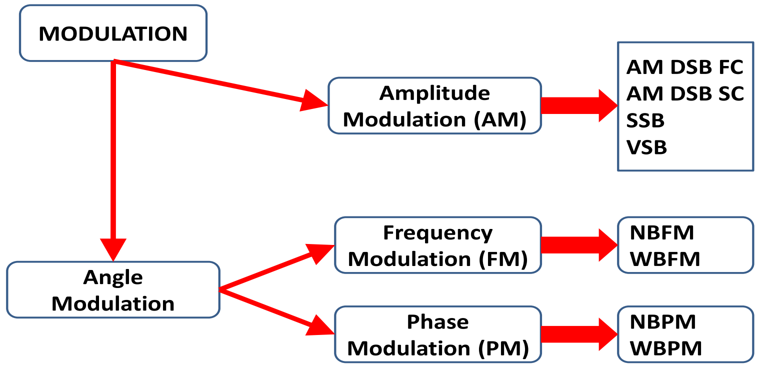
Modulation is the process of **changing the characteristics (Amplitude , Frequency , Phase) of carrier signal according to the instantaneous value of modulating signal.**

Message signal is a **low frequency signal** (voice 0-4 KHz and video 0-6 MHz)⇨ it cannot be transmitted to a long distance.

Carrier signal is a high frequency signal used to transmit the low frequency modulating signal to a long distance



Types of Modulation



The sinusoidal signal can be represented by

$x(t)=A \cos(\omega_c t+\theta)$

A –Amplitude

ω_c–Frequency

θ - Phase

Modulating signal m (t) (Message signal, Base band signal, Demodulated signal) **– Low frequency signal** -Information carrying signal.

m (t)= **$V_m \sin \omega_m t$ or $V_m \cos \omega_m t$**

Carrier signal c (t) - **High frequency signal** used to carry the information carrying signal i.e. modulating signal.

c(t)= **$V_c \sin \omega_c t$ or $V_c \cos \omega_c t$**

Amplitude modulation (AM) : AM is the process of changing the **amplitude of the carrier signal** according to the modulating signal.

Frequency modulation (FM): FM is the process of changing the **frequency of the carrier signal** according to the modulating signal.

Phase modulation (PM): PM is the process of changing the **phase of the carrier signal** according to the modulating signal.

1.1.1 Need for Modulation (or) Advantages of Modulation

Reduction in height of antenna

For efficient transmission, antenna size $\approx \lambda/2$

We know $C = f\lambda$, & $C = 3 \times 10^8$ m/s

For example if f = 5kHz, then $\lambda/2 = C/2f = 3 \times 10^8 / 2\times5\times10^3$ =30km

It is impossible to construct antenna of 30km height.

If f = 10MHz (high frequency), $\lambda/2$ = 15 m.

This antenna height can be practically achieved.

Multiplexing: Several messages are transmitted over the common channel without interference using modulation.
Adjustment of Band Width: Signal to noise ratio can be improved since it is the function of Band Width.
Ease of Radiation: Due to modulation, signals are translated to higher frequencies.
It becomes easy to design amplifier circuits and antenna systems at the increased frequencies.

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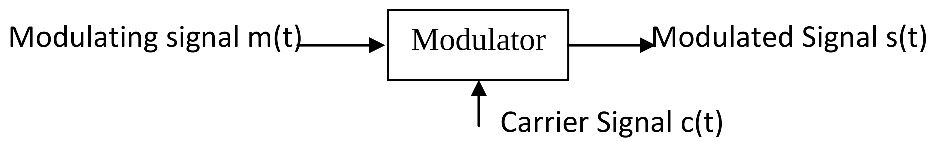
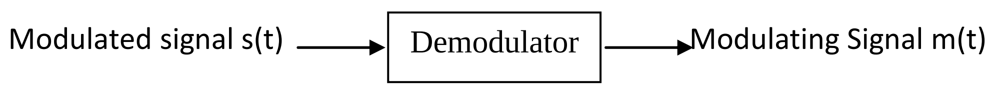
Modulator: Modulator generates modulated signal (AM, FM, PM).



Demodulator/ Detector : Demodulator recovers modulating signal from modulated signal.

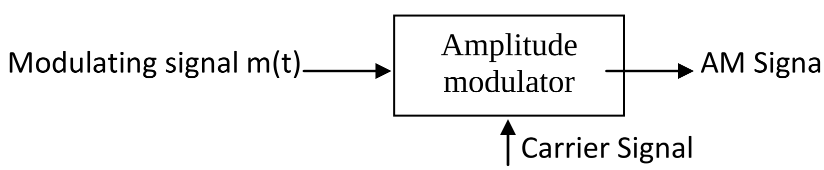


1.2 Amplitude modulation (AM/DSB FC)

In Amplitude modulation, **amplitude of the carrier is changed according** to the instantaneous value of **the modulating signal**.

Types of AM: DSB SC , SSB SC , VSB

1. **Derive the expression for Amplitude modulated signal.** **Nov 2016**



Expression for AM

The carrier signal $c(t) = V_c \sin\omega_c t$

The message signal $m(t) = V_m \sin \omega_m t$

The AM signal , $V_{AM} = V_c + V_m \sin \omega_m t$

$$= V_c\left(1+\frac{V_m}{V_c}\sin\omega_m t\right)$$

$$= V_c\left(1+m_a\sin\omega_m t\right) \quad \text{where } m_a=\frac{Vm}{Vc}$$

The instantaneous value of AM signal is $S(t) = V_{AM}(t) = V_{AM} \sin\omega_c t$

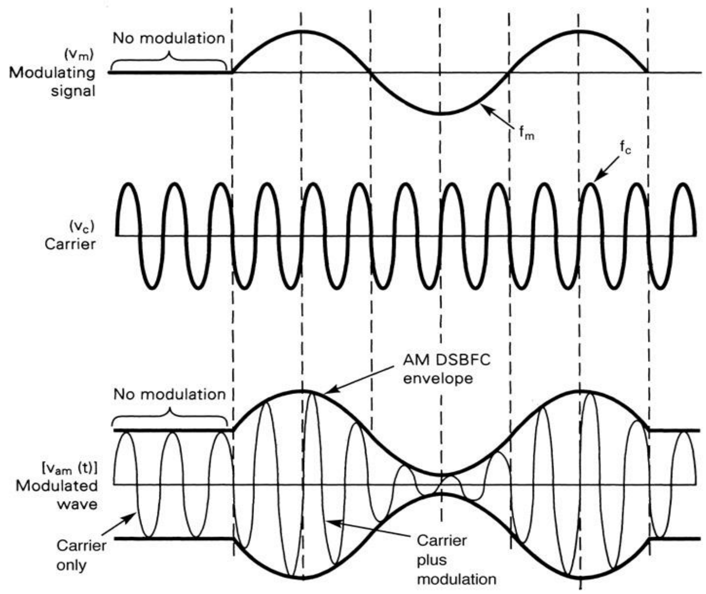
$$= V_c\left(1+m_a\sin\omega_m t\right)\sin\omega_c t$$

$$= V_c \sin\omega_c t + m_a V_c \sin\omega_c t \sin\omega_m t$$

$$= V_c \sin\omega_c t + \frac{m_a V_c}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$s(t) = [1 + k_a m(t)]A\cos(\omega_c t)$$

The AM signal consists of carrier, Lower Sideband (LSB) and Upper Sideband (USB)



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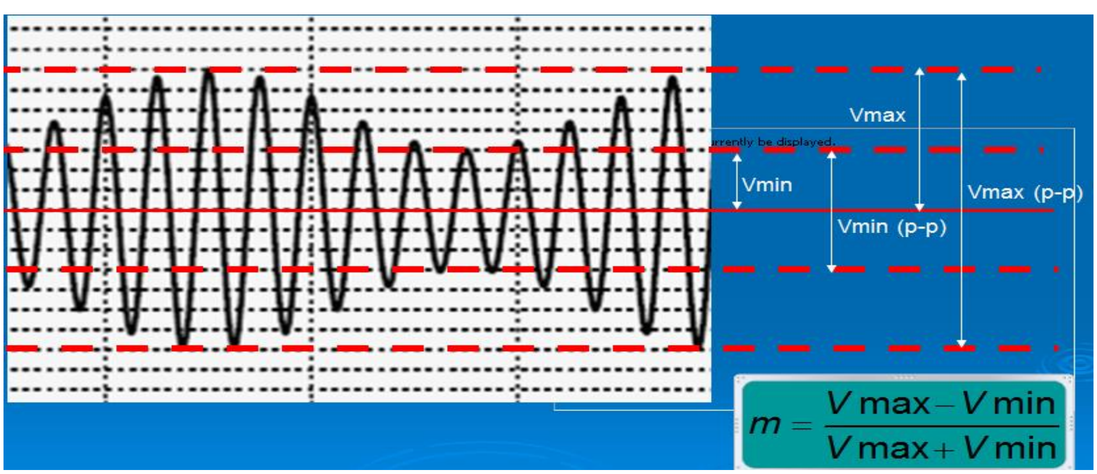
1.2.1 Modulation Index / Depth of modulation / % modulation

It is the parameter which indicates the depth of modulation (or) measure of modulation
Indicates the amount that the carrier signal is modulated.
Modulation index ranges from m= 0 to 1.
It is defined as the ratio of amplitude of modulating signal to the amplitude of carrier signal.

$$m = \frac{V_m}{V_c} \text{ or } m = \frac{V\max - V\min}{V\max + V\min}$$

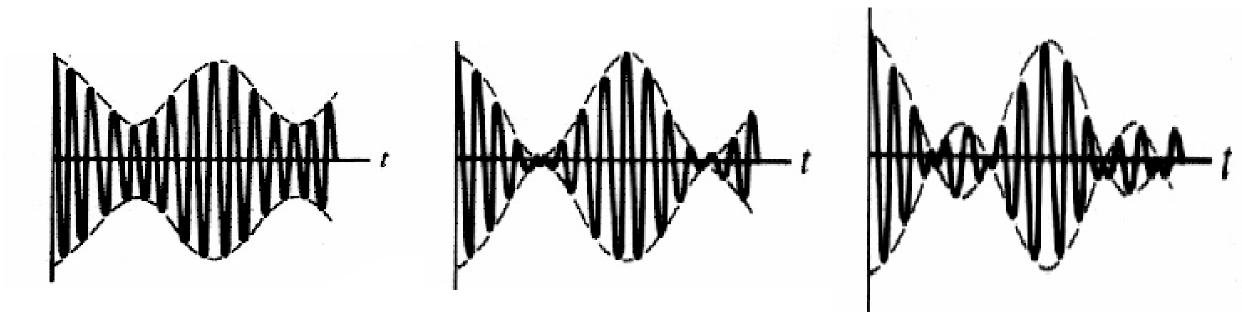


Degrees of modulation

There are 3 degrees of modulation

- m<1 under modulation
- m=1 critical modulation
- m>1 over modulation(distortion)

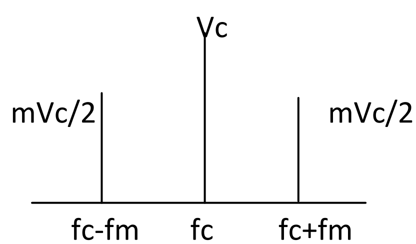
m<1 m=1 m>1



Single tone modulation- Modulation performed for a message signal with one frequency component.

Multi-tone modulation – Modulation performed for a message signal with more than one frequency component

Frequency spectrum of AM

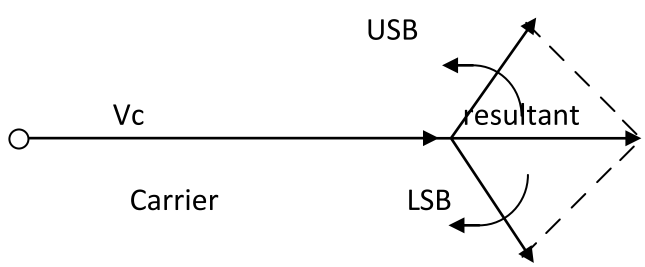


Bandwidth of AM

BW=2fm

fm- Frequency of modulating signal

Phasor representation of AM



The carrier is taken as the reference phasor and the two sideband phasors are rotating in the opposite direction.

The resultant phasor is the sum of carrier phasor and two sideband phasors rotating in the opposite direction.

Power and current relation in AM

The Power relation in AM is

$$Pt = Pc\left(1+\frac{m^2}{2}\right)$$

where Pt – total modulated power, Pc-un modulated carrier power and m- modulation index.

For **multi tone modulation** ,the modulating signal consists of more than one frequency

The transmitted power is

$$Pt = Pc\left(1+\frac{m_t^{\,2}}{2}\right)$$

$$m_t = \sqrt{m_1^{\,2}+m_2^{\,2}+m_3^{\,2}.......}$$

The Current relation in AM is

$$It = Ic\left(1+\frac{m^2}{2}\right)^{\frac{1}{2}}$$

where It – total modulated current, Ic-un modulated carrier current and m-modulation index.

2. Derive the efficiency of AM.

Transmission Efficiency: It is the ratio of the transmitted power which contains the information to the total power(i.e. the total sideband power to the total transmitted power).

It is the ratio of power in sidebands to the total power

$$\%efficiency = \frac{powerinsidebands}{Totalpower} X100$$

$$= \frac{P_{USB}+P_{LSB}}{P_t}\ x100$$

$$= \frac{\frac{m_a^2 v_c^2}{8R}+\frac{m_a^2 v_c^2}{8R}}{\frac{V_c^2}{2R}\left[1+\frac{m_a^2}{2}\right]}\ \text{x100} \quad \text{where } Pc=Vc^2/2R$$

$$= \frac{\frac{v_c^2}{2R}\left[\frac{m_a^2}{4}+\frac{m_a^2}{4}\right]}{\frac{v_c^2}{2R}\left[1+\frac{m_a^2}{2}\right]}\ \text{x 100}$$

$$= \frac{p_c\frac{m_a^2}{2}}{p_c\left[1+\frac{m_a^2}{2}\right]}\ \text{x100}$$

$$= \frac{m_a^2}{2+m_a^2}\ \text{x 100} \qquad m_a = 1$$

$$= 33.33\ \%$$

Note: Only 33.33% power is used and remaining power is wasted by transmitting the carrier along with the sidebands.
The maximum transmission efficiency of the Amplitude Modulation is 33.3%
i.e. only one-third of the total power is used by the sidebands and remaining power is wasted by transmitting carrier which does not contain information.

Advantages of AM:

- AM wave can travel a long distance
- It covers larger area than FM

Disadvantages:

- Poor performance in the presence of noise.
- Inefficient use of transmitter power.
- Wastage in Band Width.

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1.2.2 Generation of AM Waves (AM modulators)

AM modulators

Nonlinear modulator (or) low level modulator (Square law modulator, Balanced modulator)
Linear modulator (or) high level modulator (Collector modulator)

Low level modulation:

- AM generation is at low power level
- Modulation takes place prior to the output element of the final stage of the transmitter.

High level modulation

- AM generation is at high power level.
- Before modulation, the carrier and the modulating signal are amplified to an adequate power level.
- Modulation takes place in the last RF amplifier stage of the transmitter.
- Efficiency of high level modulation is greater than low level.

Nonlinear modulators

- It makes use of the non linear characteristics of nonlinear device. (diode, transistor, FET)
- To make the device to operate in the nonlinear VI characteristics (diode, transistor, FET) the input is kept low.

Linear modulator:

- It makes use of the linear region of the VI characteristics of diode.
- The input is kept high to operate the device in the linear region of VI characteristics of diode.

3. With necessary diagrams explain the non linear method of generation of AM. **Dec2008**
Explain with suitable diagrams the generation of AM using square law. **May 2015**
Explain any one method to generate Amplitude modulated wave. **Nov 2016**

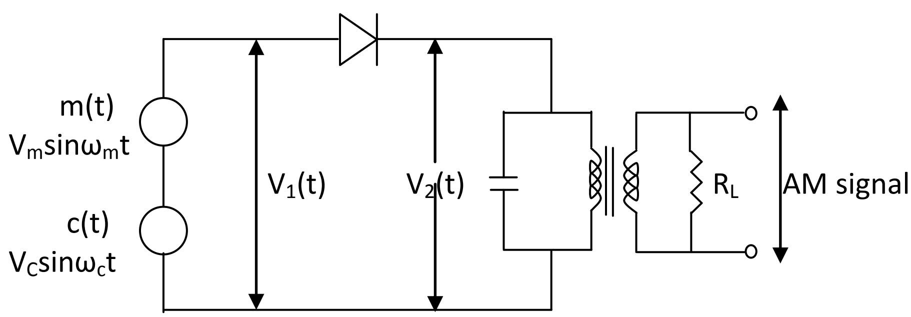
Nonlinear modulators:

i. Square law modulator
ii. Balanced modulator

i. Square law modulator

- Generation of AM.
- Nonlinear modulator.
- Input is kept low so that the device **operates in the non linear region of VI characteristics** of diode.

Circuit diagram



Circuit description:

It consists of

Summer - To add modulating and carrier signal
Nonlinear device -Diode
BPF- Tuned to ω_c

Operation:

$V_1(t)$ –sum of carrier and modulating signal is applied to the input of diode

$V_1(t)=V_C \sin\omega_c t + V_m \sin\omega_m t$ (1)

The input and output relation is given by Square law

$V_2(t) = a V_1(t) + b V_1^2(t)$ (2)

Where a and b are constants

Sub eqn (1) in eqn(2)

$V_2(t) = a V_1(t) + b V_1^2(t)$

$= a (V_C \sin \omega_c t + V_m \sin \omega_m t) + b(V_C \sin \omega_c t + V_m \sin \omega_m t)^2$

$= a V_C \sin \omega_c t + a V_m \sin \omega_m t + b V_C^2 \sin \omega_c t + b V_m^2 \sin\omega_m t + 2b V_C V_m \sin \omega_c t \sin \omega_m t$

$V_2(t)$ consists of modulating signal, carrier signal, squared modulating signal, squared carrier signal.

BPF is tuned to ω_C. It allows only ω_c & $\omega_C \pm \omega_m$ & remaining terms are eliminated.

$V_2(t) = a V_C \sin\omega_c t + 2b V_C V_m \sin \omega_c t \sin \omega_m t$

$= a V_C \sin \omega_c t + b V_C V_m [\cos (\omega_c - \omega_m)t - \cos (\omega_c + \omega_m)t]$

Carrier sidebands

AM wave with carrier and sidebands are generated.

Drawbacks:

- Heavy filtering is required to remove unwanted terms.
- Output power level is low.

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ii. Balanced Modulator

- Non linear modulator
- Common circuit for AM generation

Description:

- Two non-linear devices are connected in the balanced mode. (Here it is transistor)
- Assume two transistors are identical and the circuit is symmetrical.
- The carrier voltage across the two windings of a centre-tap transformer are equal and opposite in phase, i.e. $V_c = -V_C^1$

The input voltage to T, is $V_{bc} = V_C + V_m$

$$= V_C \sin \omega_c t + V_m \sin \omega_m t$$

(Since both V_C & V_m are in phase)

The input voltage to T_2 is $V^1_{bC} = V_C^1 + V_m$

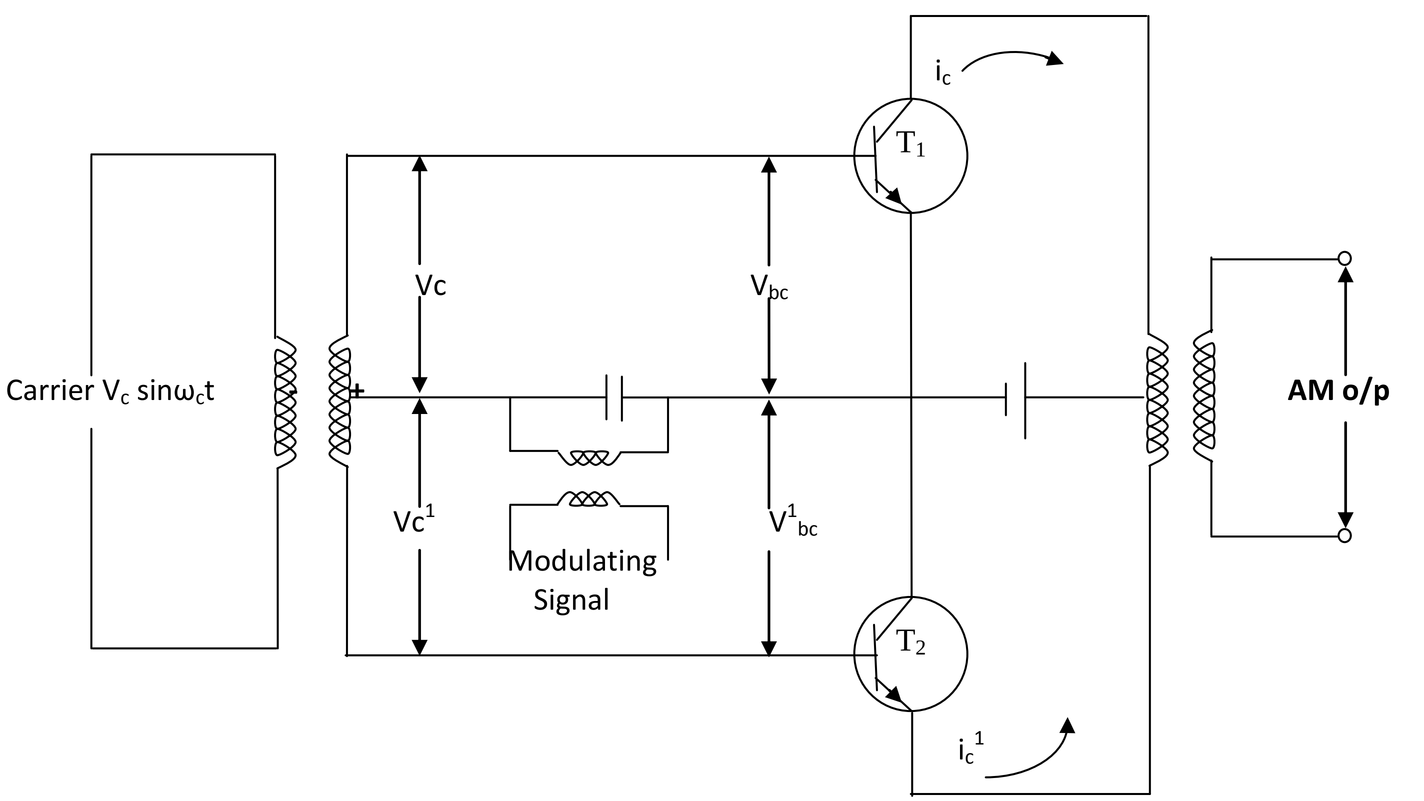
$$= - V_C \sin \omega_c t + V_m \sin \omega_m t$$

By non-linearity, the collector current is

$ic = a_1 V_{bc} + a_2 V_{bc}^2$

$ic^1 = a_1 V^1_{bc} + a_2 V^1_{bc}{}^2$

Circuit diagram:



$i_c = a_1V_C\sin\omega_ct + a_1V_m\sin\omega_mt + a_2V_C^2\sin^2\omega_ct + a_2V_m^2\sin^2\omega_mt + 2VmV_C\, a_2 \sin\omega_mt\ \sin\omega_ct$

$i_c^1 = -\, a_1V_C\sin\omega_ct + a_1V_m\sin\omega_mt + a_2V_C^2\sin^2\omega_ct + a_2V_m^2\sin^2\omega_mt - 2VmV_C\, a_2 \sin\omega_mt\ \sin\omega_ct$

The output AM voltage is

$V_0 = k(i_c - i_c^1)$ Since i_c & i_c^1 are flowing in opposite direction.

$= 2ka_1\, V_C\sin\omega_ct + 4ka_2\, V_C\, V_m \sin\omega ct\, V_C\sin\omega_ct$

$= 2ka_1\, V_C\, [1+2a_2V_m/a_1 \sin\omega_mt] \sin\omega_ct$

$V_0 = 2ka_1\, V_C\, [1+2a_2V_m/a_1 \sin\omega_mt] \sin\omega_ct$

Advantages:

- No filter is required.
- The unwanted terms are automatically balanced out.

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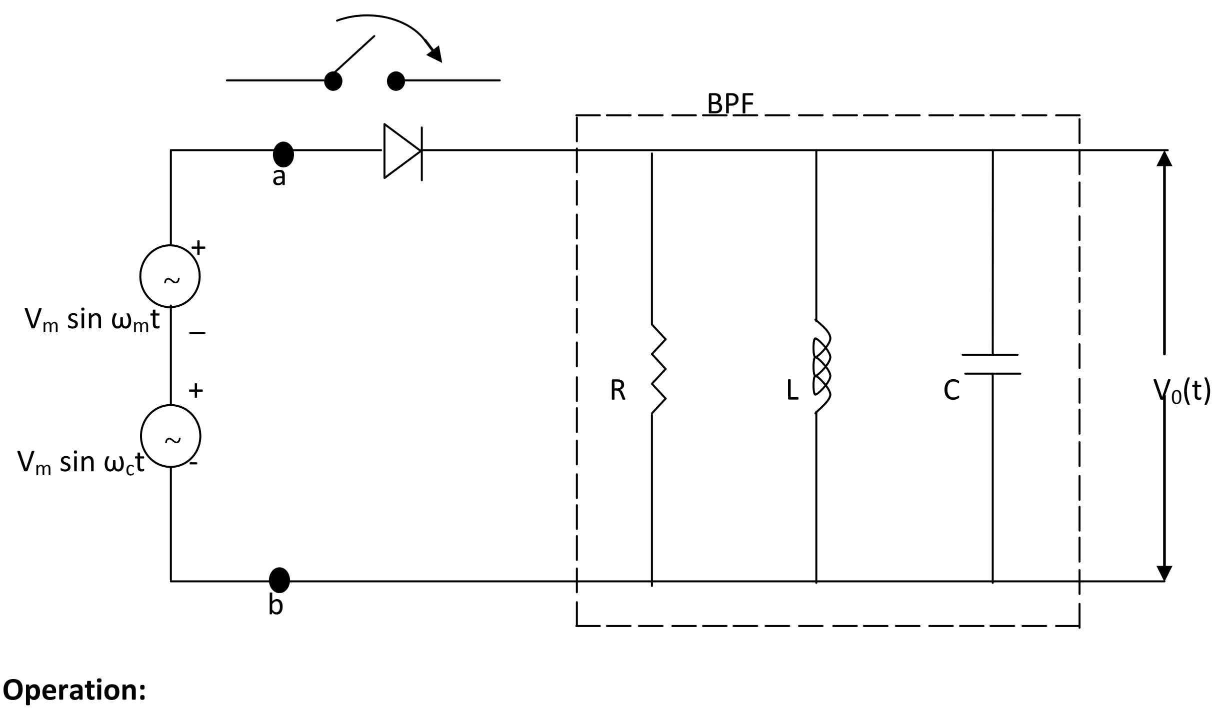
4. Explain the generation of AM using Linear modulator.

Linear modulator

- It makes use of the linear region of the VI characteristics of transistor .
- The input is kept high to operate the device in the linear region of VI characteristics of transistor.

Switching modulator

A simple diode is used for AM switching modulator



Operation:

- The diode is forward biased for every positive half cycle of the carrier, and behaves like a short circuited switch.
- When it is on, the signal appears at the input of the BPF.

- For negative half cycle of the carrier, the diode is reverse biased and behaves like a open switch (it is off). The signal does not reach the filter and no output is obtained.
- The signal is modulated at the rate of carrier frequency.
- The BPF passes frequency $\omega_c + \omega_m$, where ω_m is the maximum frequency of message signal.
- Where there is no modulating signal, the steady state o/p voltage is $\mathbf{V_0(t) = V_C \sin \omega_C t}$
- Let us consider that the diode is ideal, and carrier signal is stronger than message signal.
- The diode conducts when the combined signal (message plus carrier) is positive.
- Then the output voltage is given by $\mathbf{V_0(t) = [V_C + V_m \sin \omega_m t] \sin \omega_c t}$.

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1.2.3 Detection (or) Demodulation

Definition: It is the process of recovering of original modulating signal from the modulated signal.

Detector
- Square law detector (or) Non-linear detector
- Linear diode detector (or) Envelope detector

5. Discuss about nonlinear detection of AM **Dec2012**

Detector: Recovers the original modulating signal from the modulated signal.

Square law detector (Non – Linear detector)

- Detection of AM.
- Nonlinear detector.
- Input is kept low so that the device operates in the non linear region of VI characteristics of diode.

Description:

- Low – level modulated signal.
- Device operating in the non-linear region.
- It is similar to square law modulator – but the filter is LPF instead of BPF.

Operation:

- V_d is used to adjust the operating point.
- Because of the non-linearity of the transfer characteristics of the device, the carrier is away from the quiescent point.
- The operation is limited to the non – linear region due to which the lower half portion of the current waveform is compressed. This causes **envelope distortion**.
- The average value of the diode current varies with time.
- The distorted diode current is given by square law

$i = a_1 v_1 + a_1 v_1^2$ where $V_1 = V_C (1+m_a \sin\omega_m t) \sin\omega_c t$ V_1- input modulated voltage

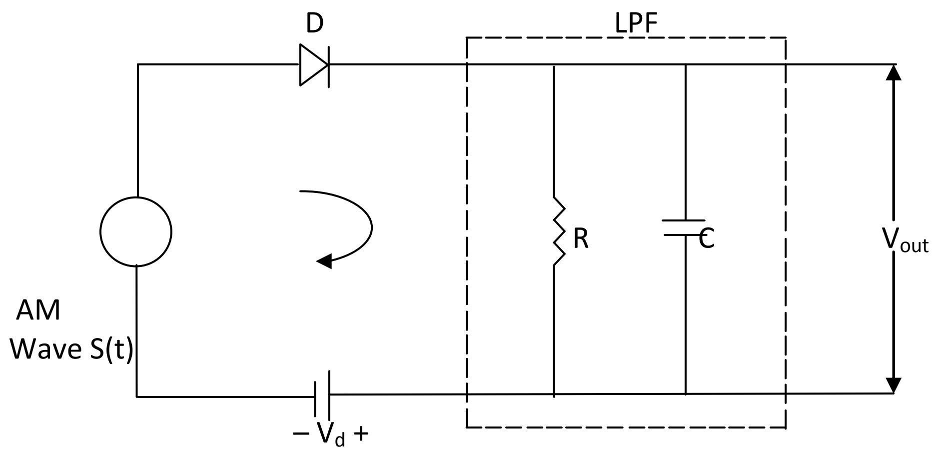
$\therefore$ $i = a_1[V_C (1+m_a \sin\omega_m t)) \sin\omega_c t] + a_2[V_C (1+m_a \sin\omega_m t) + \sin\omega_c t]^2$

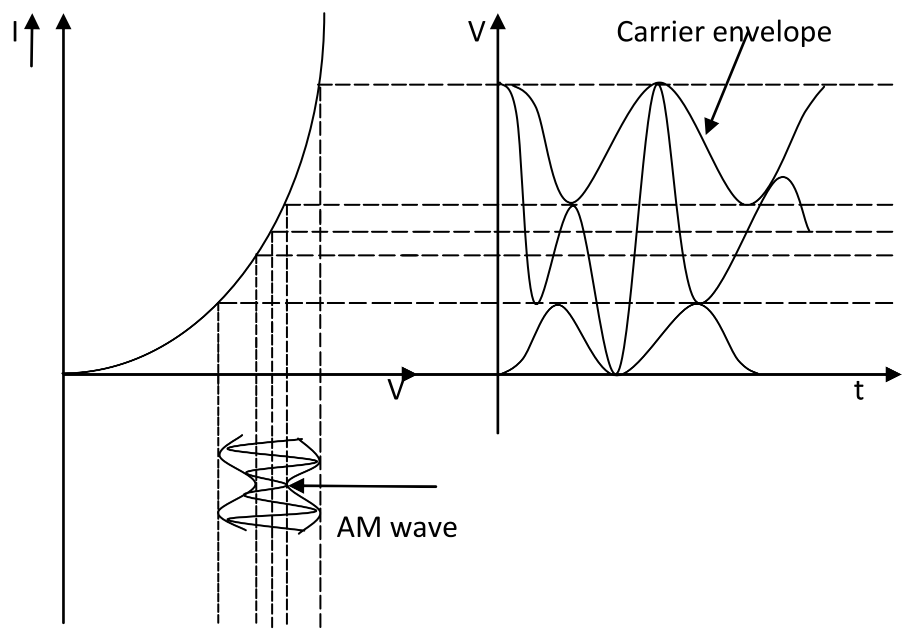
The above current equation consists of components $2\omega_c$, $2(\omega_c \pm \omega_m)$, ω_m and $2\omega_m$ besides the input frequency terms.

This diode current when passed though LPF p asses the frequencies of ω_{m}, $2\omega_m$ and suppress the other higher components.

The modulating signal with frequency ω_m is recovered.

Circuit diagram:





Distortion:

- Non – linear characteristics of the diode produces additional frequency components.
- ω_c & $2\omega_c$ are easily suppressed by LPF, since they are away from ω_m.
- But $2\omega_m$ close to ω_m cannot be totally suppressed by LPF.
  $\therefore$ Component $2\omega_m$ introduces distortion.

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6. **Draw an envelope detector circuit used for demodulation of AM and ex-plain its operation.**
May2010/May2011/May2012

Explain the demodulation of AM using envelope detection. **May 2015**

Explain any one method to demodulate Amplitude modulated wave. **Nov 2016**

Explain the operation of envelope detector. **April 2018** **Dec2017**

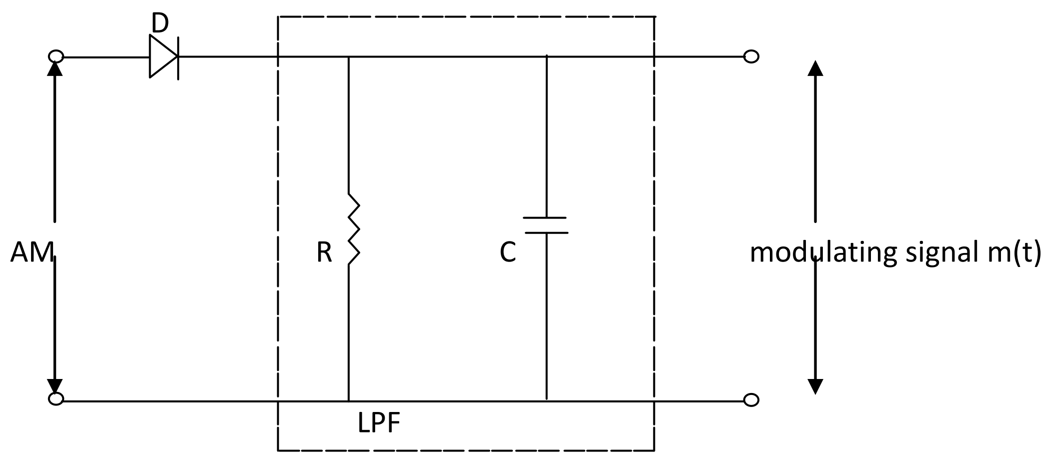
Linear diode detector (**Envelope detector/ Non coherent detector)**

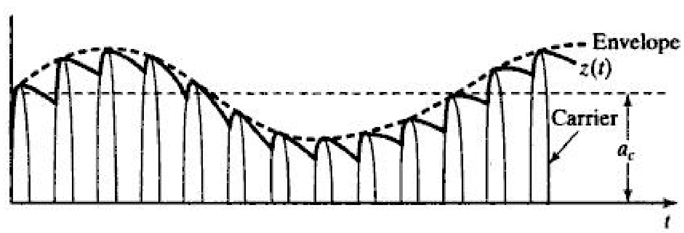
Detector: Recovers original modulating signal from the modulated signal.

Linear diode detector:

- It makes use of the linear region of the VI characteristics of diode..
- The input is kept high to operate the device in the linear region of VI characteristics of diode.
- Used in commercial **AM radio receiver circuits**.

Circuit diagram:





Description:

- A diode operating in linear region of VI characteristics can extract the envelope of an AM wave.
- Such a detector is called **envelope detector.**
- The applied modulated voltage is of large amplitude, the operation takes place in the linear region of VI characteristics.
- Circuit consists of diode and RC low pass filter.
- AM wave is applied at the input of the detector.

Operation:

- Let us assume that the capacitor 'C' is not present.
- Circuit is similar to a half wave rectifier and produces half wave rectified carrier wave.
- If the capacitor is introduced then during positive half cycle, the capacitor charges to the peak value of the input voltage (carrier voltage).
- During the negative half cycle, the diode does not conduct.
- The input voltage is disconnected from the RC circuit.
- Capacitor slowly discharges through 'R'.
- This discharging process continues until the next positive half cycle.
- when the input signal is greater than the capacitor voltage, the diode conducts again and process repeats.
- The output is spiky and follows the envelope of modulated signal.
- The spikes can be reduced if RC time constant is large , so that the capacitor discharges slowly through the load resistance R.
- But if RC is too large it produces **diagonal clipping**.

Distortion in envelope detector

- If RC is too low , discharge curve is almost vertical during the non-conducting period
- Produce large fluctuation s in the output voltage.
- If RC is too high, discharge curve is almost horizontal and several negative peaks are clipped off.
- So ,RC time constant cannot be too high or too low.
- Experimentally it is found that the amount of distortion can be reduced by selecting RC value such that $1/RC \geq \omega_m m_a / \sqrt{1 - m_a^2}$
  $m_a << 1 \quad \therefore 1/RC \geq \omega_m m_a$

Diagonal clipping:

If RC time constant is kept too high, the discharge curve becomes approximately horizontal.
In that case, negative peaks of the detected envelope may be completely or partially missed.
The recovered base band signal is distorted at negative peaks.
This type of distortion is known as diagonal clipping.

Advantages:

- Circuit is simple
- Inexpensive.

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1.3 Suppressed Carrier systems

- In AM, both Transmitted power and bandwidth is wasted.
- The transmitted power is wasted in transmitting carrier along with the sidebands which does not contain information.

Advantages of Suppressed Carrier systems (AM SC):

Both Transmitted power and bandwidth can be saved in suppressed carrier systems

Types of suppressed carrier system

DSB SC -Double sideband suppressed carrier system

SSB SC -Single sideband suppressed carrier system

1.3.1 DSB SC -Double sideband suppressed carrier system

DSB SC contains only sidebands and the carrier is suppressed.

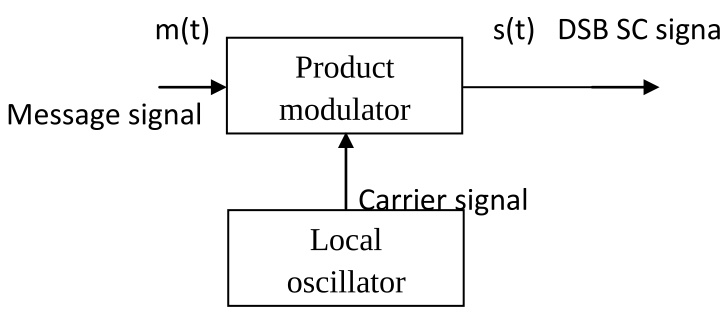
Transmitting power is saved but bandwidth remains the same as AM.

It can be generated by the product of Carrier and the message signal

7. Derive the expression for DSBSC. **Nov 2016**

Product modulator generates DSB SC i.e., multiplication of carrier signal and message signal produces DSB SC

Block diagram:



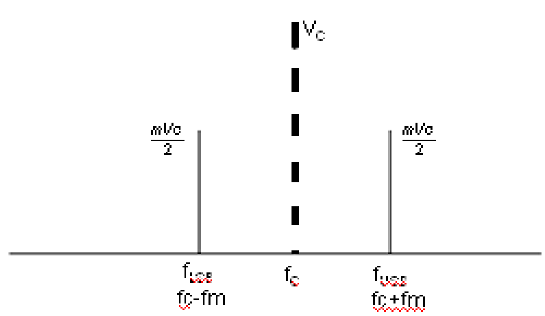
Carrier signal $c(t)= V_C \sin\omega_c t$

Message signal $m(t)= V_m \sin\omega_m t$

$$S(t)=V_C V_m \sin\omega_m t \sin\omega_c t$$

$$=\frac{V_C V_m}{2}\underbrace{[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t]}_{\text{sidebands}}$$

Frequency spectrum of DSB SC

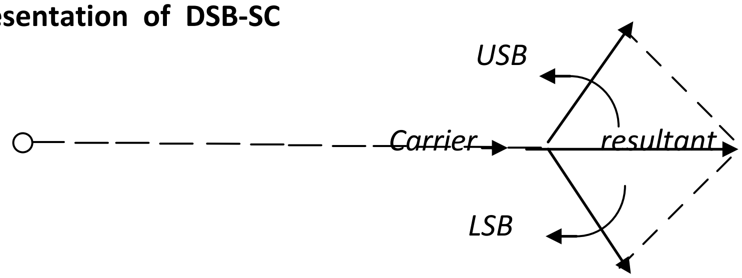


Bandwidth of DSB SC

BW=2fm

BW remains same as AM.

Phasor representation of DSB-SC



The resultant phasor is the sum of two sideband phasors only, since the carrier is suppressed shown by dotted lines

Efficiency of DSB SC

Only the sidebands are transmitted and the carrier is suppressed. Therefore the transmitting power is increased to 66.67%.

1.3.2 Generation of DSB SC AM

In this, AM wave consists of only the upper and lower sidebands. ∴ Transmitted power is saved through suppression of carrier wave.

Types of Generation of DSB-SC

i. Balanced modulator
ii. Ring modulator

8. With the help of a neat diagram, explain the generation of DSB-SC using Balanced modulator.

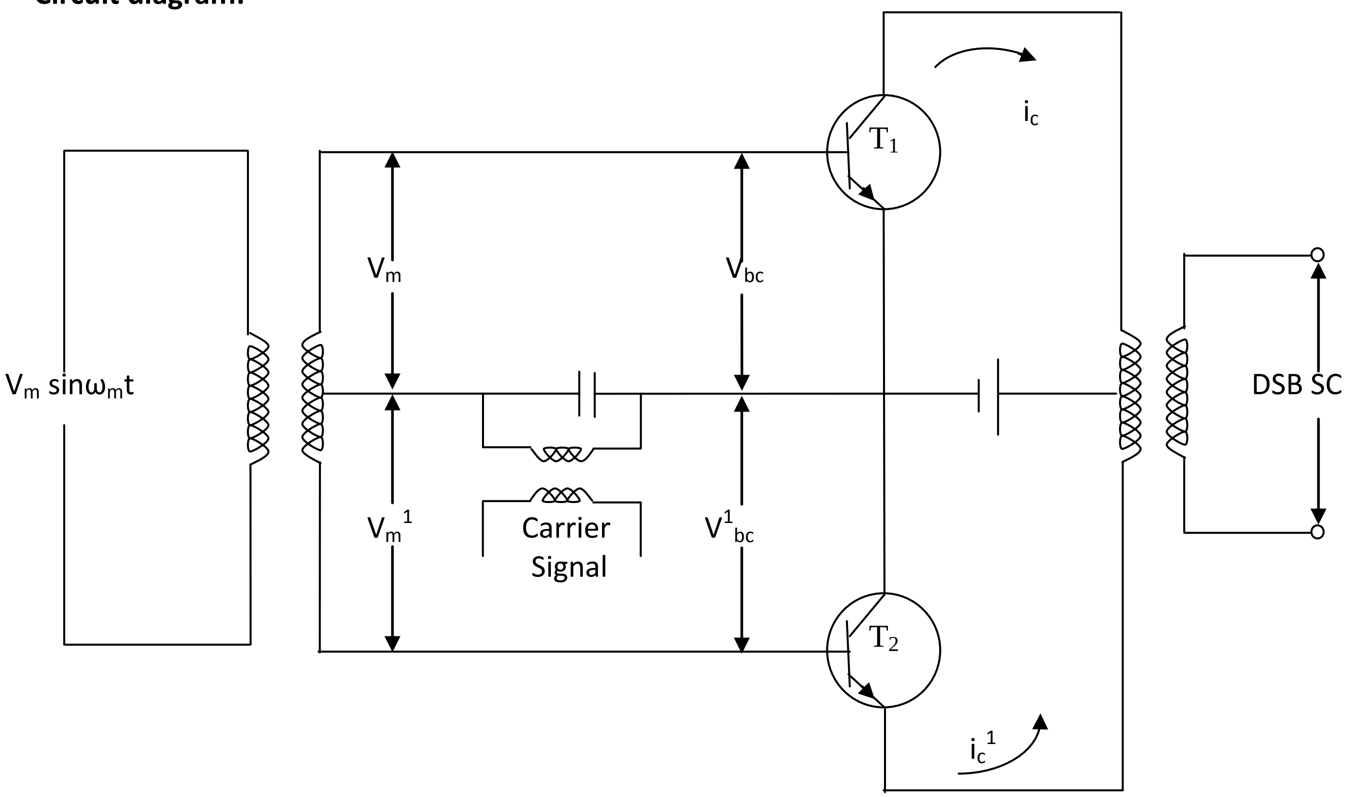
Dec2006/May2009

Derive the expression for output voltage of a balanced modulator to generate DSB SC and explain the working principle. **[Apr - 2019]** **May 2017**

i. Balanced Modulator

- Commonly used for DSB – SC generation
- Two non-linear devices are connected in the balanced mode to suppress the carrier wave.
- Operation is confined in non-linear region of its transfer characteristics.

Circuit diagram:



Operation:

- The modulating voltage applied across the two windings of a centre-tap transformer are equal opposite in phase. i.e. $V_m = -V_m^1$
- The input voltage to transistor T_1 is given by $V_{bc} = V_c + V_m$ (V_m & V_c are in phase)
$$= V_C \sin \omega_c t + V_m \sin \omega_m t \quad (1)$$
- Input voltage to transistor T_2 is given by $V_{bc}^1 = V_m^1 + V_c$ (V_m & V_c are out of phase)
$$= - V_m \sin \omega_m t + V_C \sin \omega_c t \quad (2)$$
- By the non-linearity relationship, the collector current can be written as
$$i_c = a_1 V_{bc} + a_2 V_{bc}^2 \quad (3)$$
$$i_c^1 = a_1 V^1_{bc} + a_2 V^1_{bc}{}^2 \quad (4)$$

Substituting eqn (1) & (2) in eqn (3) & (4)

$$i_c = a_1[V_C \sin \omega_c t + V_m \sin \omega_m t] + a_2[V_C \sin \omega_c t + V_m \sin \omega_m t]^2$$
$$= a_1 [V_C \sin \omega_c t + V_m \sin \omega_m t] + a_2 [V_C^2 \sin^2 \omega_c t + V_m^2 \sin^2 \omega_m t + 2V_m \sin\omega_m t\, V_C \sin \omega_c t] \quad (5)$$
$$i_c^1 = a_1 [V_C \sin \omega_c t - V_m \sin \omega_m t] + a_2 [V_C^2 \sin^2 \omega_c t + V_m^2 \sin^2 \omega_m t - 2V_m \sin \omega_m t\, V_C \sin \omega_c t] \quad (6)$$

The output DSB SC voltage is

$V_0 = k(I_c - i_c^1)$ Since I_c & i_c^1 are flowing in opposite direction.

Sub eqn 5 & 6 in V_0

$$V_0 = 2ka_1 V_m \sin\omega_m t + 4k\, a_2 V_m V_c \sin\omega_c t \sin\omega_m t$$

This circuit can also be constructed using other amplifying devices like FET

****

9. Draw the circuit diagram of ring modulator and explain its operation. **Dec2006/May 2016**

Explain any one method to generate DSB-SC AM. **Nov 2016**

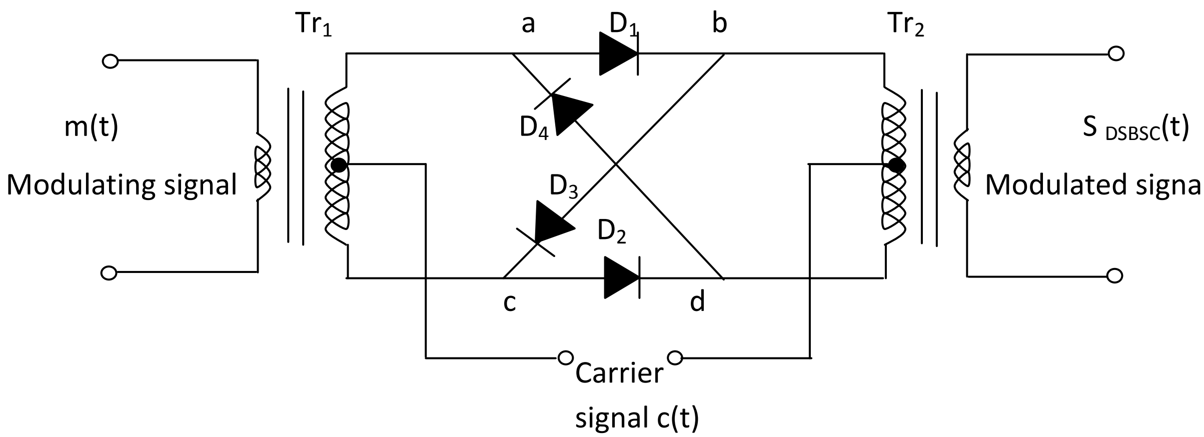
Ring modulator (or) Double Balanced Modulator

Both carrier and modulating signals are automatically balanced out and hence the name

- Most popular method
- 4 diodes are connected to form a ring and the carrier signal is connected between centre taps of the input and output transformers.
- No need for BPF at the output
- The 4 diodes are controlled by a square wave carrier $V_c(t)$ of frequency f_c.

Assumption: In switch on condition, the diodes have a constant forward resistance r_f and a constant backward resistance r_b when switched off.

Circuit diagram:



Construction:

The modulator consists of input transformer T_{r1} and output Transformer Tr_2 & four diodes.

The modulating signal is applied to the input of T_{r1} and carrier is applied to the centre tap of T_{r1} and T_{r2}.

Operation:

Carrier acts as a switching signal to alternate the polarity of m(t) at carrier frequency.

Case i. No modulating signal and only carrier signal is present.

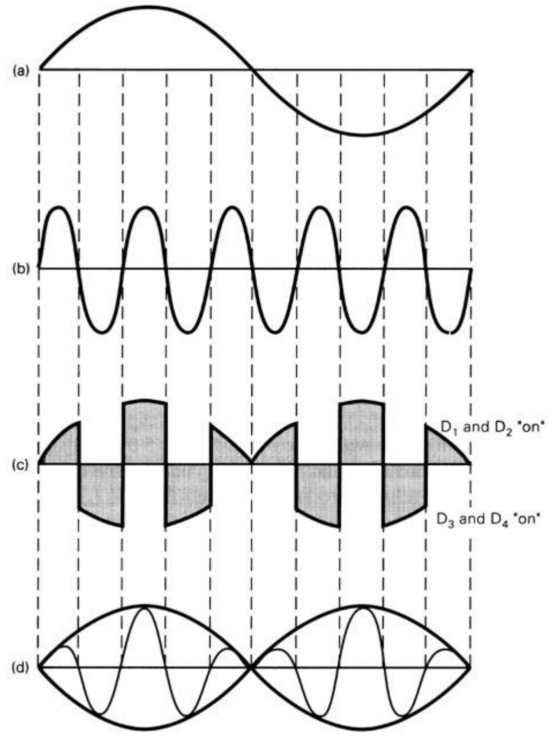
When there is no modulating signal, all the four diodes conduct depending upon the polarity of Carrier.

Positive half cycle of carrier:

- Diodes D_1 & D_2 are forward biased and D_3 & D_4 are reverse biased current divides equally in the upper & lower portions of the primary of T_{r2}.
- No output is induced in the secondary. Thus the carrier is suppressed.

Negative half cycle of carrier:

- Diodes D_1 & D_2 are reverse biased & D_3 and D_4 are forward biased, current divides equally in the upper & lower portions of the primary of T_{r2}.
- No output and carrier is effectively balanced out.



Case ii when both carrier and modulating signals are present.

- During positive half cycle of the carrier c(t)>0, diodes D_1 & D_2 conduct, D_3 & D_4 does not conduct.
- The message signal m(t) is multiplied by +1
- During negative half cycle of the carrier c(t)< 0, D_3 & D_4 conduct, D_1 & D_2 does not conduct.
- The message signal m(t) is multiplied by -1
- When polarity of modulating signal changes, 180° phase reversal takes place.

Modulating signal $m(t) = V_m \sin \omega_m t$

$c(t) = V_c \sin \omega_c t$

Output voltage $S(t) = m(t)\, c(t)$ we know that $\sin A \sin B = 1/2[\cos(A-B) - \cos(A+B)]$

$= V_m \sin \omega_m t\, V_c \sin \omega_c t$

$S_{DSBSC}(t) = [V_m V_c/2]\,[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$

The above equation shows that the output contains upper & lower sidebands only.

Advantages:

- DSB – SC is more efficient in transmitted power as compared to DSBFC.
- Better signal to noise ratio as compared to SSB.

Disadvantage:

- BW remains same as AM even though carrier is suppressed.

****

1.3.3 Detection of DSB – SC

Recover the original modulating signal from the DSB SC modulated signal.

Coherent detection (or) synchronous detection.

Costas loop detection. (Costas receiver)

10. Explain the operation of DSBSC system using coherent detection with the help of circuit diagram. **Dec2006/May2009/May 2016**

Explain any one method to detect DSB-SC AM. **Nov 2016**

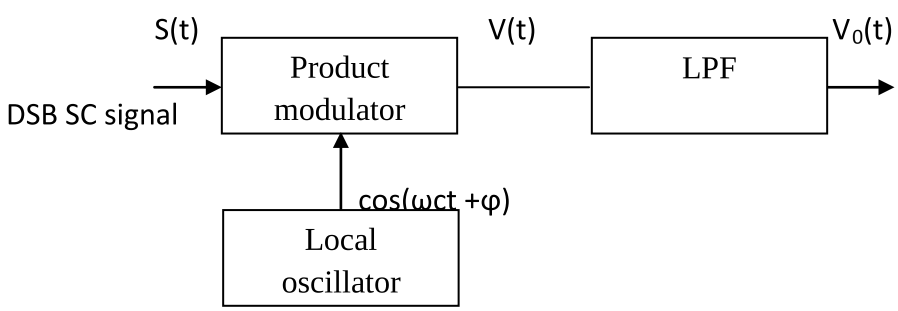
Discuss the detection process of DSB SC using coherent detector. Analyze the drawback of the suggested methodology. **Nov - 2018** **[Apr - 2019]** **May 2017**

Detection: Demodulation or detection is the process by which the original modulating signal is recovered from the modulated signal. It is the reverse process of modulation.

Coherent detection: The modulating m(t) can be recovered from DSB – SC by first multiplying locally generated carrier.

The phase and frequency of locally generated carrier and carrier at the transmitter must be exactly coherent in phase and frequency otherwise the detected signal will be distorted.

Block diagram:



Description:

- It consists of product modulator followed by an LPF.
- The product modulator multiples the DSB SC modulated signal and the locally generated carrier.
- The output of product modulator is applied to the LPF to allow the modulating signal only.

Operation:

- The input signal can be DSB – SC or SSB – SC
- It is multiplied by locally generated carrier

$V(t) = m(t)A_c Cs\ \omega_c t\ Cos(\omega_c t + \varphi)$

$= [m(t) A_c/2] Cos\ \varphi\ Cos(2\omega_c t + \varphi)]$

$= [m(t) A_c/2] Cos\ \varphi + [m(t)A_c/2] Cos(2\omega_c t + \varphi)]$

The product signal is then passed through LPF of BW ω_m.

$V_o(t) = [m(t)A_c/2] Cos\ \varphi$

The amplitude of demodulated signal is maximum, when φ = 0

Minimum, when φ = ± π/2

i.e. $V_0(t) = 0$, when φ = ± π/2

- The zero demodulated signal which occurs when φ = ± π/2 is called **quadrature null effect**.
- Phase error φ in the local oscillator causes the detector output to be attenuated by a factor Cos φ.
- When phase error is constant, the detector produces undistorted output.

Demerits:

- It requires an additional system at the receiver to ensure that the carrier at the transmitter is synchronized with the local carrier
- Receiver is complex and costly.

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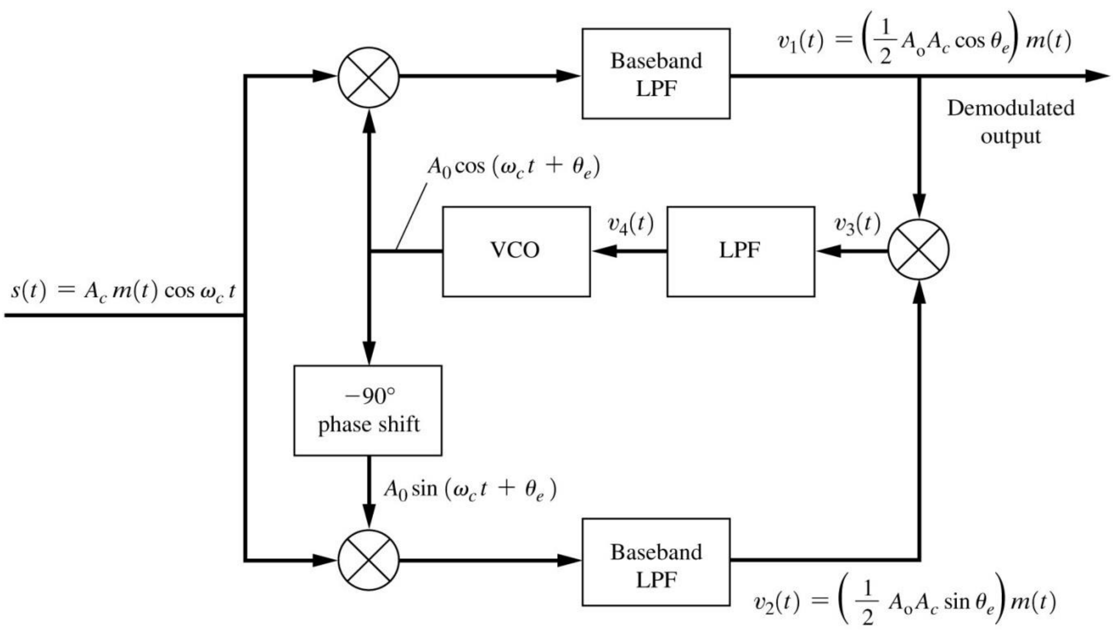
11. Explain the operation of Costas receiver **May2009**

Costas Receiver

To overcome the demerits of coherent detector , costas receiver is used

Block diagram:



(a) Costas Phase-Locked Loop

Costas receiver is used for synchronous detection of DSB – SC signal to avoid quadrature null effect.

Quadrature Null effect

The detector output is zero when θ = 90°.This is called Quadrature null effect, because the signal is zero when the local carrier is in phase quadrature with transmitted carrier

Description:

- The system has two synchronous detectors.
- One detector is called **In phase coherent detector (or) I channel**.
- It is fed with a locally generated carrier which is in phase with the transmitted carrier.
- The other synchronous detector is **Quadrature phase coherent detector (or) Q channel**.
- It is fed with a locally generated carrier which is in phase quadrature with the transmitted carrier
- The 2 detector constitute a negative feedback system which synchronizes the local carrier with the transmitted carrier.

Operation:

- If local carrier signal is synchronized with the transmitted carrier (θ = 0)
- The output of I channel is the desired modulating signal m(t) (as cos 0 =1)
- The output of Q channel is zero (as sin 0 = 0) due to quadrature null effect.
- If the local oscillator phase drifts (or) changes slightly, [θ is a small non-zero quantity].
- I channel output is almost unchanged
- Q channel output now is not a zero, (some signal will appear at its output) proportional to sin θ.
- The local oscillator is a voltage controlled oscillator, its frequency can be adjusted by an error control / dc signal.

****

1.4 Hilbert Transform

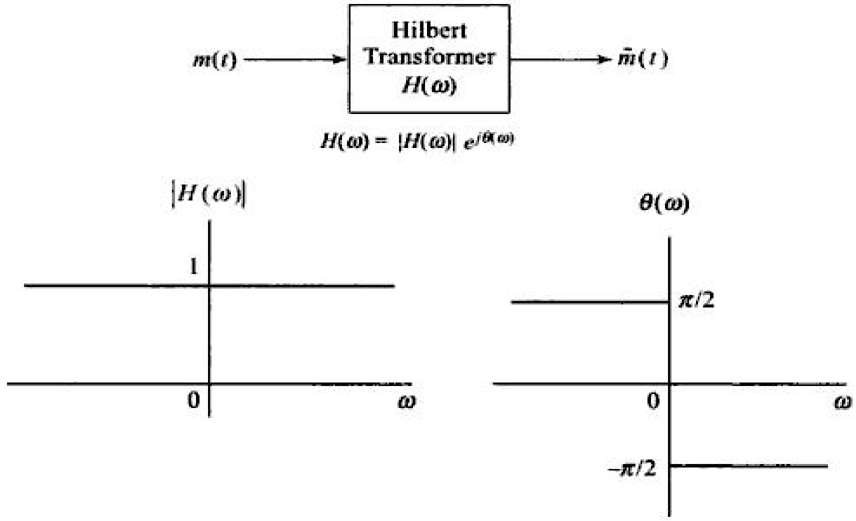
Hilbert transform is a system that produces -90^0 phase shift for all positive frequencies and 90^0 phase shift for all negative frequencies.
The amplitude of all frequency components of the input signal is unaffected.

12. Explain the Hilbert transform with an example **April 2018** **May 2015/ May 2017**

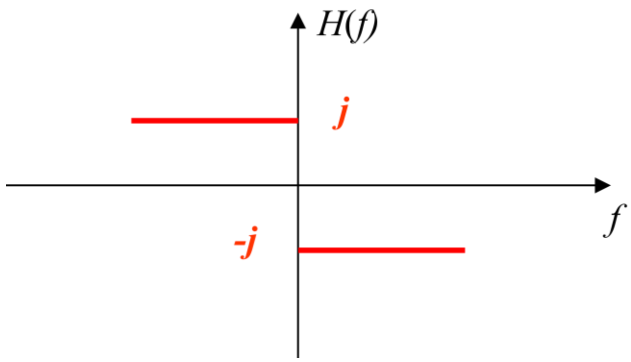
Hilbert transform is a system that produces a phase shift of -90^{0}for all positive frequencies s and a phase shift of -90^{0}for all negative frequencies.
The **amplitude** of all frequency components of the input signal is **unaffected**.

- Hilbert transform **does not involve a domain change**
  i.e., the Hilbert transform of a signal x(t) is another signal denoted by $\hat{x}(t)$ in the same domain (time domain)
- Hilbert transform of a signal x(t) is a signal $\hat{x}(t)$ whose frequency components lag the frequency components of x(t) by 90° .
- $\hat{x}(t)$ has exactly the same frequency components present in x(t) with the same amplitude–except there is a 90° phase delay

The only change that the Hilbert transform performs on a signal is changing its phase



Magnitude and phase characteristics of a Hilbert transformer.



- The amplitude of the frequency components do not change by performing the Hilbert-transform
- Hilbert transform changes cosines into sine's.
- The Hilbert transform $\hat{x}(t)$ is orthogonal to x(t)
- Since the Hilbert transform introduces a 90° phase shift, carrying it out twice causes a 180° phase shift, which can cause a sign reversal of the original signal

Properties of Hilbert transform:

- A signal x(t) and its Hilbert transform $\hat{x}(t)$ have same energy.
- The Hilbert transform of an even signal is odd and odd signal is even.
- Applying Hilbert transform to a signal twice causes sign reversal of the signal.
  If $\hat{x}(t)$ is the Hilbert transform of x(t) then Hilbert transform of $\hat{x}(t)$ is –x(t).
- A signal x(t) and its Hilbert transform $\hat{x}(t)$ are orthogonal.

The phase is -90° for the positive frequency and +90° for the negative frequency.

There are two ways of converting cosine wave into sine wave.

Hilbert transform in frequency domain

Hilbert transform in time domain

Phasor rotation to create a Sine wave out of cosine wave

Transformation process shifts all negative frequencies of signal to +90° phase shift and all positive Frequencies of signal to – 90° phase shift.

If cosine wave is applied to transformer, we get sine wave.

$\text{Cos}\omega t \rightarrow \sin\omega t$

If sine wave is applied to transformer then we get negative cosine wave from which negative sine wave can be obtained and finally it produces cosine wave.

$\text{Cos}\omega t \rightarrow \sin\omega t \rightarrow -\text{Cos}\omega t \rightarrow -\sin\omega t \rightarrow \text{Cos}\omega t$

From this, we can say Hilbert transformer is also called as quadrature filter.

****

1.5 Pre-envelope and complex envelope

The concept of pre envelope is useful in deriving the general expression of SSB SC

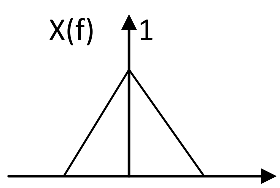
The pre-envelope of a real signal x(t) is the complex function.

$$x_p(t) = x(t) + j\hat{x}(t)$$

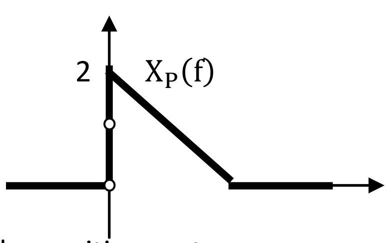
The pre-envelope is useful in treating hand pass signals and systems.

Analytical signal of the positive part of x(t)

$$X_P(f) = x(f) + \text{sgn}(f)X(f) = \begin{cases} 2X(f), & f > 0 \\ X(0) & f = 0 \\ 0, & f < 0 \end{cases}$$



Hilbert transform $\hat{x}(f) = -j\text{Sgn}(f)X(f)$



Here $X_P(f)$ doubles the positive part

The complex envelope of a band pass signal x(t) is

$\hat{x}(t) = x_+(t)e^{-j2\pi fct}$

If the pre-envelope $X_P(t)$ is band pass, it can be studied easily if we shift it to low frequency. This low pass version of $X_P(t)$ is called complex envelope.

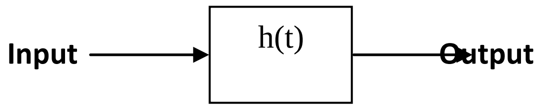
Signal Analytics and the Complex envelope

In signal processing the relationship between real and imaginary parts of a complex signal is described by Hilbert transformer.

The transform not only relates I and Q signal components but creates a class of analytical signals necessary for simulation.

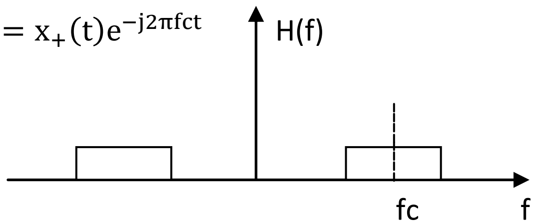
The analytical signal represents band pass signals as complex envelope.

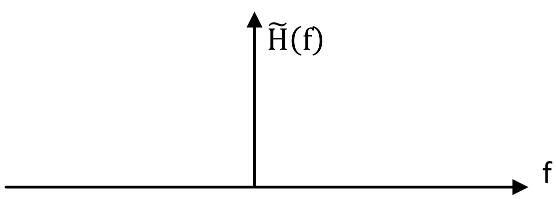
The complex envelope can be used to represent the Band Pass (BP) system by a Low pass (LP) system.



If h(t) or x(t) is a band pass filter centered around f_o we can define its complex envelope as

$$\hat{x}(t) = x_+(t)e^{-j2\pi fct}$$





Quadratic filter Hilbert transformer

$$H(jf) = F\left\{\frac{1}{\pi t}\right\}$$

$$H(jf) = |H(f)|e^{j\phi(f)}$$

$$H(jf) = -jSgn\, f$$

$$X_P(f) = H(jf) = \begin{cases} -j & f > 0 \\ 0 & f = 0 \\ j & f < 0 \end{cases}$$

$$\Phi(f) = \arg H(jf) = \frac{-\pi}{2}\, Sgn\, f$$

Phase Splitter Hilbert transformers

Analog Hilbert transformers are mostly implemented in the form of a phase splitter consisting of two parallel all-pass filter with a common input and separated output ports, each having the following transfer function respectively.

$$Y_1(jf) = e^{j\phi 1(f)}$$

$$Y_2(jf) = e^{j\phi 2(f)}$$

with $\delta(f) = \phi_1(f) - \phi_2(f)$

$$\delta(f) = -\frac{\pi}{2} \text{ for all } f > 0$$

All pass filter transformers

$$H(j\omega) = \frac{R - jX(\omega)}{R + jX(\omega)}$$

Where $\omega = 2\pi f$

$$\phi(\omega) = \arg\left\{\left(R - jX(\omega)\right)^2\right\}, \phi(\omega) = \tan^{-1}\left[\frac{-2RX(\omega)}{R^2 - X^2(\omega)}\right]$$

1.6 SSB – SC (Single sideband Suppressed Carrier System)

- In AM, both transmitting power and band width is wasted.
- In DSB – SC, power is saved by suppressing the carrier but the bandwidth remains same.
- Further saving of power is possible when one of the sideband is suppressed along with the carrier.
- In SSB-SC power is saved by suppressing one of the sideband because of its symmetry at carrier frequency.
- Here transmission B.W is also reduced to half when one of the SB is suppressed along the carrier. BW = f_m (half of DSB – SC)

Power calculation

- Power in SSB – SC – AM is

  $P_t'' = P_{SB} = [1/4]\, m_a^2 P_c$
- Power saving w.r.t AM with carrier

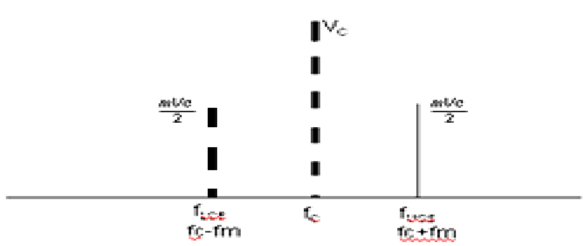
$$= [P_t - P_t''] / P_t \text{ where } P_t = \text{total power transmitted.}$$

$$= \frac{[1 + m_a^2/2]P_c - [m_a^2/4 P_c]}{[1 + m_a^2/2]P_c} = \frac{1 + [m_a^2/4]}{1 + [m_a^2/2]}$$

$$= \frac{[4 + m_a^2]/4}{[2 + m_a^2]/2} = \frac{[4 + m_a^2]}{2[2 + m_a^2]}$$

If $m_a = 1$, then % power saving = 5/6 = 83.33%

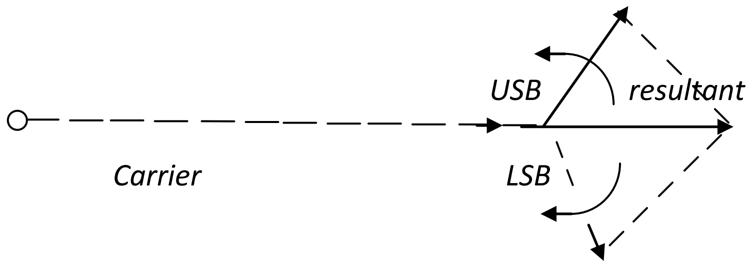
Frequency spectrum



Bandwidth of SSB SC

BW= fm (It is same as the frequency of modulating signal)

Phasor diagram of SSB SC



1.6.1 Generation of SSB – SC AM

SSB SC can be generated by

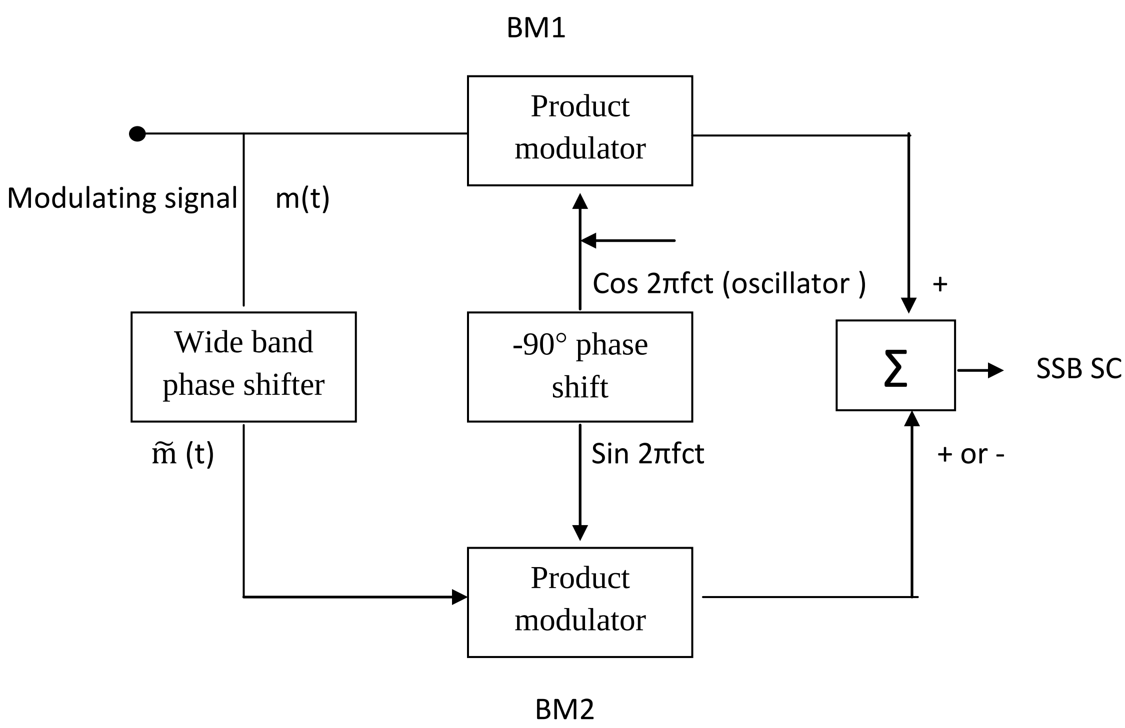
Phase shift method

Modified phase shift method (or) Weaver's method

13. Explain the generation of SSB SC signal using phase shift method. **May2009/Dec2008**
Apply the concept of Hilbert transform to generate SSB SC signal. **May 2017**
Discuss the generation of single side band modulated signal. **April 2018** **Dec 2017**

Phase shift method To overcome the drawbacks of filter method, we go for phase shift method. The filter method requires a sideband filter with a narrow transition band and it cannot be used at very low and very high frequencies

Block diagram



Operation:

- Two balanced modulators (BM1, BM2) and two phase shift networks are used in this method.
- BM1 receives the two signals directly.
- BM2 receives the two signals with a phase shift of 90^0.
- Carrier is suppressed by two balanced modulators.
- Unwanted side band is cancelled by the summer.
- Outputs of balanced modulators are added by the summer.
- Output of summer contains only USB. (Carrier is already suppressed by balanced modulator)

The output of BM1 is $m(t) \cos \omega_c t$

The output of BM2 is $\tilde{m}(t) \sin \omega_c t$

$\tilde{m}(t)$ - Hilbert transform of m(t)

$m(t) = \cos \omega_m t$

$\tilde{m}(t) = \cos(\omega_m - 90°)t = \sin\omega_m t$

The output of adder $= m(t) \cos \omega_c t + \tilde{m}(t) \sin \omega_c t$

$= \cos \omega_m t \cos \omega_c t + \cos(\omega_m - 90°) t \sin \omega_c t$

$= \cos \omega_m t \cos \omega_c t + \sin \omega_m t \sin \omega_c t$

$= \cos(\omega_c - \omega_m)t$ [Cos A Cos B + sin A sin B = Cos(A – B)]

$$\boxed{S_{SSB}\,sc(t) = \cos(\omega_c - \omega_m)t}$$

- When two signals are added at the summer LSB is generated and USB is suppressed
- When two signals are subtracted at the summer USB is generated and LSB is suppressed

Merits:

- Does not require any sharp cut off filter.
- It is possible to generate the desired side band in a single frequency translation step.

Demerits:

- Each balanced modulator need to be carefully balanced in order to suppress the carrier.
- Each modulator should have equal sensitivity to the base band signal.

****

14. How SSB can be generated using Weaver's method? Illustrate with a neat block diagram.

May2010/May 2012

Modified phase shift method / Weaver's method/ Third method

Advantages:

Generate SSB SC at any frequency and use low modulating frequencies.

No wide band phase shift network is required

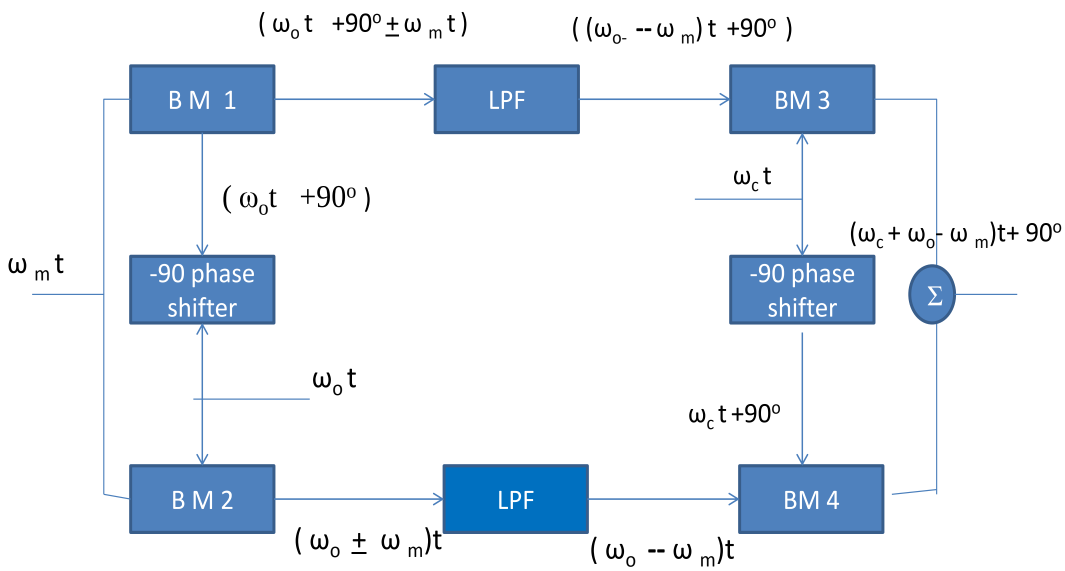
Disadvantage: Not commercially used because of its complexity.

Modulating signal $V_m(t) = V_m \sin\omega_m t$

AF carrier $V_0(t) = 2V_0 \sin\omega_0 t$

RF carrier $V_c(t) = 2V_1 \sin\omega_1 t$

Weavers method (Third method)



Output of BM1

$$\underset{A}{= 2V_0 \sin(\omega_0 t + 90^\circ)}\ \underset{B}{V_m \sin\omega_m t}$$

$$= V_m V_0 \cos(\omega_0 t + 90^\circ - \omega_m t) - \underbrace{\cos(\omega_0 t + 90^\circ + \omega_m t)}_{\downarrow\text{eliminated}}$$

Output of BM2

$$= 2V_0 \sin\omega_0 t V_m \sin\omega_m t$$

$$= V_m V_0 [\cos(\omega_0 t -- \omega_m t) - \underbrace{\cos(\omega_0 t + \omega_m t)}_{\downarrow\text{eliminated}}]$$

LPF in the BM1& BM2 eliminates the upper sidebands of the modulator.

Output of LPF1 is$V_m V_0 \cos(\omega_0 t + 90^\circ - \omega_m t)$

Output of LPF2 is $V_m V_0 [\cos(\omega_0 t -- \omega_m t)]$

Assume V m=V0 =1

Output of$BM_3 = 2\sin\omega_c t \cos(\omega_0 t + 90^\circ - \omega_m t)$ $\qquad$ $2 \sin A \cos B = [\sin(A+B) + \sin(A-B)]$

$= \sin[(\omega_c + \omega_0 - \omega_m) t + 90] + \sin[(\omega_c - \omega_0 + \omega_m)t - 90°]$ (1)

A B

Output of $BM_4 = 2\sin(\omega_c t + 90°) \cos(\omega_0 t - \omega_m) t$

$= \sin[(\omega_c + \omega_0 - \omega m)t + 90°] + \sin[(\omega_c - \omega_0 + \omega_m)t + 90°]$ (2)

The final output of summer circuit is (1) + (2) = V_0

(1) + (2) $\Rightarrow$ $V_0 = 2\sin[(\omega_c + \omega_0 - \omega_m) t + 90°]$

$V_0 = 2\cos[(\omega_c + \omega_0 - \omega_m) t]$

Note: other terms are cancelled because it is out of phase with each other

$\therefore$ Final output is the LSB of RF carrier $\omega_c - \omega_m$.

****

15. Explain the coherent detection of SSB SC signal.
Discuss the detection process of SSB SC using coherent detector. Analyze the drawback of the suggested methodology **May 2017**

Detection: Demodulation or detection is the process by which the original modulating signal is recovered from the modulated signal. It is the reverse process of modulation.

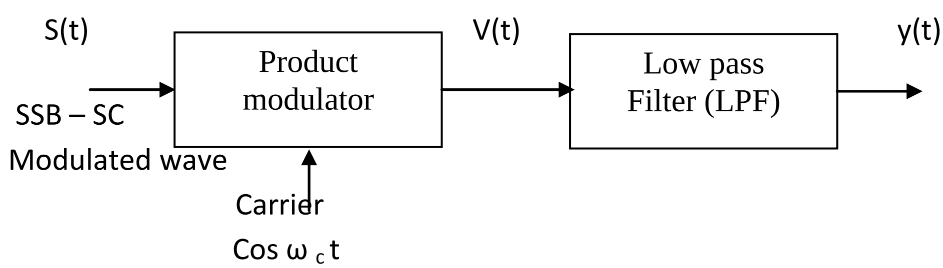
Coherent detection: The modulating m(t) can be recovered from DSB – SC by first multiplying with locally generated carrier.

The phase and frequency of locally generated carrier and carrier at the transmitter must be exactly coherent in phase and frequency otherwise the detected signal will be distorted.

Description:

- It consists of product modulator followed by an LPF.
- The product modulator multiples the SSB SC modulated signal and the locally generated carrier.
- The product modulator output is passed through LPF to recover the modulating signal

Block diagram:



The output of the multiplier (or) product modulator is

$$V(t) = S(t) \cos\omega_c t$$

$$= [m(t)\cos\omega_c t \pm \tilde{m}(t)\sin\omega_c t] \cos\omega_c t$$

Where m (t) - message signal;

$\tilde{m}(t)$ - Hilbert transform of m(t) (90° Phase shift)

$= [m(t) \cos \omega_c t \pm \tilde{m}(t) \sin \omega_c t] \cos \omega_c t$

$= m(t) [1+\cos 2\omega_c t]/2 \pm \tilde{m}(t) \sin \omega_c t \cos \omega_c t$

$= 1/2\, m(t) + 1/2\, m(t) \cos 2\omega_c t + 1/2\, \tilde{m}(t) \sin 2\omega_c t$

Low pass filter removes the terms of frequency ω_c, $2\omega_c$ and at the output we get

$$\boxed{\mathbf{Y(t) = 1/2\, m(t)}}$$

$\therefore$ Original modulating signal is recovered from modulated signal.

Advantages of SSB

- BW(f_m) is half of that required by DSBSC system
- Power of the suppressed carrier and sideband is saved.
- Due to narrow BW, effect of noise at the receiver circuit is reduced $\Rightarrow$ better quality of reception.

Disadvantages of SSB

- Transmission and reception of SSB is more complex.
- SSB receivers require precise tuning than AM receiver and frequency stability is required.

Applications:

- Point to point radio telephone communication
- SSB telegraph system
- Police wireless communication
- VHF &UHF communication

****

Explain in detail about VSB. [April 2018] [Apr - 2019]

1.7 Vestigial Sideband (VSB) Modulation

VSB overcomes the disadvantages of SSB – SC and serves as a com promise between SSB – SC & DSB – SC modulations.

- Used for TV transmission
- Vestige → means part (or) portion (or) trace.
- In VSB, the desired sideband is partially suppressed and small portion called trace (or) vestige of the undesired sideband is also transmitted to compensate for the suppression.

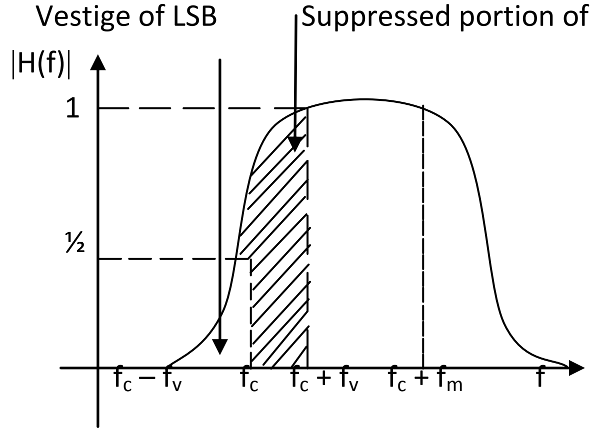
Advantages

- No need for sharp cut off filter and phase shifter.

Need for VSB:

- SSB modulation is suited for transmission of voice signal because of the energy gap that exists in the frequency spectrum of voice signal.
- When signal contain frequency component of extremely low frequency (telegraph & television signal) the USB &LSB meet at the carrier frequency and it is difficult to isolate one sideband. $\therefore$ VSB – SC is used in this case.

Magnitude Response of VSB filter (only positive – frequency portion)



USB $\rightarrow$ f_c to $f_c + f_m$

In this f_c to $f_c + f_v$ is suppressed

LSB$\rightarrow$ f_c to $f_c - f_m$ is LSB

In this, $f_c - f_v$ to f_c is transmitted as vestige

$|H(fc)| = ½$ frequency response $f_c - f_v \leq |H(f)| \leq f_c + f_v$ exhibits odd symmetry

$H(f - f_c) + H(f + f_c) = 1$

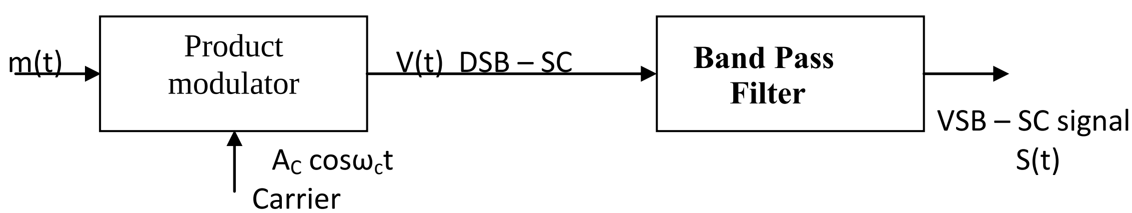
- The magnitude response at only w sum of two frequency comp in the range $f_c - f_v \leq f \leq f_c + f_v$ is equal to unity.
- Phase response is linear.
- Transmission Bandwidth of VSB modulation is $B_T = f_v + \omega$ Where $\omega \rightarrow$ message Bandwidth, $f_v \rightarrow$ width of the vestigial sideband

1.7.1 Generation and Detection of VSB

16. Discuss the generation and detection of VSB **May 2007**

Generation of VSB

Filter method (frequency discrimination method)



$$f(t) \overset{FT}{\leftrightarrow} F(\omega)$$

$$f(t)\cos\omega_c t \overset{FT}{\leftrightarrow} ½\,[F(\omega - \omega_c) + F(\omega + \omega_c)]$$

$$f(t)\cos 2\Pi f_c t \overset{FT}{\leftrightarrow} ½\,[F(f - f_c) + F(f + f_c)]$$

- Product modulator generates DSB – SC signal from the message & carrier signals.
- The output of product modulator the DSB – SC is passed through sideband shaping filter [VSB – filter]

V(t) (DSB – SC) $= A_c \cos \omega_c t\, m(t)$

$V(f) = FT[A_c \cos\omega_c t\, m(t)]$

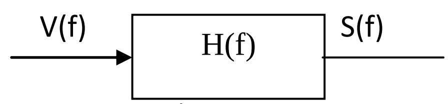
$V(f) = A_c/2\, [M(f – f c) + M(f+ fc)]$

By using the modulation property, the spectrum of VSB signal is

$S(f) = H(f)\, V(f)$

$$\boxed{S(f) = [A_c/2\ M(f – f c) + M(f + fc)]\ H(f)}$$

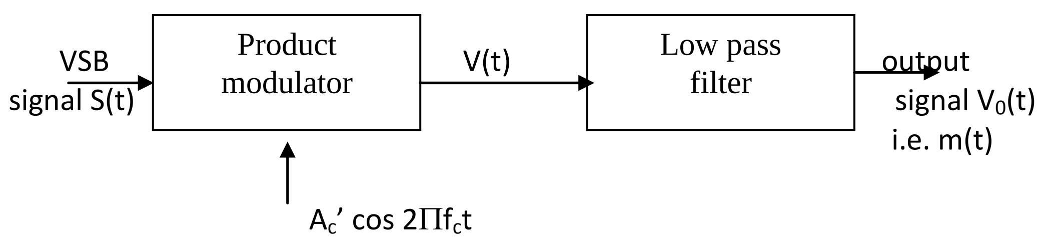
where H(f) $\rightarrow$ transfer function(or) frequency response of VSB filter



$H(f) = S(f) / V(f)$

S(f) - spectrum of VSB – SC signal

Demodulation of VSB



- Product modulator gives DSB – SC signal.
- The Fourier transform of S(t) (VSB signal) using modulation theorem is

$S(f) = [A_c/2\{M(f – f_c) + M(f+f_c)]\ H(f)$ (1)

- The output of product modulator is $V(t) = S(t)\, A_c^1 \cos(2\Pi f_c t) = A_c^1\, S(t) \cos(2\Pi f_c t)$
- Fourier transform of this using modulation theorem is

$V(f)= [A_c^{\ 1}/2]\ \{S(f – f_c) + S(f+ f_c)\}$ (2)

Sub. (1) in (2) $V(f) =[\ A_c A_c^1/4]\{M(f – 2f_c) + M(f)\}\ H(f - f_c) + A_c A_c^1/4\{M(f + M\ (f+2f_c)\}\ H(f + f_c)$

- First term $\rightarrow$ frequency spectrum of modulating signal

- Second term $\rightarrow$ frequency spectrum of VSB signal having carrier frequency $2f_c$ and it can be removed by LPF.

Frequency spectrum of signal $V_0(t)$ available at the output of VSB modulator will be

$V_o(f) = [A_c A_c^1 /4] \{H(f - f_c) M(f) + A_c A_c^1/4 M(f) H(f+f_c)\}$

$[A_c A_c^1 /4]M(f) \{H(f - f_c) + H(f+f_c) \}$

For distortion less reproduction of m(t), $V_0(t)$ is the scaled version of m(t) [scaled version $\rightarrow$ some constant multiplied by M(f)]

i.e. if $[H(f - f_c) + H(f + f_c)]$ is constant within the frequency then the output of $V_0(t)$ will be proportional to m(t). i.e. $V_0(t) = [A_cA_c^1 / 4]m(t))$

Advantages of VSB:

- Low frequencies, near f_c are transmitted without any attenuation.
- BW is reduced compared to DSB SC
  SSB-SC < BW < DSB -SC
- Filter need not have sharp cut off

Application

Mainly used for TV transmission since low frequency near f_c represent significant picture details and they are unaffected due to VSB.

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1.8 Comparison of AM systems

17. Compare the performance of amplitude modulation systems by using different attributes.

Dec2009/May 2012

Description	**AM with carrier DSB FC**	**DSB – SC – AM**	**SSB – SC – AM**	**VSB - AM**
Frequency spectrum	Carrier and 2 Sidebands	Suppressed Carrier and Only 2 Side bands	Suppressed Carrier and only one Side band	One of Side Band is partially suppressed and vestige of other SB is taken
Bandwidth	$2f_m$	$2f_m$	f_m	$f_m < BW < 2f_m$
Sidebands	Two sidebands	Two sidebands	one sideband	One of the sideband is partially suppressed and vestige of other is taken to compensate for the suppression
Power saving for sinusoidal	33.33%	66.66%	83.3%	75%
Power saving for non- sinusoidal	33.33%	50%	75%	75%
Generation methods	Easier to generate	Not difficult	More difficult to generate	Difficult but easier to generate than SSB – SC
Detection methods	Simple inexpensive	Difficult	More Difficult	Difficult
Signal to noise	$(S/N)_0 = 1/3(S/N)_f$	$(S/N)_0 = (S/N)_i$	$(S/N)_0 = (S/N)_i$	$(S/N)_0 = (S/N)_i$
Application	AM Broadcast applications	Short distance point to point communication (telephony)	Long range high frequency communication. (wireless mobile)	TV transmission and high speed data communication

1.9 Super heterodyne Receiver

A radio receiver is an electronic circuit that picks up a desired radio frequency (R.F) signal and recovers the base band signal from it.

Parameters	AM Radio	FM Radio
Frequency range	535KHz to 1065KHz	88 – 108 MHz
Intermediate frequency (IF)	455KHz	10.6 MHz
BW for IF	10KHz	200KHz.

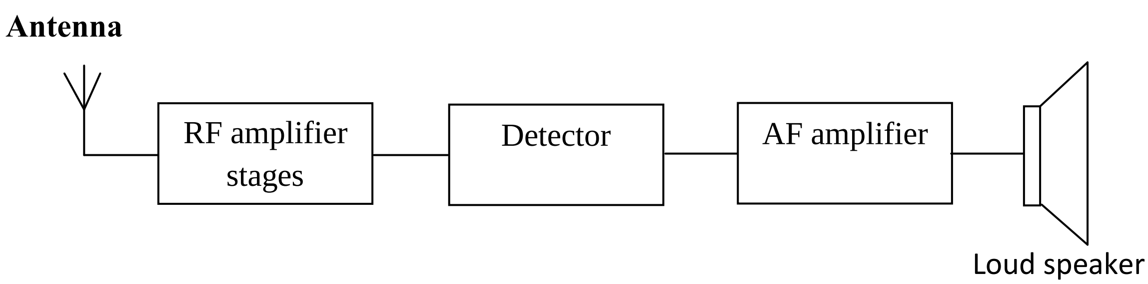
Two types of Receivers:

TRF receiver

Super heterodyne receiver

Tuned Radio Frequency (TRF) Receiver:

- Composed of RF amplifiers and detectors
- It is not often used
- No frequency conversion
- Difficult to design tunable RF stages
- Difficult to obtain high gain RF amplifier



18. Using super heterodyne principle, draw the block diagram of AM radio receiver and briefly explain it. Dec2006/May 2007/Dec2009

Explain with block diagram the super heterodyne receiver. May 2015/May 2016

Elucidate the working principle of super heterodyne receiver with the neat block diagram.

Draw signal at the output of each block. Dec2017/May 2017

Comment the choice of IF selection and image frequency elimination. Nov 2018 May 2017

[Apr - 2019] April 2018

Super heterodyne Receiver

A radio receiver is an electronic circuit that picks up a desired radio frequency (R.F) signal and recovers the base band signal from it.

A Receiver performs three important functions

Tuning (or) selection

Filtering

Amplification

Tuning (or) selection

To select the desired signal eg.TV (or) radio signal.

Filtering

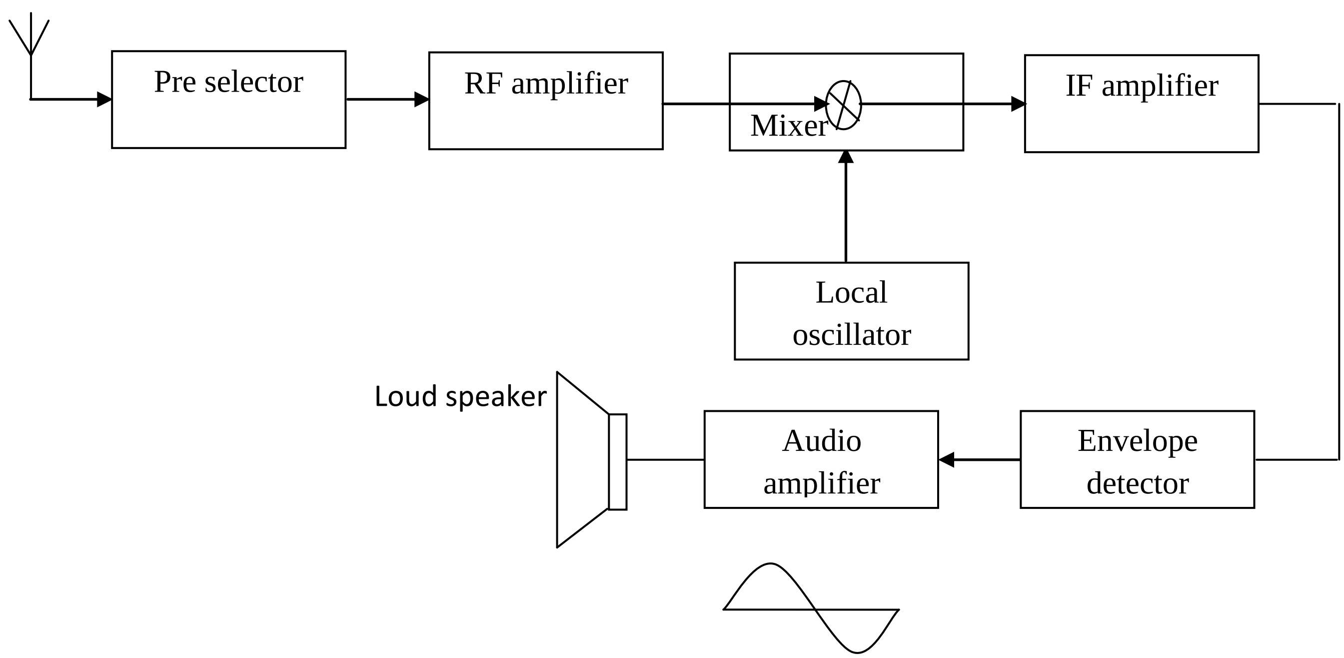
To separate the desired signal from the other modulated signal intercepted by the antenna

Amplification

To compensate the loss of signal power during the transmission from transmitter to receiver

AM super heterodyne Receiver

Block diagram:



Heterodyne Principle:

- Mixing of two frequencies (incoming and local oscillator) and producing a new frequency
- It is greater or lower than the incoming frequency called intermediate frequency (455KHz) mixer is called as first detector.
- Down convert RF signal to same fixed frequency (IF) signal and then amplification & detection is done.

Ganged tuning $\rightarrow$ constant frequency is maintained between local oscillator and RF section normally through capacitance tuning in which all the capacitors are ganged together and operated by a control knob.

RF Section:

- It contains RF amplifier and Pre-selector

- Pre-selector is a band pass filter tuned to desired frequency.
- Initial amplification is done by the RF amplifier. i.e. weak signal coming out of the antenna is amplified.

Intermediate frequency

The incoming carrier frequency is converted to a **fixed intermediate predetermined frequency** called **intermediate frequency**.

$f_i = f_{LO} - f_S$

where $f_S \rightarrow$ incoming R.F. frequency

$f_{LO} \rightarrow$ local oscillator frequency

$f_i \rightarrow$ intermediate frequency

Intermediate Frequency is **neither at the incoming carrier frequency nor at the baseband frequency.** Hence it is called as IF.

IF Section:

- It consists of one or more stages of tuned amplifier.
- This section provides most of the amplification and selectivity in the receiver.

Detector or Demodulator:

- The output of the IF section is applied to a demodulator to recover the baseband or message signal.
- If coherent detection is used, then a coherent detector is provided in the receiver.
- In AM, the information is in the amplitude variation of the carrier. So envelope detector is used.
- In FM, FSD is preceded by amplitude limiter.
- It is used to limit the amplitude variations at the FM input so as to remove noise or interference.

Audio Power Amplifier

- The output of the envelope detector is fed to the audio power amplifier.
- The output of envelope detector is few milli watts and therefore it is insufficient to drive the loud speaker. Therefore it is amplified to the required level to drive the speaker.

Loud speaker:

- The output of audio power amplifier fed to the loud speaker.
- It converts electrical signal to sound signal.

Automatic Gain Control (AGC):

- AGC is used to maintain the output voltage to a threshold level.
- There may be increase or decrease of voltage level at the input of the receiver.
- Therefore a part of the output of detector is fed to the mixer, RF amplifier and IF amplifier to maintain voltage in control level.

- AGC provides DC bias to all the sections and act as a negative Feedback system and control the overall gain.

Image Frequency

- Mixer will produce IF when the input frequency is greater or less than the local oscillator frequency by an amount equal to IF.

i.e. $f_S = f_{LO} \pm f_{if}$

$f_S = f_{LO} - f_{if} \Rightarrow f_{LO} = f_S + f_{if}$

$f_{Si} = f_{LO} + f_{if}$

$f_{Si} = f_S + f_{IF} + f_{IF}$

$$\boxed{f_{Si} = f_S + 2f_{IF}}$$

Image Frequency:

- It is a frequency other than the required selected RF frequency entering into the receiver and mix with local oscillator to produce the same IF frequency.
- When this Image frequency enters into the IF amplifier, it cannot be separated and causes interference in the receiver. $f_{Si} = f_S + 2f_{IF}$

Image Frequency Rejection Ratio (IFRR):

It is a numerical measure of ability of pre selector (BPF) to reject the image frequency. It is the ratio of gain at signal frequency to the gain at image frequency.

$$IFRR = \frac{\text{Gain at signal frequency}}{\text{Gain at image frequency}}$$

$IFRR = \alpha = \sqrt{1 + Q^2P^2}$

where $P = [f_{si}/f_s] - [f_s/f_{si}]$

where Q = quality factor of tuned circuit

fsi = image signal frequency

fs = input RF frequency

Advantages:

- Image frequency rejection
- The incoming carrier frequency is converted into predetermined higher frequency (IF).
  ∴ All the stages after the mixer have to operate in this fixed IF frequency.
- Hence the circuit becomes simple and performance also improved.

****

Characteristics of a Receiver

(i) Selectivity (ii) Sensitivity (iii) Fidelity

The performance of the radio receiver is measured by the following characteristics

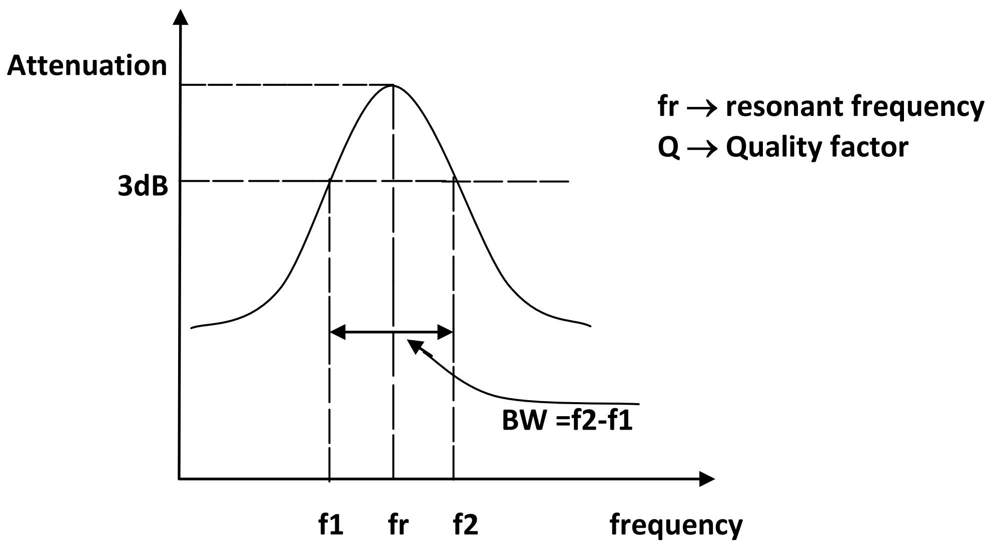
Selectivity

- It is the ability of the receiver to select the desired signal frequency and reject the unwanted signal.
- The selectivity depends upon on the tuned LC circuits used in RF and IF stages.
- The sharper resonance curves have high selectivity.
- The sharpness of Resonance curve depends on two factors.
- For better selectivity, BW should be narrow which in turn require high 'Q' of the coil.
- Selectivity is provided by both RF and IF amplifier.

BW = f r/Q

f r- resonant frequency

Q- Quality factor of the coil



Sensitivity

- It is the ability of the receiver to receive the weakest possible signal.
- It is expressed in micro volts or decibels.
- The sensitivity of the receiver is decided by the gain of the amplifier stage.
- The high gain provides better sensitivity. (If amplifier gain increases sensitivity increases)

Fidelity

- It is the ability of the receiver to reproduce all the frequency components present in the baseband signal. i.e. exact replica of the original information.
- If any of the components is altered, fidelity suffers and the reproduction of signal is distorted.
- This is mainly decided by the BW of the Audio Power amplifier which amplifies the baseband signal.

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UNIT I
Amplitude Modulation

1. ***Define modulation.***
   Modulation is defined as the process of changing the characteristics (amplitude, phase ,frequency) of high frequency carrier signal according to the instantaneous value of the modulating signal.

2. ***What is the need for modulation? (***
   What are the advantages of modulation? **May 2013/ Nov 2016**
   What are the advantages of converting the low frequency signal into high frequency s

The advantages of modulation are

- *Reduction of antenna heigh.t*
- *Ease of transmission.*
- *Multiplexing.*
- *Reduced noise.*
- *Narrow bandwidth.*
-

3. ***Define Amplitude modulation.*** ***May 2007***
   Amplitude modulation is defined as the process of changing the amplitude of the carrier signal according to the modulating signal .

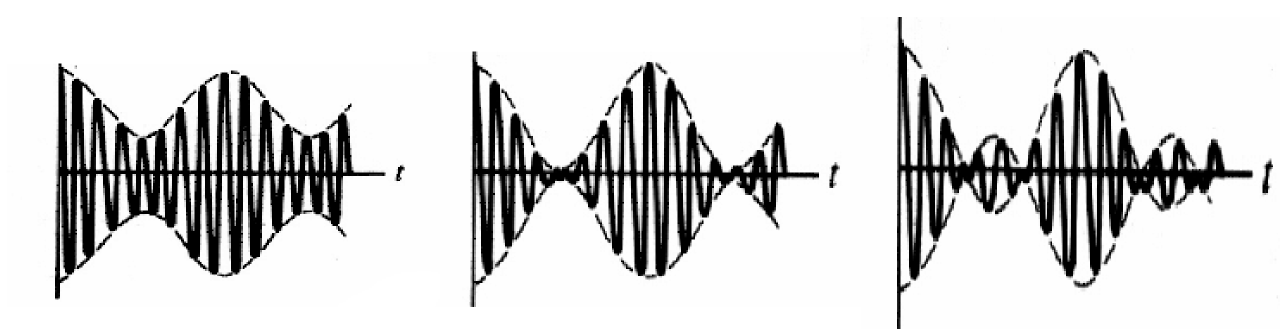
4. ***Define modulation index of AM. (or) depth of modulation (or) percentage modulation***
   May 2006/ May 2007
   Modulation index is defined as the ratio of amplitude of message signal to that of carrier amplitude.

$$m = \frac{Vm}{Vc} \text{ or } m = \frac{V\max - V\min}{V\max + V\min}$$

Where Vm- amplitude of modulating signal
Vc- amplitude of modulating signal.

5. ***What are the degrees of modulation?***
   The degrees of modulation are
   - *Under modulation. m<1*
   - *Critical modulation m=1*
   - *Over modulation m>1*

m<1 *m=1* *m>1*



6. ***Define efficiency of AM.*** ***May 2006***

Efficiency of AM is defined as the ratio of power in sidebands to the total power

$$\%efficiency = \frac{powerinsidebands}{Totalpower} X100$$

$= \dfrac{m^2}{2+m^2}$ *x 100* *m = 1 or 100%* *m-modulation index*

= 33.33 %

7. ***Give the power relation and current relation in AM***

The Power relation in AM is

$$Pt = Pc\left(1+\frac{m^2}{2}\right)$$

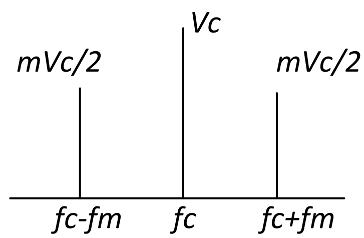
where Pt – total modulated power, Pc-un modulated carrier power and m- modulation index

The Current relation in AM is

$$It = Ic\left(1+\frac{m^2}{2}\right)^{\frac{1}{2}}$$

where It – total modulated current, Ic-un modulated carrier current and m-modulation index.

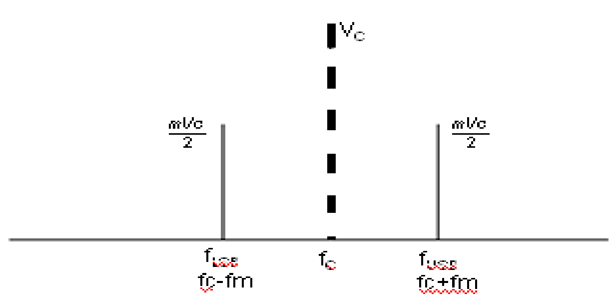
8. ***Draw the frequency spectrum of AM.***



fc- carrier frequency
fc--fm -LSB frequency
fc-+fm -USB frequency

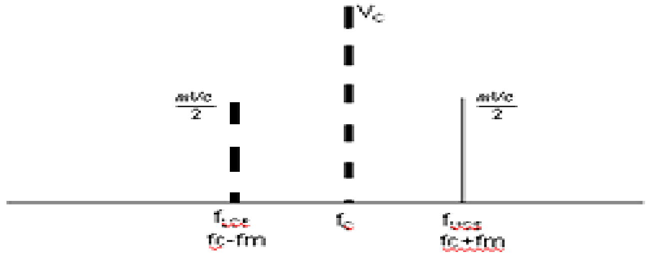
BW=2fm

9. ***Draw the frequency spectrum of DSB SC.***



BW=2fm

10. Draw the frequency spectrum of SSB SC.



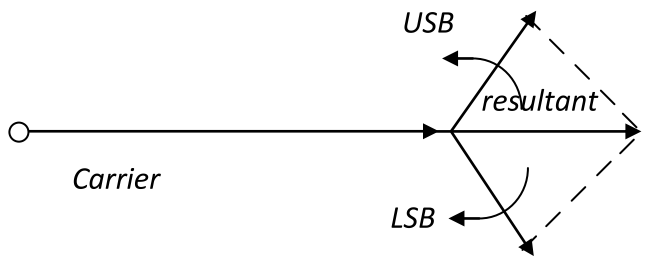
BW=fm

fc- suppressed carrier frequency

fc--fm - suppressed LSB frequency *fc-+fm -USB frequency*

11. Draw the phasor representation of AM,DSB SC,SSB SC

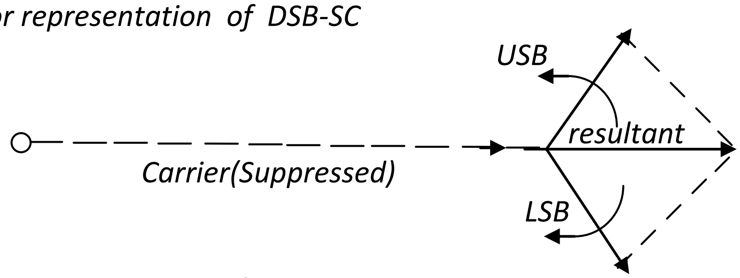
Phasor representation of AM



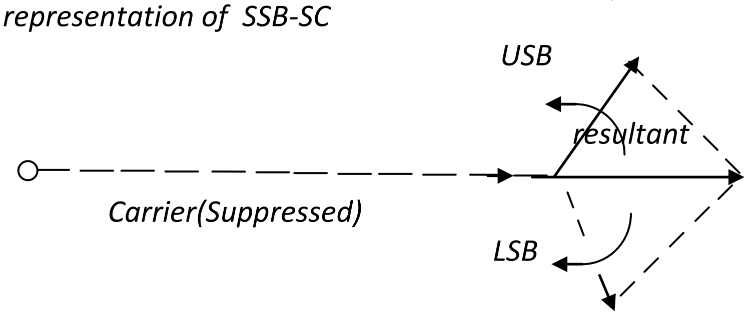
Carrier is the reference phasor and the two side band phasors rotating in the opposite direction.

The resultant is sum of carrier and two side band phasors

Phasor representation of DSB-SC



Phasor representation of SSB-SC



12. What are the advantages of Suppressed Carrier (SC) systems?

The advantages of Suppressed Carrier (SC) systems are

In DSB SC BW remains same as AMFC but power is saved.

In SSB SC BW is reduced to half when compared to AMFC and power is also saved.

13. What is the bandwidth of AM signal?

The bandwidth of AM signal is twice the maximum frequency of modulating signal.

BW=2fm

fm- modulating signal frequency

14. What is the bandwidth of DSB SC , SSB SC AM signal?

The BW of DSB SC=2fm

The BW of SSB SC= fm

fm- modulating signal frequency

15. Compare AM with DSB-SC and SSB-SC. **April 2018** ***May 2013***

AM	DSB-SC	SSB-SC
Bandwidth=2fm	*Bandwidth=2fm*	*Bandwidth=fm*
Contains USB, LSB, carrier	*Contains USB,LSB*	*Contains LSB or USB*
More power is required for Transmission	*Power required is less than that of AM.*	*Power required is less than AM &DSB-SC*

16. Define demodulation or detection.

Demodulation or detection is defined as the process of recovering the modulating signal from the modulated signal. It is the reverse process of modulation.

17. What is diagonal clipping?

In the envelope detector, if RC time constant is kept too high, the discharge becomes approximately horizontal. In that case, negative peaks of the detected envelope may be completely or partially missed. The recovered base band signal is distorted at negative peaks. This type of distortion is known as diagonal clipping.

18. Define coherent (or) Synchronous detection.

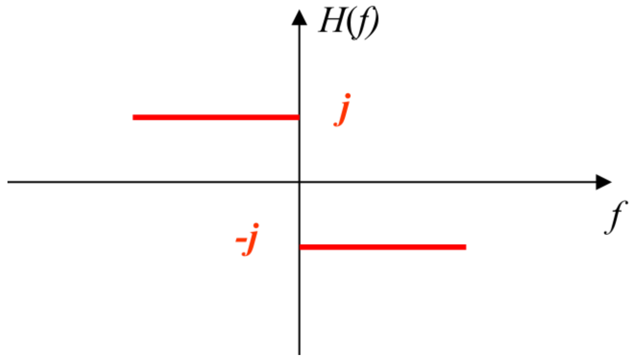
The phase and frequency of locally generated carrier must be exactly coherent with that of phase and frequency of transmitted carrier otherwise the detected signal will be distorted called coherent detection.

19. Comment on Hilbert Transform.

- *Hilbert Transform corresponds to a 90^0 phase shift.*
- *The only change that the Hilbert transform performs on a signal is changing its phase.*

- *The amplitude of the frequency components of the signal do not change by performing the Hilbert- transform.*
- *Hilbert transform changes cosines into sines, the Hilbert transform* $\hat{x}(t)$ *of a signal x(t) is orthogonal to x(t).*

$$H(f) = \Im[h(t)] \quad H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \end{cases}$$



20. What are the properties of Hilbert transform?

The properties of Hilbert transform ,

- A signal x(t) and its Hilbert transform $\hat{x}(t)$ have same energy.
- The Hilbert transform of an even signal is odd and odd signal is even.
- Applying Hilbert transform to a signal twice causes sign reversal of the signal. If $\hat{x}(t)$ is the Hilbert transform of x(t) then Hilbert transform of $\hat{x}(t)$ is –x(t).
- A signal x(t) and its Hilbert transform $\hat{x}(t)$ are orthogonal.

21. Determine the Hilbert transform of Cos ω t. ***Dec2017***

The Hilbert transform of cos ω t is sin ω t. Since it produces 90^0 *phase shift.*

22. What is pre envelope and complex envelope? ***May 2016***

The pre-envelope of a real signal x(t) is the complex function.

$x_p(t) = x(t) + j\hat{x}(t)$

The pre-envelope is useful in treating hand pass signals and systems.

The complex envelope can be used to represent the band pass (BP) system by a low pass (LP)

23. What are suppressed carrier (SC)systems?

In AM ,the transmitting power is wasted in transmitting the carrier which does not contain any information. The transmitting power is saved by suppressing the carrier called suppressed carrier systems.

24. What are the types of suppressed carrier systems?

The types of suppressed carrier systems are

- *Double side band Suppressed carrier system (DSB SC)*
- *Single side band Suppressed carrier system (SSB SC)*

25. What are the advantages of SSB SC?

The advantages of SSB SC are both transmitting power and bandwidth is saved.

26. Mention the applications of SSB.

The applications of SSB are

- *Point to point radio telephone communication.*
- *Police wireless communication.*
- *VHF &UHF communication.*

27. What are advantages and disadvantages of SSB? **[Apr - 2019]** ***May 2007***

Advantages of SSB are

BW(fm) is half of that required by DSB SC system

- *Transmitter power requirement in SSB is reduced.*
- *Due to narrow BW, effect of noise at the receiver circuit is reduced.*
- *This gives better quality of reception in SSB.*

Disadvantages of SSB

- *Transmission & reception of SSB is more complex.*
- *SSB receivers require precise tuning than AM receiver.*

28. What is VSB? Where it is used? ***Dec 2017***

Vestigial side band is a compromise between SSB and DSB. In VSB one of the sideband is partially suppressed and vestige(portion) of other side band is also transmitted.

VSB is mainly used in TV broadcasting for the video transmission .

29. What are the advantages of VSB-AM?

What are advantages and applications of VSB? ***May 2011***

Advantages of VSB:

- *Low frequencies, near fc are transmitted without any attenuation.*
- *BW is reduced compared to DSB SC.*
- *SSB-SC< BW < DSB – SC.*

Application : *Mainly used for TV transmission.*

30. For television signal transmission vestigial sideband modulation is selected. Justify your answer.

Suggest a modulation scheme for the broadcast of video transmission and justify

Nov 2009/Nov 2014/ Nov 2016

VSB is mainly used in TV broadcasting for the video transmission .

TV signals contains frequency component of extremely low frequency, the USB and LSB meets the carrier frequency and is difficult to isolate one of the side band since low frequency near fc represent significant picture details and they are unaffected due to VSB. Therefore VSB is used for TV transmission.

31. What are the parameters used to evaluate the ability of a radio receiver?

The parameters commonly used to evaluate the ability of a receiver to successfully demodulate a radio signal are

- *Selectivity*
- *Sensitivity*
- *Fidelity*

32. Define sensitivity. ***May 2014***

*Sensitivity of a receiver is the **ability to receive or detect weak signals** and amplify them.*

33. Define selectivity.

***S**electivity of a receiver is the **ability to select the desired signal** among the various signals and reject the unwanted signal.*

34. Define Fidelity.

Fidelity of the receiver is the ability to reproduce all the frequency components of modulating signal at the output of the receiver.

35. Define super heterodyne principle.

Define heterodyning. ***May 2015***

Heterodyne principle is defined as the process of mixing two signals having different frequencies to produce a new frequency. This process uses a locally generated carrier wave, which determines the change of frequency.

36. What is called image frequency? ***Dec 2014***

Image frequency is defined as the signal frequency plus twice the intermediate frequency. This has the effect of two stations being received simultaneously and hence it is undesirable.

fsi = fs + 2 fi

fsi - image frequency

It can be eliminated by providing adequate image signal selectivity (pre selector)between antenna and mixer input.

37. What is intermediate frequency?

In super heterodyne receiver, all the incoming frequencies are converted to a pre determined fixed frequency called Intermediate frequency.

(IF) is defined as the difference between the signal frequency and the local oscillator frequency.

IF = fs – fo when fs > fo (or) IF = fo –fs when fo > fs

38. Define image frequency rejection ratio.

Image frequency rejection ratio defined as "the ratio of the gain at the signal frequency to the gain at the image frequency".

$$IFRR\ (\alpha) = \frac{\text{Gain at the signal frequency}}{\text{Gain at the image frequency}}$$

$$IFRR = \alpha = \sqrt{1 + Q^2P^2}$$

where P = fsi/fs – fs/fsi

where Q = quality factor of tuned circuit

fsi = image signal frequency

fs = input RF frequency

39. What is the value of standard intermediate frequency for AM radio and FM radio?

The most common intermediate frequency used in AM radio receivers is 455 kHz.
The most common intermediate frequency used in FM radio receiver is 10 M Hz.

40. What are the advantages of super heterodyne receiver over TRF? [Apr - 2019]
What are the characteristics of super heterodyne receiver? **May 2010**

The advantages of super heterodyne receiver over TRF are

- *High selectivity and sensitivity.*
- *Uniform bandwidth because of fixed intermediate frequency.*
- *It eliminates image frequency.*
- *Improved stability.*

41. What theorem is used to calculate the average power of a periodic signal $g_p(t)$?
State the theorem. **May 2016**

Parseval's Theorem is used to calculate the average power of a periodic signal.
Parseval 's theorem states that the total average power in a periodic signal equals the sum of average of power in all of its harmonic components.

$$1/_T \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

42. Do the modulation techniques decide the antenna height? **May 2017**

Yes , the modulation decides the antenna height .The antenna height is inversely proportional to frequency. So by modulation the antenna height is reduced.

SOLVED PROBLEMS

1. ***A transmitter supplies 8 Kw to the antenna when modulated. Determine the total power radiated when modulated to 30%.*** [Apr - 2019]

   Given data:

   % modulation m=0.3, carrier power, Pc=8 kw

   Formula: *Pt=Pc(1+m^2/2)*

   Pt=8.36 kw

2. ***A 500 W carrier is modulated to a depth of 60 percent. Calculate the total power in modulated wave.*** ***Nov 2008***

   Given data:

   Carrier power pc=500W

   Depth of modulation ma =60%

   Find total power pt.

   Solution :

   P t=pc (1+ma^2/2)

   P t=512.5W

3. ***The antenna current of an AM transmitter is 8A when only carrier is sent. It increases to 8.93A when the carrier is modulated by a single sine wave. Find the percentage modulation.***

    Given data:

    Un modulated carrier current ,Ic =8A , modulated current It=8.93A

    Formula: $It=Ic\,(1+m^2/2)^{½}$

    m=0.701

    %m=70%

4. ***A transmitter radiates 9 kW without modulation and 10.125 KW after modulation. Determine depth of modulation.*** **Nov 2007**

    Given data :

    p_c*-un modulated carrier power=9kw*

    p_t*- transmitted power.=10.12kw*

    Solution:

    $Pt=Pc\,(1+m^2/2)$

    $m = \sqrt{2}(p_t/p_c - 1)$

    m=0.5

5. ***How many AM broadcast stations can be accommodated in a 100KHz bandwidth if the highest frequency modulating a carrier is 5 KHz?*** **April 2010 / Nov 2011**

    Given data:

    Modulating frequency, fm =5 KHz

    Total bandwidth=100 kHz

    Solution:

    B w =2fm

    = 2x 5 kHz

    = 10 kHz

    Total bandwidth is 100 KHz

    The number of AM broadcast stations

    .’. 100/10=10

6. ***An amplitude modulation transmitter radiates 1000w of un modulated power. If the carrier is modulated simultaneously by two tones of 40% and 60% respectively. Calculate the total power radiated.*** **Nov 2012**

    Given data :

    Un modulated carrier power Pc=1000W

    m1=40%

    m2=60%

    Find transmitted power Pt.

Solution :

$m_t=\sqrt{m1^2+m2^2}$

m1=40/100=0.4

m2=60/100=0.6

m t =0.721.

$Pt=pc(1+mt^2/2)$

P t=1259.9W

7. ***Compute the bandwidth of the amplitude modulated signal given by S(t)=23[1+0.8cos(310t)]cos(230000πt). May2012/April2009***

   Given data:

   Carrier amplitude Ac =23

   Modulation index ma=0.8

   ωm = 310

   ωc=230000

   Solution :

   2πfm=310

   fm=310π/2π=155Hz

   Bandwidth=2fm=98.67 Hz.

8. ***Obtain the BW of AM signal S(t) = 23 cos(23000Πt) (1+0.8cos310Πt)***

Given :

AM signal S(t) = 23 cos(23000Πt) (1+0.8cos310Πt)

Solution:

General expression for AM $A_C \cos \omega_c t[1+m_a \cos \omega_m t]$

$\omega_c = 230000\Pi$ $\quad A_C = 23$ $\quad ma = 0.8$ $\quad \omega_m = 310\Pi$

$2\Pi f_c = 23{,}0000\Pi$

$f_c = 23{,}000/2 = 11.5KHz$ $\qquad 2\Pi f_m = 310\Pi$

$BW = 2f_m = 2 \times 155$ $\qquad f_m = 155Hz$

BW = 310Hz

9. A carrier of 6kV is amplitude modulated by an audio signal of 3 kV. Find the modulation index. Nov - 2018

Vm = 3kV
Vc= 6kV
Modulation index m= Vm/Vc = 3k/6k
m=0.5

43. What are advantages of converting low frequencing signal to high frequency signal? Nov 2018

In multiplexing, low frequency signals are converted to high frequency signals and combined with other high frequency signals so that you can pack multiple signals into a single signal, although this combined siganl will have a greater bandwidth.

44. What are the advantages of coherent detection? April 2018

Coherent detection therefore offers several key advantages compared to direct detection:

(1) Greatly improved receiver sensitivity.

(2) Can extract amplitude, frequency, and phase information from an optical carrier, and consequently can achieve much higher capacity in the same bandwidth.

UNIT – II ANGLE MODULATION

Phase and frequency modulation, Narrow Band and Wide band FM – Modulation index, Spectra, Power relations and Transmission Bandwidth - FM modulation –Direct and Indirect methods, FM Demodulation – FM to AM conversion, FM Discriminator - PLL as FM Demodulator.

Limitations of AM

- More affected by noise due to amplitude variations produced by lightning, spark plug ignition system.
- Wastage of transmitted power.
- Less efficiency.

To overcome the limitations in AM, Frequency modulation is used.

By increasing the Band width, the noise is reduced in FM.

2.1 Angle Modulation

Definition

It is the process by which the **total phase angle of the carrier is changed according to the modulating signal** keeping amplitude constant

$$\omega_i = \frac{d\theta_i}{dt}$$

The angle modulated wave is mathematically expressed as $s(t) = A_c [\cos(\omega_c t + \theta)]$

$= A_c [\cos(\theta(t))]$

Classification of Angle modulation

Angle Modulation

Frequency Modulation **Phase Modulation**

Frequency Modulation -FM

It is the process by which the frequency of the carrier is changed according to the modulating signal [Amplitude of carrier remains constant]

Phase Modulation -PM

It is the process by which the phase of the carrier is varied according to the modulating signal [Amplitude of carrier remains constant]

2.2 Phase modulation

It is the process by which the phase of the carrier is varied according to the modulating signal [Amplitude of carrier remains constant

$\theta(t) = 2\pi\, f_c\, t + K_p\, m(t)$

$2\pi\, f_c\, t$ - angle of the un modulated carrier

K_p - phase sensitivity constant represented in radians/ volt

PM signal $S(t) = A_c \cos \theta(t)$

$= A_c \cos [2\pi\, f_c\, t + K_p\, m(t)]$

Phase deviation

The phase angle of the carrier varies from its un modulated signal during the modulation process.

$\Delta\omega = K_p V_m$ or $K_p A_m$

Modulation index of PM

Ratio of maximum phase deviation to the phase of the modulating signal.

2.3 Frequency modulation

It is the process by which the frequency of the carrier is changed according to the modulating signal [Amplitude of carrier remains constant]

$f(t) = f_c + K_f\, m(t)$

f_c- frequency of un modulated carrier

K_f- frequency sensitivity in Hertz/ volt

$S(t) = 2\pi\, f_c\, t + 2\pi K_f \int m(t)\, dt$

1. **Derive the expression for FM signal.** **Nov 2018** **May2013/May2016**

   How the phase and frequency modulation are related? Explain. **May2010**

   When frequency of the carrier varies, phase of the carrier also varies and vice versa

   Un modulated carrier is $A_c \cos(\omega_c t + \theta) = A_c \cos \varphi$.

   where φ is the total phase angle of the carrier

   ω_c- carrier frequency

   Instantaneous frequency $\omega_i = \frac{d\theta_i}{dt}$ (1)

   $$2\pi f_i = \omega_i = \frac{d\theta_i}{dt}$$

   $$\therefore f_i = 1/2\pi \frac{d\theta_i}{dt}$$

   from (1) $\theta_i = \int \omega_i\, dt$ (2)

   Instantaneous value of phase angle is given by

   $\theta_i = \omega_c t + K_P\, m(t)$ (3)

Where

ω_c – un modulated carrier

K_P - phase deviation constant (or) phase sensitivity expressed in radians / volt

m(t) –message or modulating signal.

The phase deviation $\Delta p = K_p A_m$

The phase modulated signal is given by

$S(t) = A_c \cos \theta$ (4)

Sub (3) in (4)

$$\boxed{S(t) = A_c \cos[\omega_c t + K_P m(t)]}$$

Instantaneous frequency is given by

$\omega_i = \omega_c + K_f m(t)$ (5)

where

ωc-un modulated carrier frequency

k_f – frequency deviation constant or frequency sensitivity expressed in Hz/Volt

Integrating equation (5) to obtain phase of the FM wave

$\int \omega_I dt = \int \omega_c t + K_f \int m(t)$

$\int \omega_I dt = \theta_i = \omega_c t + K_f \int m(t)$

$$\boxed{\therefore \theta_i = \int \omega_I dt}$$

Frequency modulated signal is

$S(t) = A_c \cos\theta i$

$$\boxed{S(t) = A_c \cos[\omega_c t + K_f \int m(t)]}$$

Where $m(t) = A_m \cos\omega_m t$

$\therefore S(t) = A_c \cos[\omega_c t + K_f A_m \sin\omega_m t / \omega_m]$

$K_f A_m = \Delta\omega \rightarrow$ frequency deviation

$\therefore \Rightarrow S(t) = A_c \cos[\omega_c t + \Delta\omega/\omega_m \sin\omega_m t]$

$$\boxed{S(t) = A_c \cos[\omega_c t + \Delta f/f_m \sin\omega_m t]}$$

$\Delta f / f_m = m_f$ (or) β (or) $\mu \rightarrow$ Modulation index

Note: In PM , phase angle varies linearly with m(t).

In FM, phase angle varies linearly with integral of m(t)

Modulation Index for FM:

It is defined as the ratio of frequency deviation to the modulating frequency

m_f (or) $\beta = \Delta f / f_m$

Δf -frequency deviation

f_m -Modulating signal frequency

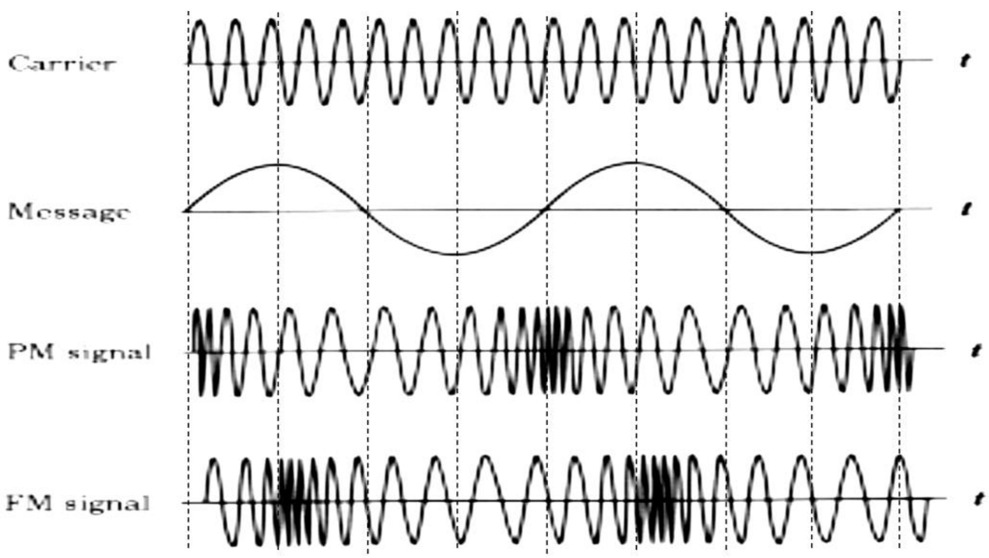
The frequency modulated signal is

$$S(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t)$$

Frequency deviation:

The deviation of frequency from original carrier frequency is called frequency deviation.

FM and PM waveforms



- For FM signal, the max frequency deviation takes place when modulating signal is at positive and negative peaks.
- For PM signal, the max frequency deviation takes place near zero crossings of the modulating signal.
- Both FM and PM waveforms are identical except for the phase shift.
- From modulated waveform, it is difficult to find whether modulation is FM or PM

$$\% \text{ Modulation} = \frac{\text{Actual frequency deviation}}{\text{max. allowable frequency deviation}}$$

Deviation Ratio:

$$DR = \frac{\text{max. frequency deviation (hertz)}}{\text{max. modulating signal frequency (hertz)}}$$

$$\frac{\Delta f(\max)}{f_m(\max)}$$

Note: DR is basically the modulation index corresponding to maximum modulating frequency.

****

2.2.1 Comparison of FM and PM

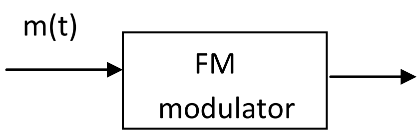
Sl.No	Characteristics	FM	PM
1.	Definition	Frequency of carrier is changed with m(t)	Phase of carrier is changed with m(t).
2.	Bandwidth	BW=2(Δf + fm)	BW ≈ 2Δf
3.	Noise	More affected by noise when compared to PM	Less affected by noise when compared to FM
4.	Applications	It is used in radio broadcasting	Used for data & voice transmission

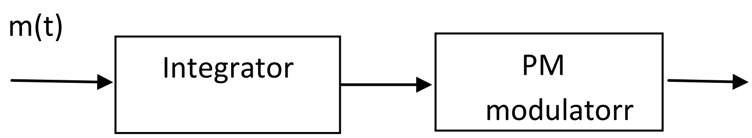
2. How can you generate an FM from PM and PM from FM? **April 2018**

2.2.2 Generation of FM from PM (or) Conversion from PM to FM.

- PM and FM are closely related in the sense that the net effect of both is variation in total phase angle.
- In FM, phase angle varies linearly with the integral of m(t).

By integrating the modulating signal m(t) and then applied to phase modulator, FM is generated from PM





Let $m(t) = A_m \cos \omega_m t$

After Integration

$$m(t) = \int A_m \cos \omega_m t dt = \frac{A_m}{\omega_m} \sin \omega_m t$$

After phase modulation $\theta \; \alpha \; m(t)$

$$\theta \; \alpha \; m(t)$$

$$\theta = km(t) = \frac{KA_m}{\omega_m} \sin \omega_m t$$

The instantaneous value of modulated voltage is given by

$$s(t) = A_c(\omega_c t + \theta)$$

$$s(t) = A_c\left(\omega_c t + \frac{KA_m}{\omega_m} \sin \omega_m t\right)$$

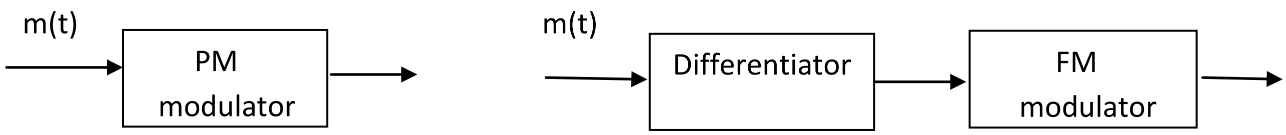
$$s(t) = A_c(\omega_c t + \beta \sin \omega_m t)$$

where $\beta = \frac{\Delta f}{f_m} = \frac{KA_m}{f_m}$ as $[\Delta f = KA_m]$

$$s(t) = V_C \cos[\omega_c t + \beta \sin \omega_m t]$$

This is the expression for FM wave.

Generation of PM from FM (or) Conversion from FM to PM



m(t) is first differentiated and then applied to frequency modulator to generate PM from FM.

We know that, $m(t) = A_m \cos \omega_m t$

After differentiation : $\frac{d}{dt} m(t) = -\omega_m A_m \sin \omega_m t$

After frequency modulation

$$\omega_i = \omega_C + \frac{dm(t)}{dt}$$

$$= \omega_C + K[-\omega_m A_m \sin \omega_m t]$$

$$\omega_i = \omega_C - K.\omega_m A_m \sin \omega_m t]$$

We know that the instantaneous phase angle of frequency modulated signal is

$$\phi_i = \int \omega_i \, dt = \int (\omega_C - K\omega_m A_m \sin \omega_m t) \, dt$$

$$= \omega_C t + \frac{K\omega_m A_m}{\omega_m} \cos \phi \, \omega_m t$$

$$\phi_i = \omega_C t + KA_m \cos\omega_m t$$

The instantaneous voltage after modulation is given by $S_{pm}(t) = V_C \sin \phi_i$

$$s(t) = A_C \cos(\omega_C t + KA_m \cos \omega_m t)$$

This is the expression for phase modulated wave. The process of integration and differentiation are linear. Therefore no new frequencies are generated.

****

2.3 Types of Frequency Modulation:

BW of FM depends on Modulation index (β). If β is high, BW is large

FM
- Narrow band FM $\rightarrow \beta << 1$ is small compared to one radian.
- Wide band FM $\rightarrow \beta >> 1$ is large compared to one radian

Narrowband FM

- When β is small, then BW is narrow.
- NB FM is also called as low-index FM.
- BW of a narrow band FM is same as that of AM, which is twice the baseband signal frequency. BW = $2f_m$.

Wideband FM

- When β >>1 (e.g 10) then the FM signal has wide BW.
- BW of wideband FM is too large; ideally infinite.

3. Draw the block diagram of generation of narrow band FM and derive an expression for single-tone narrow band FM. May2009/Dec2011/May2011

Narrowband FM

- When $\beta<<1$, then BW is narrow
- NB FM is also called as low-index FM
- BW of a narrow band FM is same as that of AM, which is twice the baseband signal frequency. BW = $2f_m$.

Generation of Narrowband FM:

FM signal $S(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$

$= A_c \cos\omega_c t \cos(\beta\sin\omega_m t) - A_c \sin\omega_c t . \sin(\beta \sin\omega_m t)$

Where Cos (A+B) = Cos A Cos B – Sin A Sin B

For NBFM, $\beta<<1$

$\therefore$ $S(t) = A_c \cos\omega_c t - A_c \sin\omega_c t\, \beta\sin\omega_m t$ (If θ is small ,Cos θ =1, sin $\theta = \theta$)

The modulated signal consists of carrier and two side bands. It is similar to AM and it is not widely used.

$m(t) = A_m \cos\omega_m t$

$S(t) = A_c \cos 2\pi f_c t - A_c\beta \sin 2\pi f_m t\ \sin 2\pi f_c t$

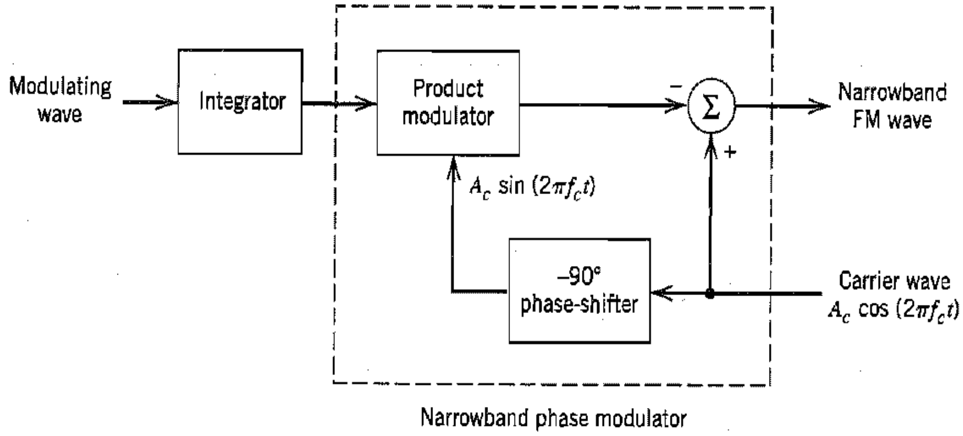
$S(t) = A_c \cos(2\pi f_c t) - 1/2\, A_c\beta\, [\cos 2\pi(f_c - f_m) t + \cos 2\pi(f_c + f_m) t]$

Spectrum of NBFM consists of carrier frequency f_c , upper sideband ($f_c + f_m$) and lower sideband

($f_c - f_m$ } .It is similar to AM

Bandwidth of narrow band FM (NBFM)=$2f_m$

Block diagram



Operation:

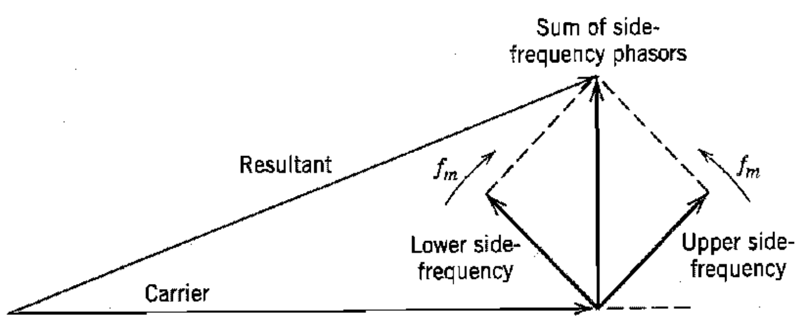
- The block diagram consists of product modulator, -90^{0}phase shift network , adder.
- This modulator splits the carrier signal in two paths.
- One path is direct and the other path contains the --90^{0}phase shifter.
- The product modulator generates DSB SC signal.
- The difference between these two signals produce Narrow band FM with some distortion.
- Ideally the envelope of FM is constant but the envelope of NBFM has amplitude modulation and varies with time.
- It produces some harmonic distortion.
- It can be reduced by restricting $\beta \leq 0.3$ radians

****

Phasor representation of NBFM:

- Here carrier phasor is taken as the reference phasor and the resultant of two sideband phasor is at right angle to the carrier phasor.
- The resultant phasor representing the narrowband FM.
- It has same amplitude as the carrier phasor but out of phase with respect to carrier.

  [In AM, the resultant phasor has amplitude different from carrier phasor].



4. Derive an expression for a single tone FM (WBFM) signal with necessary diagrams and draw its frequency spectrum. **Dec2012/ May 2016/ Nov 2016**

Obtain the mathematical expression WBFM signal. Also compare and contrast its characteristics with NBFM. **May 2017**

Frequency spectrum (or) Frequency analysis of single tone FM (WBFM)

The FM signal $S(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$

In complex form, $S(t) = A_c$ Real Part of $e^{j[\omega c\, t + \beta \sin \omega m\, t]}$

$$= A_c \, RP \, e^{j\omega c\, t} \, e^{j\beta \sin\omega m\, t} \qquad (1)$$

The complex envelope of FM signal is given by

$$\tilde{S}(t) = A_c \; e^{j\beta \sin\omega m\, t} \qquad (2)$$

Sub (2) in (1) $\quad S(t) = R\,P\,\tilde{S}(t) e^{j\omega c\, t} \qquad (3)$

The complex envelope is a periodic function of time 't' with fundamental frequency f_m

Since S(t) is a periodic function it can be expressed in complex Fourier series.

$$\tilde{S}(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{j2\pi n f m\, t} \qquad (4)$$

where $C_n = 1/2f_m \int_{-½ f_m}^{½ f_m} \tilde{S}(t) \, e^{-j2\pi n f m\, t} dt$

$$C_n = f_m \int_{-½ f_m}^{½ f_m} A_c \, e^{j\beta \sin\omega m\, t} \, e^{-j2\pi n f m\, t} dt$$

Let $2\pi f_m t = x$

$2\pi f_m \, dt = dx$

$t = x / 2\pi f_m$

when $t = 1/2f_m \Rightarrow x = \pi$

$t = - 1/2f_m \Rightarrow x = -\pi$

$$C_n = f_m A_c \int_{-\pi}^{\pi} e^{-j n x} \, dx / 2\pi f_m \quad e^{j\beta \sin\omega m\, t}$$

$$= A_c/2\pi \int_{-\pi}^{\pi} e^{j\beta \sin\omega m\, t} \quad e^{-j n x} \, dx$$

$$= A_c \, J_n(\beta) \qquad (5)$$

Bessel function $J_n(\beta) = 1/2\pi \int_{-\pi}^{\pi} e^{j(\beta \sin x - n x)} dx$

Sub (5) in (4)

$$\tilde{S}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi f m n t}$$

$$S(t) = RP \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi fm n t} e^{j\omega c t}$$

$$S(t) = RP A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(n\omega m + \omega c)t}$$

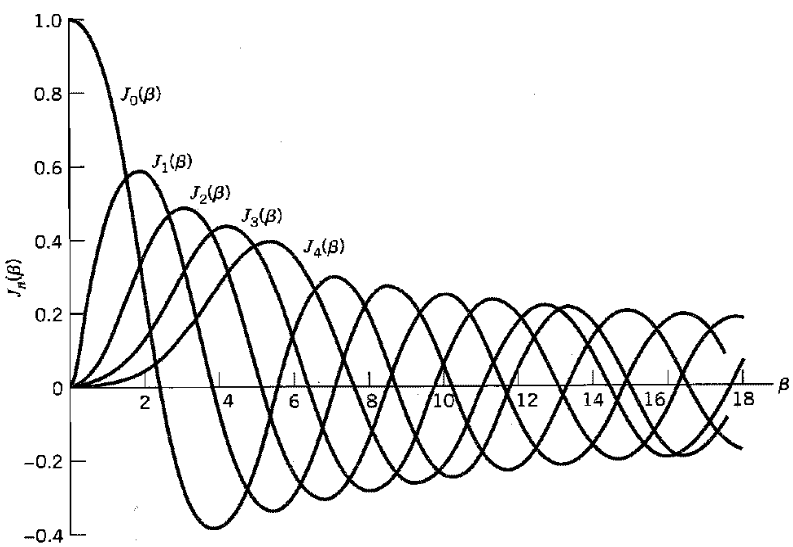
$$\boxed{S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t}$$

Spectrum of FM signal

$S(t) = A_c J_0(\beta) \cos\omega_c t + A_c J_{-1}(\beta) \cos(\omega_c - \omega_m)t + A_c J_1(\beta) \cos(\omega_c + \omega_m)t + \ldots\ldots\ldots$

It consists of carrier and infinite number of side bands

Bessel function of n th order first kind



For even values of n

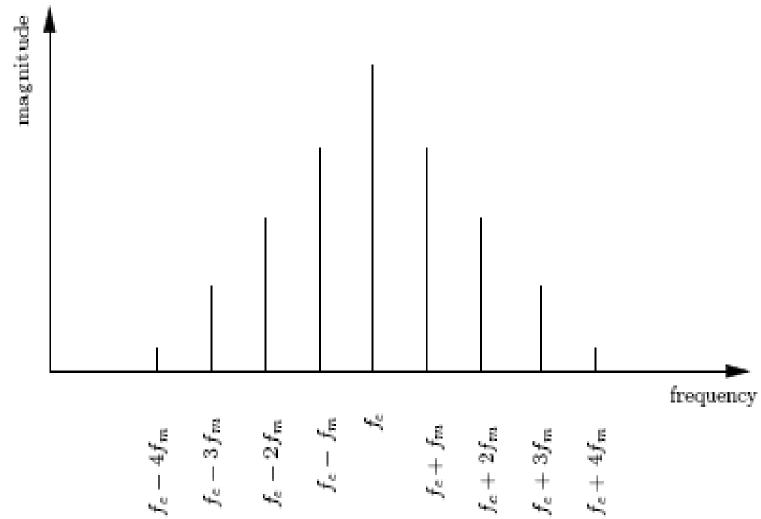
$J_{-n}(\beta) = J_n(\beta)$

For odd values of n

$J_{-n}(\beta) = -J_n(\beta)$

$\therefore$ $S(t) = A_c J_0(\beta) \cos\omega_c t + A_c J_1(\beta)[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] + A_c J_2(\beta)[\cos(\omega_c + 2\omega_m)t + \cos(\omega_c - 2\omega_m)t] + \ldots\ldots$

Frequency Spectrum



- The spectrum of WBFM consists of infinite no of sidebands which are centered around the carrier (or) separated from the carrier by $\pm \omega_m$, $\pm 2\omega_m$ $\pm$ …………….
- The modulation index determines how many sidebands have significant amplitude.
- If β is large, more number of significant sidebands.
- If β is small, then lesser no of sidebands.
- The infinite number of sidebands makes the BW infinite.
- If least significant sidebands are ignored, the BW is finite.

  [least significant sidebands is the SB with amplitude $\leq$ 1% of the carrier amplitude]
- The amplitude of FM is unchanged.
- Hence the power of FM is same as that of the un modulated carrier power.
- The average power of FM wave is $A_c^2/2R$ which is equal to carrier power.
- Total power = sum of carrier power and sideband power.

  Total = $P_c + P_1 + P_2 + P_3 + \ldots\ldots\ldots + P_n$

  Where $P_c \rightarrow$ carrier power

  $P_1 \rightarrow$ power in first set of sidebands

  $P_2 \rightarrow$ power in second set of sidebands

****

2.4 Transmission Band Width of WBFM

Theoretically there is simplest method to calculate the BW = $2f_{mxx}$ n radians/sec
where n = no of significant sidebands

$$[n \approx \beta : \beta >> 1]$$

$$BW \approx 2f_m n \approx 2f_m \beta$$

Carson's rule:

- Practical BW of FM can be found out by Carson's rule.
- An empirical formula for the BW of a single tone wideband FM is given by Carson's rule.

Carson's rule

$BW \approx 2(\Delta\omega + \omega_m)$ radians

Where $\Delta\omega \rightarrow$ frequency deviation

$$\boxed{BW \approx 2(\Delta f + f_m)}$$

$$= 2\Delta f\,[1+f_m/\Delta f]$$

$$\boxed{\therefore\ BW \approx 2\Delta f\,(1 + 1/\beta)}$$

we know $\beta = \Delta f / f_m$

Case (i) $\Delta f << f_m$

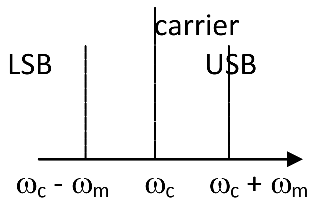
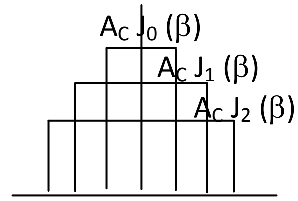
$$\boxed{B.W \approx 2f_m} \rightarrow \text{NBFM}$$

Case (ii) $\Delta f >> f_m$

$$\boxed{B.W \approx 2\,\Delta f} \rightarrow \text{WBFM}$$

This is the practical BW for large β.

5. Comparison of NBFM and WBFM

Sl.No	**Characteristics**	**NBFM**	**WBFM**
1.	Modulation Index	$\beta << 1$	$\beta >> 1$
2.	Frequency Spectrum	It consists of carrier & 2 sidebands. 	It consists of Infinite no of sidebands. 
3.	Bandwidth	$BW = 2f_m$	$BW = 2(\Delta f + f_m)$
4.	Maximum frequency deviation	5KHz	75 KHz
5.	Noise	Less suppression of noise.	Noise is more suppressed.
6.	Range of modulating frequency	30Hz to 3KHz	30Hz to 15KHz
7.	Pre-emphasis & De emphasis	Not needed.	Needed.
8.	Applications	Police wireless	Radio Broadcasting

2.5 Generation of FM

Methods of FM Generation
- Direct method
  - Varactor diode Modulator
  - Reactance Modulator
- Indirect method → Armstrong method

Direct Method:

In the direct method, the carrier frequency is directly varied in accordance with input base-band signal.

Indirect method:

In the indirect method of producing FM, the modulating signal is first used to produce a narrow-band FM signal.

Frequency multiplication is next used to increase the frequency deviation to the desired level.

6. Describe with neat diagram the method of generation of direct FM signal. May2017
Explain with diagram the generation of FM using direct method. May 2015/ Nov 2016

Varactor diode Modulator- Direct Method

- In the direct method, the carrier frequency is directly varied in accordance with input modulating signal.
- It is easily accomplished by VCO (voltage controlled oscillator)
- In L-C oscillator, frequency is determined by tuning capacitor and inductor.

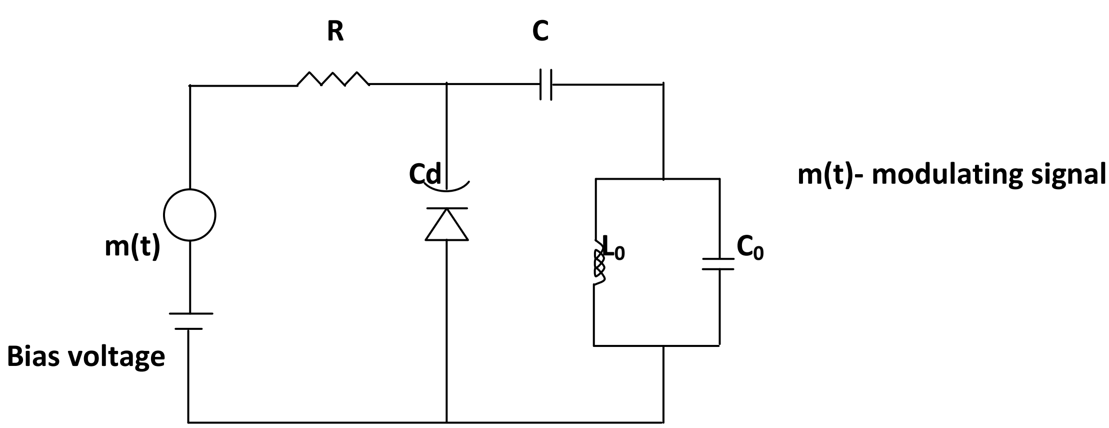
Principle :

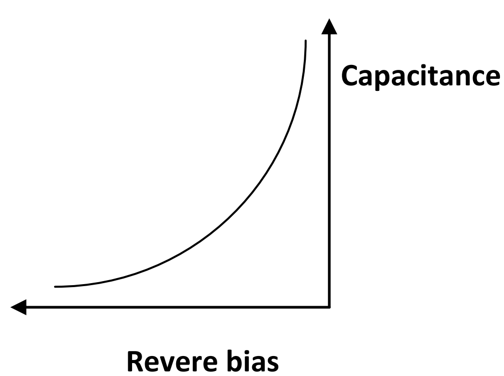
- The frequency of the oscillator is varied by varying the reverse bias of varactor diode
- Capacitance can be varied which in turn varies the frequency.
- Thus FM is generated.
- This is the concept behind varactor diode & reactance modulator.

Description :

- Varactor diode is a PN junction diode used as a variable capacitor in the reverse biased condition.
- The variable capacitor is dependent upon the magnitude of the reverse bias $C \alpha 1/\sqrt{V}$.
- Capacitor C isolates the varactor diode from the oscillator as far as dc is concerned and acts as short circuit for operating frequencies.
- The dc bias to the varactor diode is regulated so that the oscillator frequency is not affected by supply fluctuations.

Circuit diagram:





Operation:

- The modulating signal is fed in series with the regulated supply

$\therefore$ Effective bias to the varactor diode = DC bias voltage (V) + instantaneous value of the modulating signal

- The varactor capacitance varies with the modulating signal and frequency of the oscillator output changes and thus FM is generated.
- For positive half cycle m(t) increases , reverse bias increases .
- If rev bias increases capacitance decreases and frequency increases.
- For negative half cycle m(t) decreases , reverse bias decreases .
- If rev bias decreases capacitance increases and frequency decreases.
- The capacitance C_d of the diode is $C_d = K(V_D)^{-½}$

  where $V_D \rightarrow$ total instantaneous voltage across the diode.

  $K \rightarrow$ const of proportionality

- $V_D = V_0$ + Modulating signal voltage

  $= V_0 + V_m \sin\omega_m t$

  Where $V_0 \rightarrow$biasing voltage to maintain reverse bias across the varactor diode.

- Total capacitance of the oscillator tank circuit = $C_0 + C_d$

  Instantaneous frequency of oscillation

  $\omega_i = 1/\sqrt{L_0(C_0 + C_d)}$

  ω_i depends on V_D

  V_D depends on m(t)

- $\therefore$ ω_i (oscillator frequency) depends on m(t) (modulating signal)
- Thus frequency modulation is generated.

Applications

- Automatic frequency control circuits.
- Remote tuning.

****

7. Explain the working of Reactance modulator. **April 2018**

Explain with diagram the generation of FM using direct method.

May 2015/ Nov 2016/May 2017/ Dec 2017

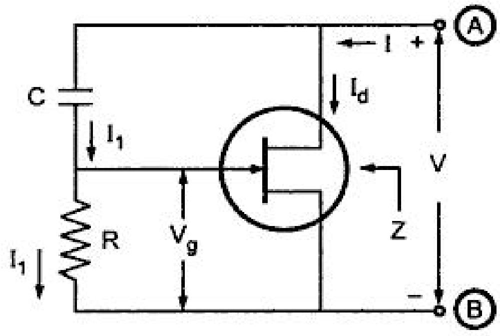
Reactance modulator

- Direct method of FM generation
- Frequency of carrier is directly varied according to message signal.

Principle:

- FET is made to act as capacitive reactance.
- For that an external voltage 'V' is applied & corresponding current is calculated to find [V/I = Z]

Circuit diagram:



$X_C >> R$

$V_g = V_{gs}$

Assumptions:

- Bias network current Ib is negligible as compared to the drain current of the FET. (Id>>Ib**)**
- Drain to gate impedance (Xc) must be greater than the gate to source impedance (R) by more than 5:1 (Xc >> R).

Description:

- Reactance obtained across terminals A – B.
- Terminals A – B of the circuit is connected across the tuned circuit of the oscillator to get FM output.
- The varying voltage (modulating voltage) Vg changes the reactance of the FET across terminals A – B.
- This change in reactance varies the frequency of the tank circuit.

Gate voltage $V_{gs} = I_b R$

$I_b = V/(R - j X_c)$ $\quad \therefore V_{gs} = VR / (R - j X_c)$ (1)

- The drain current of the FET is given as $Id = g_m V_{gs}$ (2)

  Sub. (1) in (2)

  $I_d = gm. RV/ (R - j X_c)$ (3)

- Impedance between AB assuming Id >> Ib

  $Z = V/ Id$ (4)

  Sub (3) in (4)

  $=V/ [gm R V /(R - j X_c)] = R - j X_c /gm R$

  $Z = 1/ g_m (R - j X_c)$

  If Xc >> R, then

  $Z = - j X_c/ g_m R =-j X_{eq}$

  Where $X_{eq} = X_c /gm R$

  $= 1/ 2\pi fC\, gm R$

  $= 1/ 2\pi f C_{eq}$ where $C_{eq} = gm RC$

  $\therefore$ FET behaves as capacitive reactance

- The equivalent capacitance (Ceq depends on the device trans conductance gm = Id/Vgs

$\therefore$ it can be changed by changing V gs.

- Ceq can be set to any original value by adjusting R&C values.
- If Xc >> R is not satisfied, then Z will not be purely reactive.
- It will have a resistance part in it which is added with Xc.
- Xc must be 5 or 10 times larger than R.
- Xc = nR at carrier frequency. when n = 5 to 10.

  [The value of reactance is proportional to g_m of FET which can be made to depend on gate bias and its variations]

  $X_c = 1/\ \omega C = nR$

  $C = 1/\ \omega nR$

  $C = 1/\ 2\pi fnR$

  C eq= gm R/2πfn R (since C eq = gm RC)

  C eq = gm /2πfn [C eq is made independent of R. So amplitude variations can be avoided]

Advantages

- Simple.
- Low cost.

Disadvantages:

- High frequency instability due to LC oscillator.
- Crystal oscillator has higher order stability.
- Even if crystal oscillator is used, frequency cannot be varied.
- So this method cannot be used for broadcast and communication purpose.

****

8. Explain the Armstrong method to generate FM signal. May2013/May 2016

Explain the Indirect method of FM Generation. May2007

Indirect method

In the indirect method of producing FM, the modulating signal is first used to produce a narrowband FM signal.

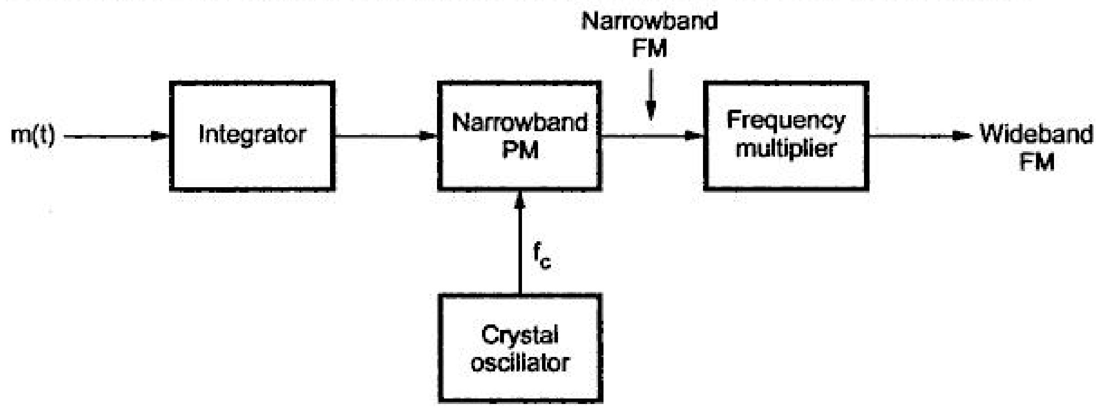
Then frequency multiplication is used to increase the frequency deviation to the desired level.

- Used in commercial broad casting.
- Frequency stability is achieved by using crystal oscillator.

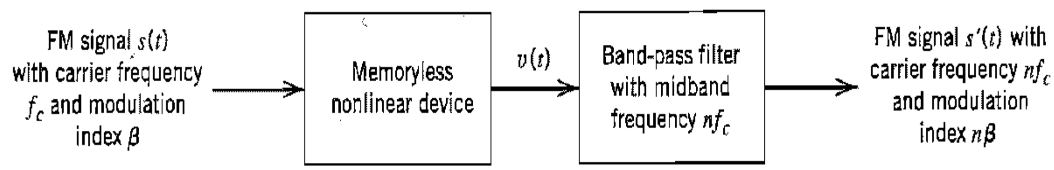
Principle of operation

- Use PM for Generating FM.
- Generate NB FM and frequency multiplier is used to generate WB FM.
- Multiplication process is performed in several stages.
- To increase the carrier frequency as well as the frequency deviation to the desired value.

Block diagram:



Block diagram of frequency multiplier



Operation:

- PM is used because it is easy to generate.
- But PM is inherent to distortion.
- To minimize distortion, modulation index is kept small.
- The phase modulated signal is $S(t)_{PM} = A_C \cos(\omega_c t + m_P \sin\omega_m t)$

  where $m_P \rightarrow$ modulation index for phase modulation.
- Instantaneous angular frequency ω_i of the phase modulated signal is $\omega_p = d\theta(t) / dt$
- As long as the modulating frequency does not change, phase modulation produces FM output.
- This technique is employed in indirect method.
- NBFM generated by this method is multiplied by frequency multiplier to produce the desired WBFM.

Frequency Multiplier:

- The frequency multiplier consists of memory less non – linear device followed by BPF
- Frequency multiplier not only increases the frequency but also increases the β(modulation index)
- If S(t) is an FM input signal, then

  $V_0(t) = a_1 S(t) + a_2 S^2(t) +$ ----------- $a_n S^n(t)$

  [any non – linear device obeys square law]

  Where a_1, a_2 ----- a_n are co efficients determined by the operating point of the device

  'n' $\rightarrow$ denotes the highest order of non – linearity

Input FM $S(t) = A_C \cos[2\Pi f_c t + 2\Pi k_f \int m(t)dt]$

- The instantaneous frequency of this FM signal is

  $f_i = f_c + k_f\, m(t)$

- Consider the max non – linearity of equation

  $nf_i = nf_c + nk_f m(t)$

- Therefore WBFM S(t) is

  $S^1(t) = A_c^1 \cos[2\Pi nf_c t + 2\Pi k_f \int m(t)dt]$

  $\therefore$ where $A_c^1 = nA_c$

  BPF is tuned to nf_c where f_c is the carrier frequency of incoming FM signal, S(t).

****

2.6 Demodulation of FM signal

Detection (or) Demodulation

The process of recovering the original modulating signal from the frequency modulated signal.

Types of FM detector:

FM detector (discriminator)

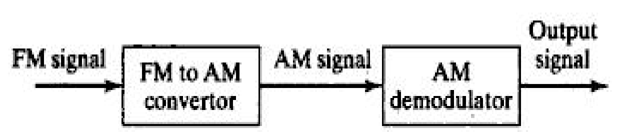
Frequency discriminator

1. **Slope detector**
2. **Balanced slope detector**

Phase discriminator

1. **Foster – Seeley discriminator**
2. **Ratio detector**

Frequency discriminator



Principle:

- Converts FM to AM by using frequency selective circuit (or) frequency discriminator circuit whose **output voltage depends on input frequency.**
- The original signal **m(t) is recovered from AM using envelope detector**.

2.6.1 Frequency Discriminator

Slope detector

Balanced slope detector

[Apr - 2019]

9. With necessary diagrams explain the operation of slope detector for demodulating FM signal.
Dec2012

Explain the FM demodulation process using frequency discrimination method. Dec 2017

Demodulation of FM signal

The process of recovering the original modulating signal from the frequency modulated signal.

Frequency discriminator

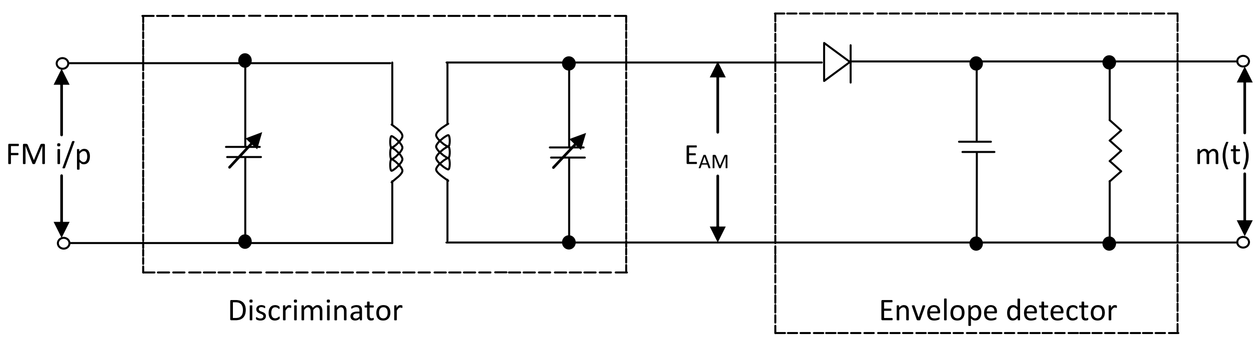
Principle:

- Convert FM to AM by using frequency selective circuit (or) frequency discriminator circuit whose output voltage depends on input frequency.
- The original signal m(t) is recovered from AM using envelope detector.

Slope detector

- It depends on the slope of the frequency response characteristic of frequency selective circuit.
- It uses single tuned circuit.
- It is tuned to frequency which is slightly away from carrier frequency f_c.

Circuit diagram:



Operation:

- When the input carrier frequency f_c increases, amplitude variations also increases.
- When the input carrier frequency f_c decreases, amplitude variations also decreases.
- The frequency variation at the input produces amplitude variations at the output.
- The small variation in the frequency Δf of the input signal will produce change in the amplitude of e_{AM}.

$$e_{AM} = \alpha(\Delta\omega), \text{ where } \alpha = de_{AM}/d\omega$$

- In this way, FM signal is converted into AM signal which is detected by envelope detector to recover the modulating signal m(t).

Advantages

- Simple and Inexpensive.

Disadvantages

- The non-linear characteristic of the circuit causes harmonic distortion since the slope is not same at all point of the characteristics.
- It does not eliminate amplitude variations

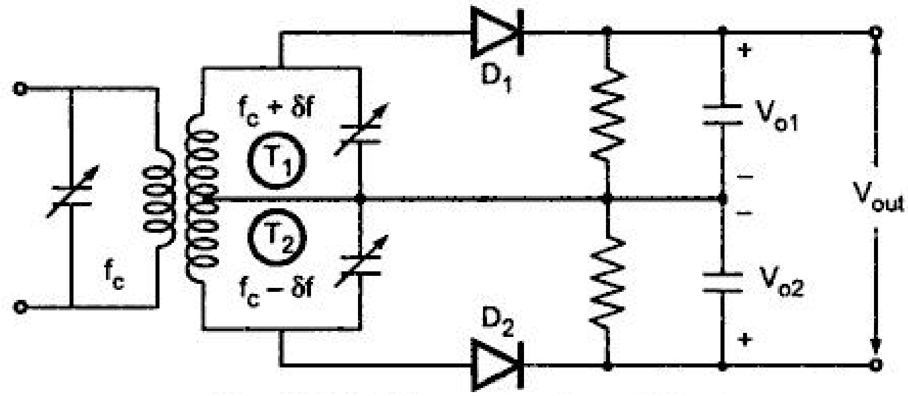
Balanced Slope Detector (or) Discriminator

To eliminate harmonic distortion produced in simple slope detector, balanced slope detector is used.

Principle:

- **Convert FM to AM by using frequency selective circuit** (or) frequency discriminator circuit whose output voltage depends on input frequency.
- The original signal **m(t) is recovered from AM using envelope detector.**

Circuit diagram:



Description:

- Balanced slope detector consists of slope detector circuits.
- Due to center tapped secondary, the input voltage to the 2 slope detectors, T_1 & T_2 are 180° out of phase.
- Primary is tuned to fc,
- T_1 (upper tuned circuit of secondary) is tuned above fc i.e., fc+ Δf and T_2 is tuned below fc i.e. fc - Δf
- R_1 C_1 and R_2 C_2 are the filters used to bypass the RF ripples.
- V01, V02$\rightarrow$ output voltages

  V0 = V01 – V02

Operation:

Circuit operation can be explained by providing the input frequency in 3 ranges as follows,

Case i. At fin= fc

Induced voltage in the T_1 winding of secondary = induced voltage in the winding T_2.

Input voltage to Diode D1 = input voltage to Diode D2.

$VD1 = VD2$

$V0 = V01 - V02$

$\therefore V0 = 0$

Case ii. At fin > fc i.e., **fin**= **fc** +Δf

Induced voltage in the winding T1 > induced voltage in T2

Input voltage to Diode > input voltage to Diode D2

$VD1 > VD2$

$V0 = V01 - V02$

$\therefore$ V0 = positive maximum

Case iii At fin< fc i.e., **fin**= **fc-** Δf

Induced voltage in the winding T2 > induced voltage in T1

Input voltage to diode D 2 > input voltage to Diode D1

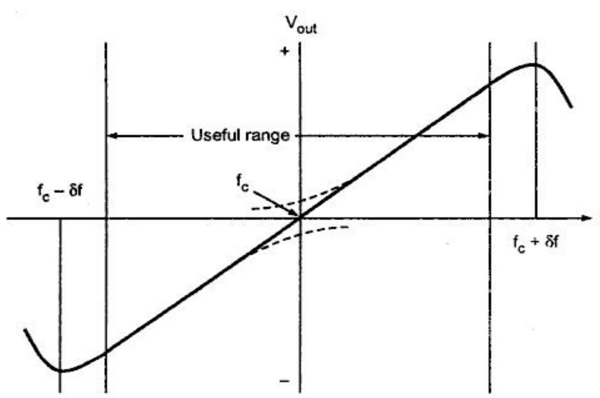
$VD2 > VD1$

$V0 = V01 - V02$

$\therefore$ V0 = negative maximum

If output frequency goes outside the range of (fc - Δf) to (fc + Δf), the output voltage will fall due to the reduction in tuned circuit response.

S shaped discriminator characteristics



Advantages

- More efficient than simple slope detector.
- Better linearity than slope detector.

Disadvantages

- Does not provide enough linearity.
- Difficult to tune 3 different frequencies fc, fc + Δf & fc- Δf
- Amplitude limiting is not provided.

****

2.6.2 Phase Discriminator

Foster – Seeley Discriminator

Ratio detector

10. Draw the circuit diagram of a Foster – Seeley discriminator and explain its working with relevant phasor diagrams. **Nov 2018** **Dec2006/May2012/May 2016**

With the phasor representation explain the Foster Seeley discriminator. May 2015

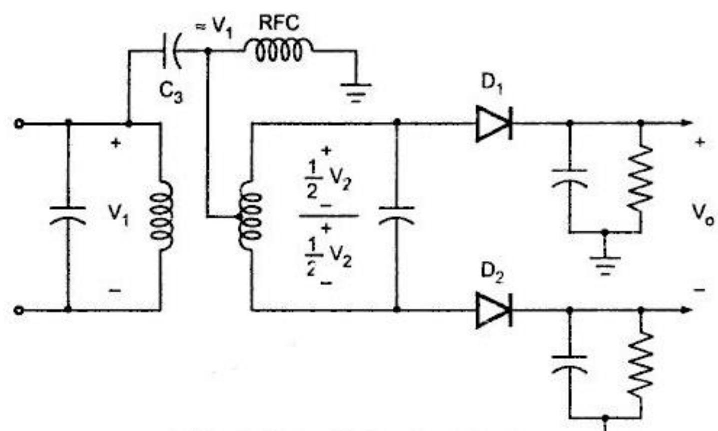
Foster – Seeley Discriminator- Phase Discriminator

- Most widely used FM detector.
- Both primary & secondary are tuned to the same center frequency 'f_c' of the incoming signal.
- Phase shift between primary and secondary voltages of the tuned transformer are a function of frequency.
- Secondary voltage lags primary voltage by 90° at the centre frequency because of C_3.
- $f_c \rightarrow$ resonant or tuned frequency of the transformer.

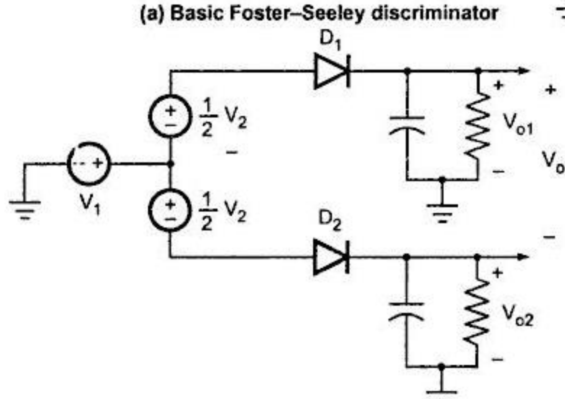
Principle:

- Convert FM to AM by using frequency selective circuit (or) frequency discriminator circuit whose output voltage depends on input frequency.
- The original signal m(t) is recovered from AM using envelope detector.

Circuit diagram:



(a) Basic Foster–Seeley discriminator



(b) Voltage generator equivalent circuit

Description:

- Primary is coupled to centre tap of the secondary through C_P.
- RFC offers high impedance to frequency of FM.
- The secondary voltage V2 is equally divided across upper half and lower half of the secondary coil.

  VD1 = V1+ 0.5V2 , VD2 = V1 – 0.5V2

  V0 = VD1 – VD2

  V0 ≈ | V01 | - | V02 |

$$\boxed{\therefore V0 \approx | V01 | - | V02 |}$$

- The primary and secondary tuned circuits are tuned to the same center frequency.
- The voltages applied to the two diodes D_1 and D_2 are not constant and vary depending on the frequency of the input signal.
- This is due to the change in phase shift between the primary and secondary windings depending on the input frequency.

Operation:

Case 1: fin=fc**,** phase shift between V1 and V2 is 90°.

V1- Primary voltage V2- secondary voltage

| V01 | = | V02 |

V0 = V01 – V02

∴ V0 = 0

Case2: fin > fc**,** phase shift reduces

The phase shift

| V01 | > | V02 |

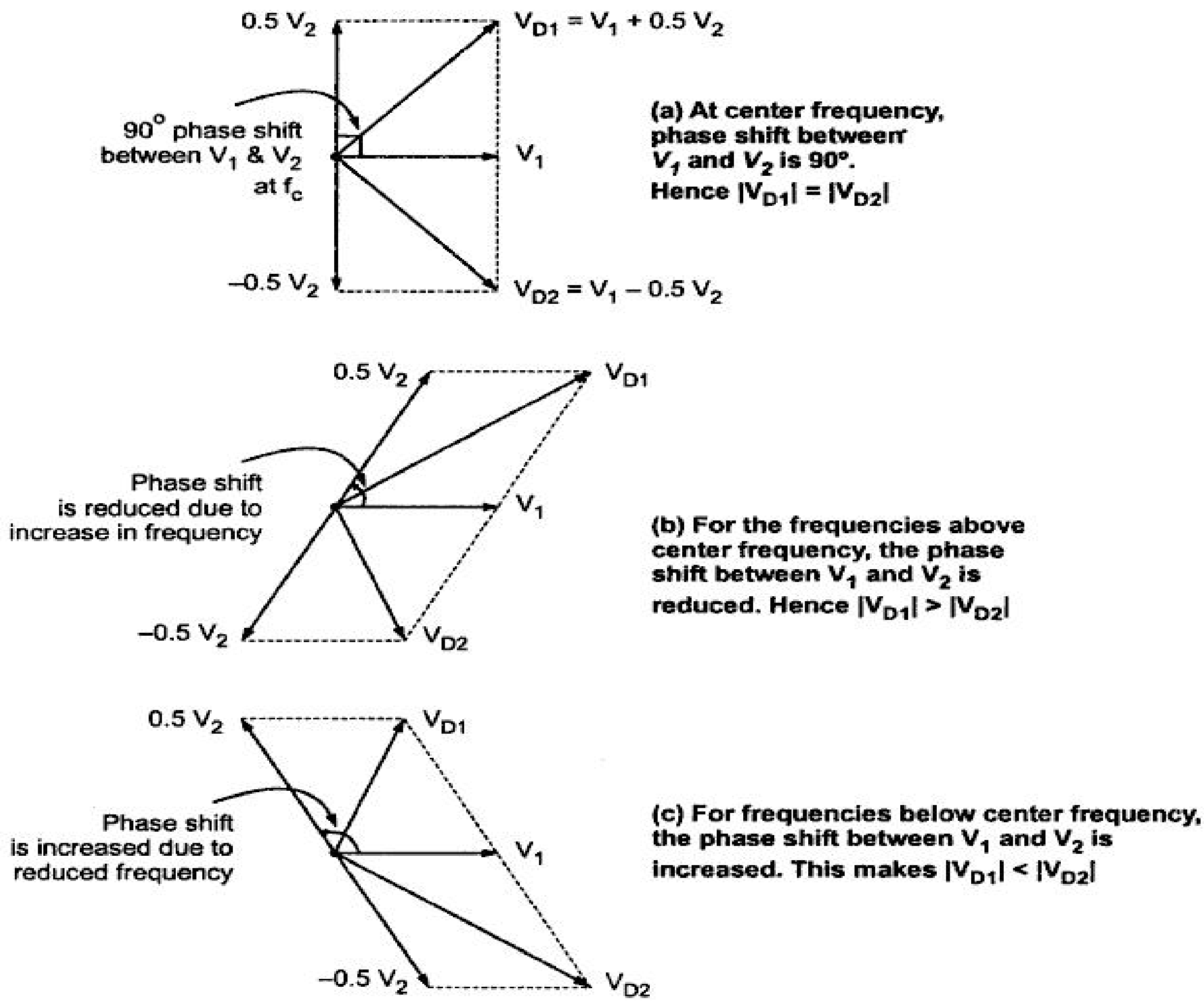
V0 is positive maximum.

Case3: fin **< fc**, phase shift increases

| V01 | < | V02 |

V0 ≈ negative maximum

Phasor diagram:



Advantages

- Much easier to align than balanced slope detector.
- Only two tuned circuits necessary and both are tuned to same frequency.
- Linearity is better.

Disadvantages

- Does not provide any amplitude limiting.
- The demodulator output responds to any amplitude variations and produce errors and modify the discriminator characteristics.
- The distortion is decreased using a limiter circuit in the FM receiver.

****

11. Explain the working of Ratio detector. **Dec2011**

Write about the basic principles of FM detection and explain about Ratio detector. **Nov 2016**

Analyze and brief how ratio detector suppresses the amplitude variations caused by the communication media without using amplitude limiter circuit. **May 2017**

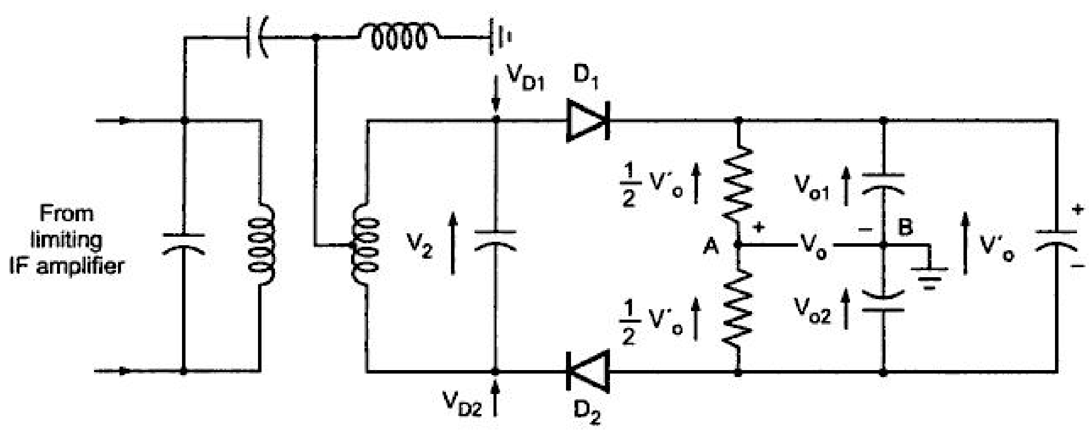
Ratio Detector

- It is a phase discriminator circuit used in TV receiver .
- It is an improvement over foster – seeley discriminator and widely used.
- Both are identical expect for the following changes
  1. Direction of diode D_2 is reversed.
  2. A large value capacitor C has been included in the circuit.
  3. Output is taken from the centre tap of a resistor 'R'

Principle:

- **Convert FM to AM by using frequency selective circuit** (or) frequency discriminator circuit whose output voltage depends on input frequency.
- The **original signal m(t) is recovered from AM using envelope detector**.

Circuit diagram:



Operation:

- The polarity of voltage V_{02} is reversed since D_2 is reversed.
  ∴ output voltage V_o is sum of voltage appears across the combined load
- $V_o = |V_{01}| - |V_{02}|$. (when $V_{01}\uparrow$, $V_{02}\downarrow$ & vice-versa)
- Output voltage V_0 is taken across the terminal AB. From the circuit diagram,

$$V_o = -\frac{1}{2}V_o' + V_{o1}$$

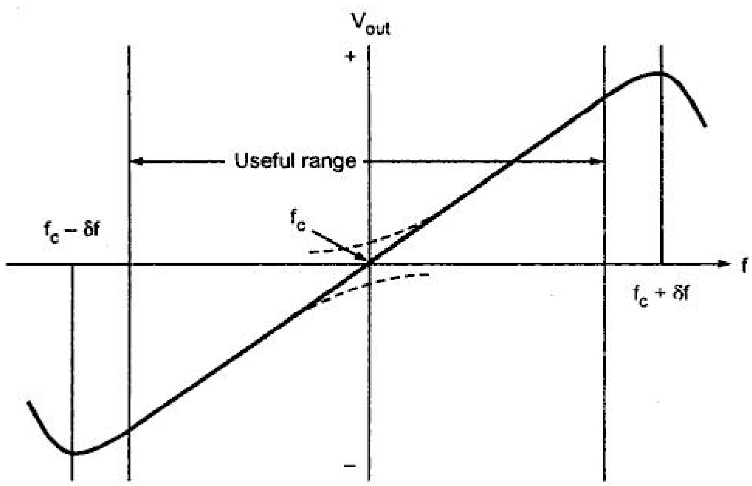
$$V_o = \frac{1}{2}V_o' - V_{o2}$$

Adding these two equations

$2V_0 = V_{01} – V_{02}$

$V_0 = 1/2[V_{01} – V_{02}] = 1/2[|V_{01}| - |V_{02}|]$

- The output is only half of that given by Foster – Seeley discriminator,
- Ratio detector has exactly same behavior except that its output is reduced.
- A large capacitor ‘C’ is connected across ‘V_0 .
- ‘C’ is mainly used to improve the constancy.
- If the input voltage decreases or increases suddenly, the output voltage does not respond immediately.
- since it is held constant by means of large capacitance.
- Since the two diodes are in series, they have the large time constant.
- It cannot respond to fast changes in input voltage.
- Therefore no need for separate amplitude limiting circuit.



Advantages

- Does not respond to amplitude variations present in the input FM. It is suppressed by shunt capacitor ‘C’
- Very good linearity due to linear phase relationship between primary and secondary.
- Reduced fluctuations in the output voltage compared to Foster seeley circuit.

Disadvantage

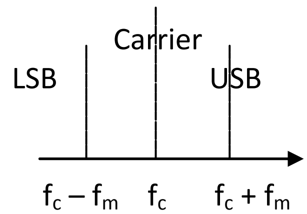
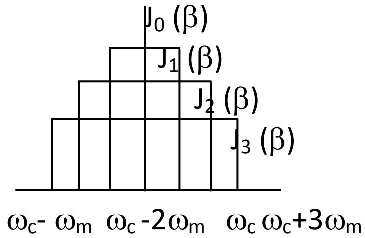
- It does not tolerate the long-period variation in signal strength. This requires an AGC signal

****

12. Compare AM and FM.

List the advantages of frequency modulation over amplitude modulation. **Dec2017**

Sl.No	Characteristics	AM	FM
1.	Definition	Amplitude of the carrier is changed with modulating signal	Frequency of carrier is changed with modulating signal
2.	BW	BW= $2f_m$	Theoretically BW=$2(\Delta f+f_m)$

		BW required is less compared to FM	More BW compared to AM.
3.	Modulation index	$m_a = V_m/V_c$ and cannot be greater than 1.	$\beta = \Delta f/ f_m$ and can be greater than 1
4.	Frequency spectrum	 In AM, the spectrum consists of carrier & 2 sidebands.	 In FM, the spectrum consists of carrier & infinite sidebands
5.	Power	Power depends on sidebands $P_t = P_c[1+m_a^2 /2]$	Power remains constant and depends on carrier only $P = A_C^2/2R$
6.	Efficiency	Less efficient than FM	More efficient than AM
7.	Noise	Noise interference is more	Less noise interference
8.	No of channels	More number of channels accommodated since BW less in AM	Less number of channel accommodated since BW is more
9.	Adjacent channel interference	Present	Absent
10.	Frequency Range	Medium Frequency & High Frequency (300 – 3000) KHz	VHF & UHF (30 – 3000 MHz)
11.	Application	It is used in AM Broad coasting It is used in TV for picture transmission	It is used in FM Broad casting. It is used in TV for sound transmission

****

2.7 PLL as FM demodulator / detector

PLL is Phase locked Loop. It consists of phase detector, Low pass filter and VCO.

Lock in range: It is defined as the range of frequencies over which PLL will track the input frequency signal and remains locked.

Dynamic range: It is the range of input frequencies over which PLL will capture the input signal.

13. Explain the operation of PLL as FM demodulator [Apr - 2019] **Dec 2014**

Explain the detection of FM wave using PLL detector. April 2018 **May 2017**

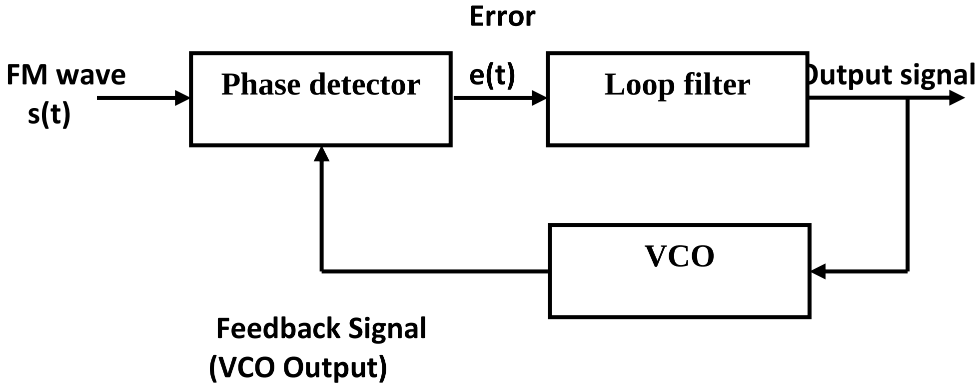
PLL FM demodulator / detector

The PLL (Phase locked loop) FM demodulator is one of the **most widely used FM demodulator** or detectors.

Elements of PLL

The elements of the phase locked loop system are a phase detector or comparator, low pass filter and voltage controlled oscillator (VCO).

Block Diagram



Description

- Input signal applied to Phase Locked Loop (PLL) is an FM signal S(t)
- The Voltage Controlled Oscillator (VCO) connected in the form of feedback system has a frequency proportional to an externally applied voltage.
- Any frequency modulator may serves as a VCO.
- The phase detector (or) Comparator produces a low frequency signal proportional to the phase difference between the incoming signal and the VCO output signal.

  $\therefore s(t) = A_c \sin[2\pi f_c t + \phi(t)]$

  where A_c is of carrier Amplitude, k_f is the frequency sensitivity of FM, $\phi(t)$ is phase angle.

  $$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

- The error signal e(t) or low frequency signal from phase detector is fed to loop filter.
- This output is fed to VCO as control input.

Operation:

- If the f frequency of Incoming signal shifts slightly, the phase difference between the VCO signal and incoming signal will increase with time.
- This will change the control voltage on the VCO and the VCO frequency loops back to same value as the incoming signal.

VCO output, r (t) = $A_c \cos[2\pi f_c t + \phi^1(t)]$

where, A_c = VCO signal Amplitude, K_f is the frequency sensitivity of VCO.

$$\phi^1(t) = 2\pi k_f \int_0^t V(\tau)\, d\tau$$

- The loop can maintain lock until the input signal frequency changes.
- The VCO input voltage is proportional to the frequency of the incoming signal.
- In FM signal, the instantaneous frequency varies in accordance with the modulating signal.
- When VCO is locked to fc, the error signal is zero.
- VCO frequency is also equal to zero.
- If an FM signal is applied to the phase detector, there will be a difference in the phases of the VCO output and the input FM signal.
- Control signal is produced in proportion to phase difference.
- This control voltage will modify the VCO frequency, which is again compared with the incoming frequency.
- VCO tracks the instantaneous frequency of the applied FM signal.
- The control signal produced is proportional to the frequency deviation in the FM signal.
- Since the frequency deviation is proportional to the modulating signal.
- The control signal appearing at the output of LPF is the modulating signal.

  Thus, FM signal is demodulated by PLL.

Advantages of PLL

- Simple circuit that can be implemented in an integrated circuit.
- No need of tuned circuits.
- Small number of external components required and less cost.
- Linearity is good.
- Distortion is less.

Applications of PLL

- Frequency multiplication / Division.
- AM Detection.
- FSK Demodulation.

****

UNIT II
Angle modulation

1. ***Define Angle modulation.***
   Angle modulation is defined as the process of changing the total phase angle of the carrier according to the modulating signal.

2. ***Define frequency modulation.***
   Frequency modulation is defined as the process of changing the frequency of the carrier according to the modulating signal.

3. ***Define phase modulation.***
   May2007
   Phase modulation is defined as the process of changing the phase of the carrier according to the modulating signal.

4. ***Define modulation index of FM.*** ***May2013 /Nov 2016***
   Define the modulation index of FM wave and specify how you will distinguish narrow band and wide band FM respectively. ***Nov 2013***
   It is defined as the ratio of maximum frequency deviation to the modulating frequency
   $$\beta = \frac{frequency\ deviation}{modulating\ frequency}$$
   β= Δf /fm

   β <<1 Narrow band FM
   β>>1 wide band FM

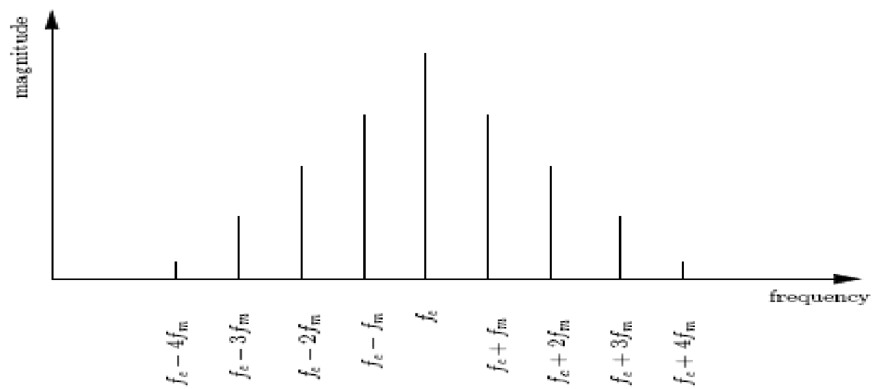
5. ***Define frequency Deviation.***
   Define carrier swing. ***May 2017***
   The maximum departure of the instantaneous frequency from the carrier frequency is called frequency deviation.
   Δf = fc max- fc =fc- fcmin
   Carrier swing=2Δf
   Δf =$A_m K_f$
   A_m – amplitude of modulating signal(Volts)
   K_f – Frequency deviation constant or Frequency sensitivity (Hz/Volts)

6. ***Define deviation ratio D.***
   The deviation ratio D is defined as the ratio of the frequency deviation Δf, which corresponds to the maximum possible amplitude of the modulation signal m(t),to the highest modulation frequency .
   D = Δf / fmax
   Δf- frequency deviation
   fmax- highest modulation frequency

7. ***Write the expression for the spectrum of single tone FM signal.***
   The expression for the spectrum of single tone FM signal
   $$s(t) = Ac \sum_{n=-\infty}^{\infty} Jn(\beta) cos(\omega ct + n\omega mt)$$

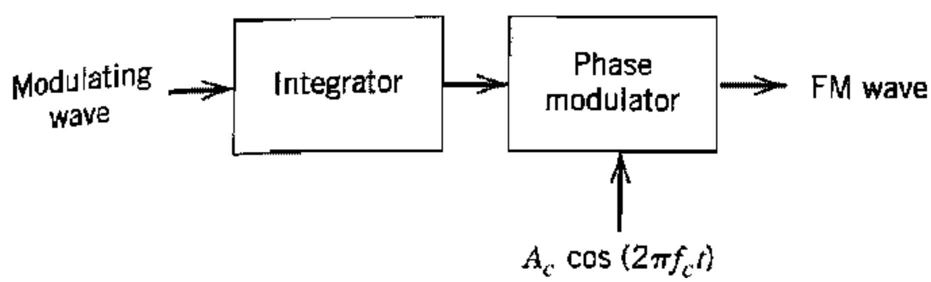
Ac-Carrier Amplitude
Jn(β)- Bessel coefficient
ωc -carrier frequency
ωm -modulating signal frequency

8. ***Draw the frequency spectrum of FM signal***

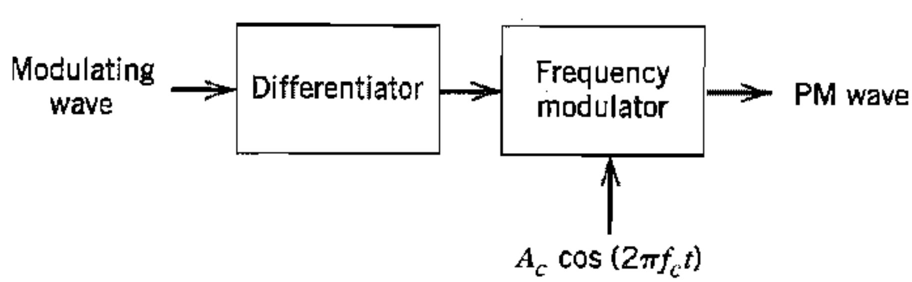


9. ***Give the average power of an FM signal.***
The amplitude of the frequency modulated signal is constant .The average power of the FM signal is same as that of the un modulated carrier power. P=Vc² / 2R
Vc – un modulated carrier amplitude
R –Resistor across which power is measured

10. How to convert PM to FM?
Illustrate the relationship between FM and PM with the block diagram. ***May 2012***



11. How to convert FM to PM?



12. Compare PM and FM . **[Apr-2019]** ***May2007/Nov2007/Nov 2010***

FM	***PM***
Frequency of the carrier is changed according to the modulating signal	*Phase of the carrier is changed according to the modulating signal*
Frequency deviation is proportional to modulating voltage and modulating frequency	*Phase deviation is proportional to modulating voltage only*
Noise immunity is better than AM &PM	*Noise immunity is better than AM but worse than FM*

13. What are the types of Frequency Modulation?

Based on the modulation index, FM can be divided into two types. They are Narrow band FM and Wide band FM. If the modulation index (β) is greater than one then it is wide band FM and if the modulation index I s less than one then it is Narrow band FM.

β <<1 Narrow band FM

β>>1 wide band FM

14. State Carson's rule. **[Apr - 2019]** ***May 2017***

State Carson's general rule for determining the bandwidth of an angle modulated wave?(or)

What is Carson's rule ?

May 2013

An approximate rule for the transmission bandwidth of an FM Signal generated by a single tone-modulating signal of frequency fm is defined as

$BW = 2(\Delta f + f_m)$ or $2\,\Delta f\,(1+1/\beta)$

Δf –Frequency deviation

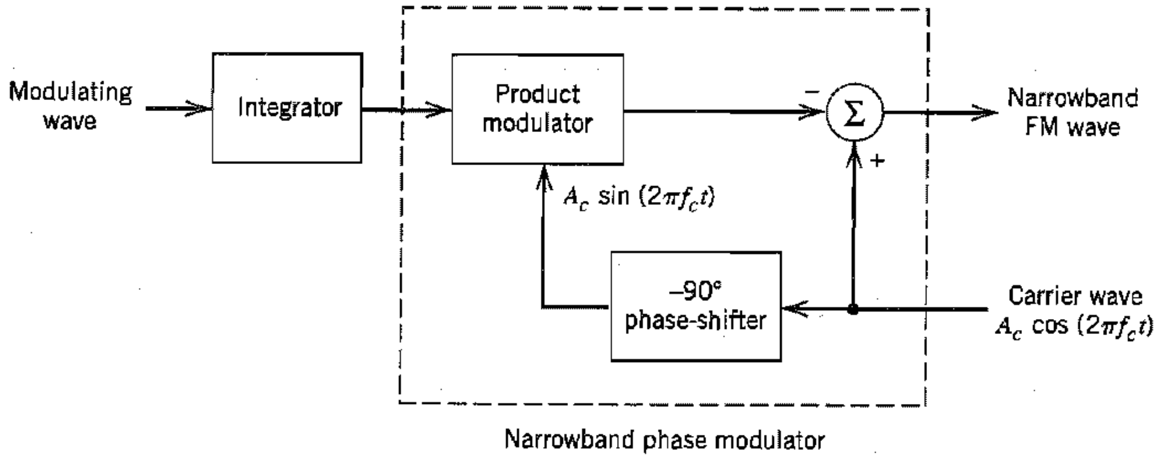
fm- Modulating frequency

β – modulation index

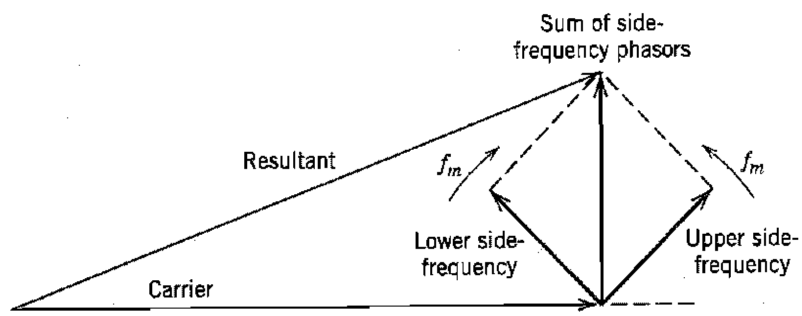
15. Distinguish between NBFM and WBFM. **[Apr - 2019]** ***April 2011/Dec2017***

Parameter /Characteristics	***NBFM***	***WBFM***
Modulation index (β)	*β <<1*	*β>>1*
Spectrum	*Carrier and two side bands*	*Carrier and Infinite number of sidebands*
Frequency deviation	*5 KHz*	*75 KHz*
Bandwidth	*2fm*	*BW = 2(Δf + fm)*
Noise	*Less suppression of noise*	*Noise is more suppressed*
Range of modulating frequency	*30 Hz to 3 KHz*	*30 Hz to 15 KHz*
Application	*Mobile communication*	*Broadcasting*

16. Draw the block diagram of a method for generating a narrowband FM signal. ***April 2010***

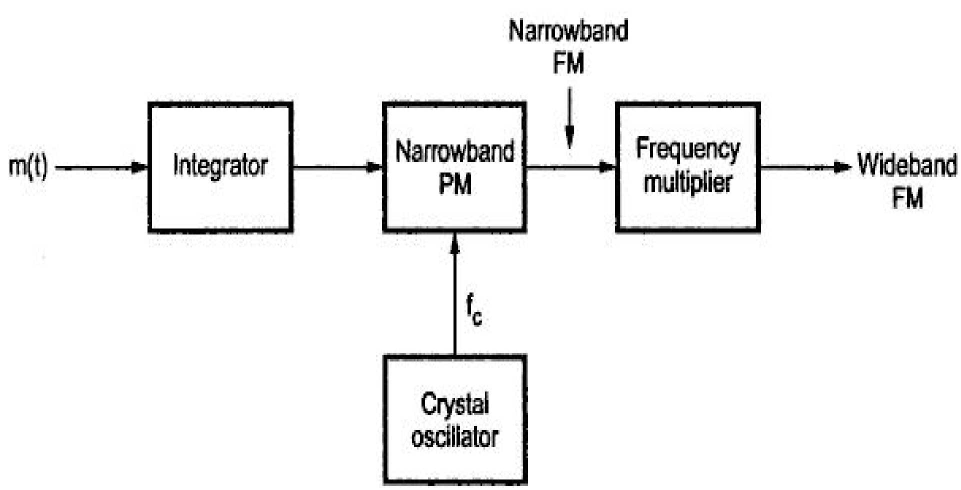


17. Draw the phasor representation of NBFM. **Nov 2018** ***Nov 2006***



18. How is the narrowband FM converted into wideband FM. ***Nov 2011 /Nov 2012***

The modulating signal is first used to produce a narrow-band FM signal and frequency multiplication is next used to increase the frequency deviation to the desired level to generate WBFM



19. What are the methods of generating an FM wave? **April 2018**

There are two methods of generating an FM wave. They are,

Direct method

In this method the frequency of the carrier is varied directly as the function of the modulating signal. It is used for the generation of NBFM

Indirect method

In this method the modulating signal is first used to produce a narrow-band FM signal and frequency multiplication is next used to increase the frequency deviation to the desired level. It is used for the generation of WBFM

20. What is the use of crystal controlled oscillator?

The crystal-controlled oscillator always produces a constant carrier frequency there by enhancing frequency stability.

21. What are the generation methods of direct FM?

In direct FM, two generation methods are available.

- *Varactor diode modulator.*
- *Reactance tube modulator.*

22. What are the disadvantages of direct method? ***Nov 2009***

The disadvantages of direct method are

- *Frequency instability due to the use of LC oscillator. Therefore we have to use the Automatic Frequency control (AFC) Scheme.*
- *Even if crystal oscillator is used the frequency cannot be changed*
- *Due to frequency instability direct methods cannot be used for broadcast and communication purposes.*

23. What are the advantages of indirect method of FM generation?

The advantages of indirect method of FM generation are

- *High frequency stability is achieved by the use of crystal oscillato.r*
- *Used for commercial broadcasting.*

24. What is meant by detection or demodulation? ***May 2011 /May 2012***

The process of recovering original message signal from the modulated signal is called as detection.

25. What is the principle of FM detection ? **April 2018**

- *FM signal is converted to corresponding AM signal using frequency selective circuits. i.e., circuits whose output voltage depends on input frequency.*
- *The original modulating signal is recovered from this AM signal by using envelope detector.*

26. Mention the types of FM discriminators or detectors.

Name the methods for detecting FM signals. ***May 2011 / May 2012***

FM discriminators can be divided into two types.

Frequency discriminators(Slope Detectors)

The principle of operation depends on the slope of the frequency response characteristics of frequency selective circuits.

Single tuned discriminator (or) slope detector.
Stagger tuned discriminator (or) Balanced slope detector.

Phase Discriminator

Foster-Seeley discriminator.
Ratio detector.

27. Mention the advantages of balanced slope detector.

The advantages of balanced slope detector are

- *This circuit is more efficient than simple slope detector.*
- *It has better linearity than the simple slope detector.*

28. What are the disadvantages of balanced slope detector?

The disadvantages of balanced slope detector are

- *Amplitude limiting is not provided*
- *Linearity is not sufficient*
- *This circuit is difficult to tune since the three tuned circuits are to be tuned to different frequencies, i.e., f_c, $(f_c+\Delta f)$ and $(f_c-\Delta f)$.*

29. What are the advantages and disadvantages of Foster Seeley discriminator?

Advantages:

- *Linearity is better than slope detector*
- *Two tuned circuits tuned to same frequency*

Disadvantages:

- *An amplitude limiter is required*
- *Linearity is not sufficient.*

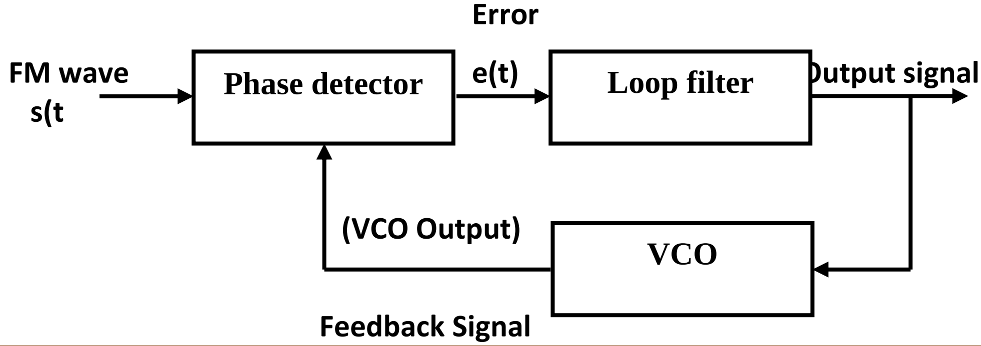
30. What are the advantages of ratio detector? **Nov 2011**

The advantages of ratio detector are

- *It does not respond to amplitude variations present in the input of FM. It is suppressed by shunting capacitor 'C'*
- *Very good linearity due to linear phase relationship between primary and secondary.*
- *Reduced fluctuations in the output voltage compared to Foster – seeley circuit.*

31. Draw the block diagram of PLL FM demodulator. **Nov2007/May 2007**

Draw a simple schematic of a PLL demodulator. **Nov 2013**



32. PLL FM demodulator is widely used for FM detection. Justify
What are the advantages of PLL FM demodulator?

The PLL FM demodulator is widely used for FM detection because

- *Simple circuit that can be implemented in an integrated circuit*
- *No need of tuned circuits.*
- *Small number of external components required and less cost.*
- *Linearity is good*
- *Distortion is less*

33. What are the applications of PLL? ***Nov 2006***

The applications of PLL are

- *AM detection*
- *Multiplication / Division*
- *FSK demodulation*

34. List the advantages of FM over AM. ***Nov 2014***

The advantages of FM over AM are

- *Noise interference is less*
- *Less adjacent channel interference*
- *High efficiency*

35. Write the applications of FM.

The applications of FM are

- *Radio broadcasting.*
- *Sound broadcasting in T.V.*
- *Satellite communication.*

36. What are the disadvantages of FM system compared to AM?
The disadvantages of FM are

- *A much wider Bandwidth is required by FM.*
- *FM transmitting and receiving equipments are more complex and costly.*
- *Less coverage area*

37. Compare AM and FM.

Amplitude Modulation	***Frequency Modulation***
Amplitude of the carrier is changed according to the modulating signal	*Frequency of the carrier is changed according to the modulating signal*
Carrier and two sidebands are present	*Carrier and infinite number of sidebands Are present*
Noise interference is more	*Noise interference is less*
Less efficient	*Efficiency is more*
Adjacent channel interference is more	*Adjacent channel interference is less*

38. Why FM is used for voice transmission?

FM is widely used for mobile applications because the amplitude variations do not cause a change in audio level. As the audio is carried by frequency variations rather than amplitude ones and interference is less.

39. Define lock in range and dynamic range of a PLL. ***May 2015***

Lock in range: *It is defined as the range of frequencies over which PLL will track the input frequency signal and remains locked.*

Dynamic range: *It is the range of input frequencies over which PLL will capture the input signal.*

40. Distinguish the feature of Amplitude modulation (AM) and Narrow band frequency modulation (NBFM). ***May 2017***

NBFM is similar to AM
NBFM consists of two sidebands and carrier as AM.
But noise is less in NBFM than AM

Solved problems

1. A carrier wave of frequency 100MHz is frequency modulated by a sinusoidal wave of amplitude 2 volts and frequency 100 KHz. The frequency sensitivity of the modulator is 2.5 KHz/volt. Determine the bandwidth of FM signal.
Nov2009

Given data:

f_c =100MHz A_m = 2V f_m =100KHz K_f = 2.5 KHz/V

Solution:

$\Delta f = K_f . A_m$

$= 2.5 \times 10^3 \times 2 = 5KHz$

$BW = 2(\Delta f + f_m)$
$= 2(5+100) = 210$ KHz

2. A carrier wave of frequency 100MHz is frequency modulated by a signal 20 sin ($200\pi x 10^3 t$). What is the bandwidth of FM signal if the frequency sensitivity of the modulation is 25 Khz/v. ***April 2010***

Given data:

Ω m=$200\pi x 10^3$ Am=20
carrier Frequency =100MHz
Frequency sensitivity K_f=25KHz

Solution:
$2\pi f$ m=$200\pi x 10^3$, fm=$100X10^3$
BW=2(Δf +fm)

$\Delta f = K_f A_m$ =25000x20 , Δf=500KHz

BW=2(500+100)

=1200 KHz

3. ***A Carrier signal is frequency modulated with a sinusoidal signal of 2KHz resulting in a maximum frequency deviation of 5KHz. Find modulation index, Band Width of the modulated signal.***
Nov 2012/May 2015
Given data:

f_m = 2KHz, Δf = 5 KHz

Solution:

Modulation index, $\beta = \Delta f / f_m = 5/2 = 2.5$

$BW = 2(\Delta f + f_m) = 2(5+2) \approx 14$ KHz

4. ***If the maximum phase deviation in a phase modulation system when a modulating signal of 10V is applied as 0.1 radian , determine the value of phase deviation constant.***
May 2014
Given data:

A_m = 10V, Δp = 5 KHz

Solution:

$K_p = A_m \Delta p$ = 10x5 =50 radian/volt

5. ***A carrier signal is frequency modulated by a sinusoidal signal of 5 Vpp and 10 KHz. If the frequency deviation constant is 1KHz/V, determine the maximum frequency deviation and state whether the scheme is narrow band FM or Wide band FM. Nov 2014/May 2016***
Given data:

A_m = 5V (V pp), f m=10K Hz and K_f = 1 KHz /V

=2.5 V

Solution:

$\Delta f = K_f A_m$ =1x2.5 =2.5

$\beta = \Delta f / f_m$ = 2.5/10=0.25

Since the modulation index β is less than 1 it is NBFM.

Exercise Problems:

1. ***The carrier frequency of broadcast signal is 100MHz and if the audio signal modulating the carrier is 15 KHz & frequency deviation is 75KHz. Find BW of FM.***

Given : Δf = 75 KHz, f_m = 15 KHz , f_c = 100 MHz

Solution:

$B.W = 2\,(\Delta f_m + f_m)$

$= 2\,(75 + 15) = 180$ KHz

2. ***A single tone modulating signal cos ($15\Pi 10^3 t$) frequency modulates the carrier 10MHz & produces frequency deviation of 75KHz. Find the modulation index.***

Given: $\Delta f = 75$ KHz $\quad 2\Pi f_m = 15\,\Pi 10^3 \quad f_m = 7.5 \times 10^3$

Solution:

$\beta = \Delta f/f_m = 75 \times 10^3 / 7.5 \times 10^3 = 10$

3. ***Carrier signal is frequency modulated with the sinusoidal signal of 2KHz resulting of max. frequency deviation 5KHz. Find modulation index and BW.***

Given: $f_m = 2$KHz, $\Delta f = 5$ KHz

Solution:

$\beta = \Delta f / f_m = 5/2 = 2.5$

$BW = 2(\Delta f + f_m) \approx 15$ KHz

4. ***Find carrier frequency, modulating frequency, modulation index, frequency deviation of FM signal represented by S(t) = 12 sin($6x10^8$ + 5 sin1250t). Also find the power dissipated for 10Ω resistor.***

Given:

$S(t) = 12 \sin(6x10^8 + 5 \sin 1250t)$

Solution:

FM signal is represented by $S(t) = A_C \sin(\omega_c t + \beta \sin \omega_m t)$

$\therefore A_C = 12,\ \omega_c = 6x10^8,\ \omega_m = 1250,\ \beta = 5$

$2\Pi f_c = 6x10^8 \quad , f_c = 6x10^8/2x3.14 = 95.5MHz$

$\beta = m_f = 5$

$2\Pi f_m = 1250 \Rightarrow f_m = 1250/2\Pi = 199Hz$

$\beta = \Delta f/f_m \Rightarrow \Delta f = \beta \times f_m = 5 \times 199 = 995Hz$

Power dissipated by 10Ω *resistor is* $P_d = A_c^2/2R = 12^2/ 2 \times 10 = 7.2W$

$[A_C \rightarrow$ *un modulated carrier amplitude*$]$

5. ***Obtain the BW of FM signal S(t) = 10cos[2x10$^7\Pi$t + 8 cos(100Πt)].***

Given : $A_c = 10$, $\omega_c = 2x10^7\Pi$, $\beta = 8$, $\omega_m = 1000$

Solution:

$2\Pi f_c = 2x\Pi x10^7$

$f_c = 10^7 = 10MHz$

$2\Pi f_m = 1000\Pi$

$f_m = 500Hz$

$\beta = \Delta f/f_m \Rightarrow \Delta f = \beta \, x \, f_m = 8 \, x500 = 4000Hz$

$BW = 2(\Delta f + f_m)$

$= 2(4000+500) = 9 \, KHz$

6. ***A carrier wave of frequency 1000 Mz is frequency modulated by a sine wave of amplitude 2V, frequency 100Hz. If the frequency sensitivity of modulator is 2.5 KHz/V, calculate BW.***

Given: f_c =1000MHz ,A_m = 2V , f_m =100KHz , K_f = 2.5 KHz/V

Solution:

$\Delta f = K_f.A_m$

$= 2.5 \, x \, 10^3 \, x \, 2$

$\Delta f = 5KHz$

$BW = 2(\Delta f + f_m) = 2(5+100) = 210 \, KHz$

7. ***An angle modulated wave is described by the equation***

V(t) = 10 cos (2x10$^6\Pi$t+ 10 cos2000Πt). Calculate

(i) Power of the modulated signal

(ii) Maximum frequency deviation (iii) BW ***May 2016***

Given : $A_c = 10$ **,** $\omega_c = 2 \, x \, 10^6\Pi$ **,** $\omega_m = 2000\Pi$, fm=1000 Hz =1KHz

(i) $P = A_c^2/2R = 10^2/2x1 = 50W$

(ii) $\Delta f = ?$

$\beta = \Delta f/f_m \Rightarrow \Delta f = \beta f_m$

$\therefore \Delta f = 10x1 = 10 \, KHz$

(iii) $B.W = 2(\Delta f + f_m)$

$= 2(10+1) = 22 \, KHz$

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UNIT – III
RANDOM PROCESS

Random variables, Random Process, Stationary Processes, Mean, Correlation & Covariance functions, Power Spectral Density, Ergodic Processes, Gaussian Process, Transmission of a Random Process Through a LTI filter.

Introduction

- Random signals are non-deterministic i.e., it cannot be predicted.
- Many signals and noise in communication system are random signals.
- Probability is the mathematical tool to analyze the random signal.
- Probability is the study of random experiments.

Random Experiments:

An experiment whose outcome cannot be predicted exactly is called random experiment.

Example: Flipping a coin, drawing a card from a deck of playing cards, etc.

Sample space: All possible outcomes of a random experiment are called a sample space.

Sample: A particular outcome is called a sample point or sample.

Event: Events are subsets of sample space. In other words an event is a collection of outcomes.

An event is the outcome of getting an odd number in throwing a die.

Probability of an event

Probability is a set function assigning non negative values to all events E such that the conditions are satisfied

$0\leq P(E) \leq 1$ for all events

$P(\Omega) = 1$

Some of the most important properties are

$P(E^1) = 1-p(E)$

$P(\phi)= 0$

$P(E_1 \cup E_2) = P(E_1) + P(E_2)- P(E_1 \cap E_2)$

$E_1 C E_2$, $P(E_1) \leq P(E_2)$

Conditional probability.

Conditional Probability of an event E_1, given the event E_2 is

$$\mathbf{P(E_1/E_2) = P(E_1 \cap E_2)/P(E_2)} \quad \textbf{provided } \mathbf{P(E_2) \neq 0}$$

For independent event, $\mathbf{P(E1 \cap E2)=P(E1)\ P(E2)}$

Example:

In throwing a fair die, the probability of

A = {The outcome is greater than 3} is

$$P(A) = P(4) + P(5) + P(6) = \frac{1}{2}$$

The probability of

B = {The outcome is even}

$$P(B) = P(2) + P(4) + P(6) = \frac{1}{2}$$

In this case,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(4) + P(6)}{\frac{1}{2}} = \frac{2}{3}$$

Baye's Theorem

If the events $\{E_i\}_{i=1}^{n}$ are disjoint and their union makes the sample space, then they make a partition of the sample space Ω. Then, if for an event A, we have the conditional probabilities$\{P(A|E_i)\}_{i=1}^{n}$. $P(A)$can be obtained by applying the total probability theorem states as

$$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A|E_i)$$

Baye's rule gives the conditional probabilities $P(E_i|A)$ by the following relation

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)}$$

Example:

In a certain city 50% of the population drives to work, 30% take the subway and 20% take the bus. The probability of being late for those who drive is 10%, for those who take the subway is 3%, and for those who take the bus 5%

i. What is the probability that an individual in this city will be late for work?

ii. If an individual is late for work, what is the probability that he drove to work?

Solution:

Let D, S and B denote the events of an individual driving, taking the subway, or taking the bus. Then P(D)= 0.5, P(S) = 0.3, and P(B) = 0.2.If L denotes the event of being late, then from the assumptions, we have

$$P(L|D) = 0.1;$$
$$P(L|S) = 0.03;$$
$$P(L|B) = 0.05;$$

i. From the total probability theorem,

$$P(L) = P(D)P(L|D) + P(S)P(L|S) + P(B)P(L|B)$$

= 0.5 x 0.1 + 0.3 x 0.03 +0.2 x 0.05

= 0.069

ii. Applying Baye's rule, we have

$$P(D|L) = \frac{P(D)P(L|D)}{P(D)P(L|D) + P(S)P(L|S) + P(B)P(L|B)}$$

$$= \frac{0.05}{0.069}$$

$$= 0.725$$

Exercise Problem:

In a binary communication system, the input bits transmitted over the channel are either 0 or 1 with probabilities 0.3 and 0.7 respectively. When a bit is transmitted over the channel,it can be received correctly or incorrectly (due to channel noise). Let us assume that if a 0 is transmitted, the probability of it being received in error (i.e., being received as 1) is 0.01 and if a 1 is transmitted, the probability of it being received in error (i.e., being received as 0) is 0.1.

3.1 Random Variable

- Random variable is defined as a function which maps the outcome of a random experiment to a set of real numbers.
  i.e. assigning a real number to the outcome of a random experiment.
- It is also known as **stochastic variable**
- Random variables are denoted by upper case letter X,Y etc., and values assumed by random variables are denoted by lower case letters with subscripts. x1, x2, y1, y2, etc.,

Discrete Random Variable:

A Random variable that takes on a finite number of values is known as discrete random variable
e.g Tossing of two coins
Sample space **S = {HH, HT, TH, TT}**
Let number of heads be X, then the random variable **X = {2,1,1,0}**

Discrete probability distribution

Let X be the discrete RV and let x1,x2… be the values that X assumes
Let P (X = x j) = f(x j) j = 1,2,3, …… be the probability of xj
Let there be a function f(x) such that **f(x) > 0**
Σ f(x j) = 1
where f(x j)is probability function (or) probability distribution of the discrete random variable.

Continuous Random Variable:

A RandomVariable that takeson infinite number of values is called continuous Random Variable

Continuous probability distribution

Let there be a function f(x) such that f(x)>0 then $\int_{-\infty}^{\infty} f(x))dx = 1$

Where f(x) $\rightarrow$probability density function.

Joint Distributions

- So far only one random variable is considered.
- But the study of communication systems may involve more than one random variable.

  i.e. 2,3, etc … (2 $\rightarrow$ one for transmitter one for receiver).
- Both random variable may be discrete or continuous or one discrete and one continuous.

(But communication systems involve either both discrete or both continuous random variables)

Discrete case

Let X and Y be 2 discrete random variables.

The joint probability function of x & y is defined as

f(x,y) = P(X = x, Y = y)

Where f(x,y) satisfies the following properties

$$\mathrm{f(x,y)} \geq 0$$

$$\sum_{x}\sum_{y} \mathrm{f(x,y)} = 1$$

Continuous case

Let X and Y be two continuous random variables. The joint probability function of x &y is defined as

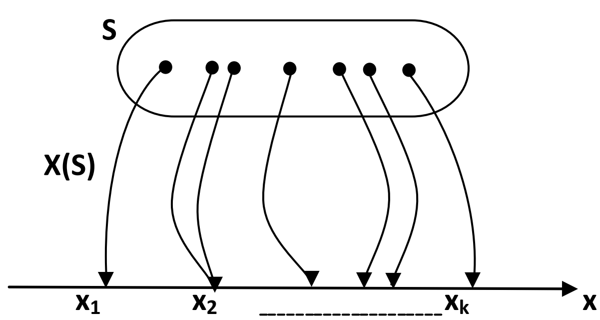
$$\mathrm{f(x,y)} \geq 0$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{f(x,y)dx\,dy} = 1$$

1. **What is CDF and PDF? State their properties. Also discuss them in detail by giving examples of CDF and PDF for different types of random variables.** **Dec 2015**

Discrete Random Variable and CDF (Cumulative Distribution Function)

If sample space "S" holds a countable number of sample points, then X is a discrete random variable having countable number of distinct values.



Sample points mapped by discrete random variable X(s)

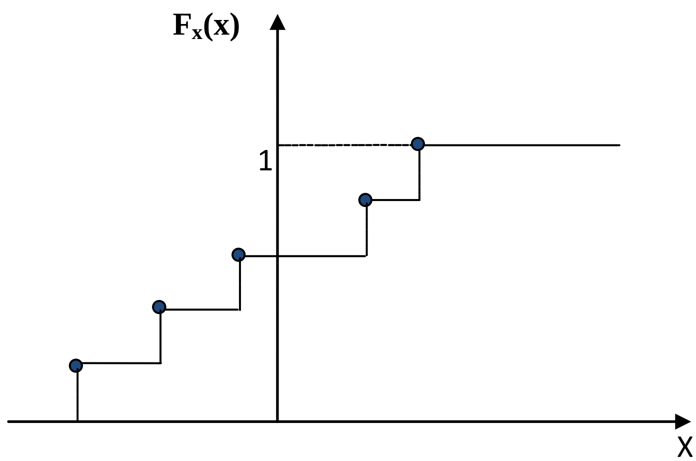
Example: X = {1,2,3,..........}

Note: Each outcome produces a single number, but two or more outcomes may map into the same number.

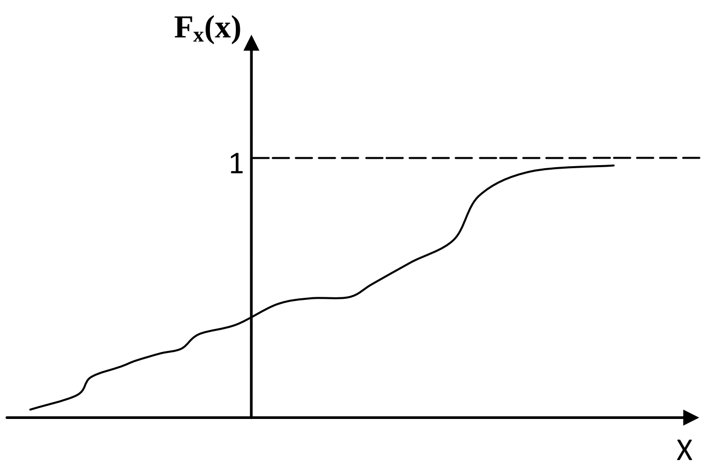
Cumulative distribution function (CDF)

The cumulative distribution function $F(X \leq x)$ is defined as the probability that the outcome of an experiment will be one of the outcome for which $X \leq x$, where x is any given number.

Symbolized by $F_X(x) = P(X \leq x)$.



The CDF for a discrete random variable



The CDF for acontinuous random variable

Properties of CDF are

(i) $0 \leq F_X(x) \leq 1$

(ii) $F_X(x)$is non-decreasing

(iii) $F_X(-\infty) = 0$

$F_X(\infty) = 1$

(iv) $P(a < X \leq b) = F_X(b) - F_X(a)$

Probability Density Function (PDF)

A continuous Random Variable may take on any value within a certain range of the real line, instead of being to a countable number of distinct points.

More common description of continuous Random Variable is its probability density function.

Probability density function is defined as the derivative of cumulative distribution function denoted by

$$f_X(x) = \frac{d}{dx} F_X(x)$$

Properties of PDF are

(i) $f_X(x) \geq 0$ for all x

i.e. probability density function is non zero for all values of x.

$$\int_{-\infty}^{\infty} f_x(x)\, dx = 1$$

(ii) The area under the probability density function is equal to 1

$$\text{i.e.},\, f_x(x) = \frac{d}{d_x} F_X(x)$$

Integrating on both sides

$$\int_{-\infty}^{\infty} f_x(x)\, dx = \int_{-\infty}^{\infty} \frac{d}{d_x} F_X(x)$$

$$= [F_X(x)]_{-\infty}^{\infty}$$

$$= F_X(\infty) - F_X(-\infty)$$

$$= 1 - 0$$

$$\int_{-\infty}^{\infty} P_x(x)\, dx = 1 \text{ Hence proved}$$

(iii) CDF is obtained by integrating PDF

$F_X(x) = \int_{-\infty}^{\infty} f(x) dx$(or)

$$P(a < x \leq b) = F_X(b) - F_X(a) = \int_a^b f_x(x) dx$$

****

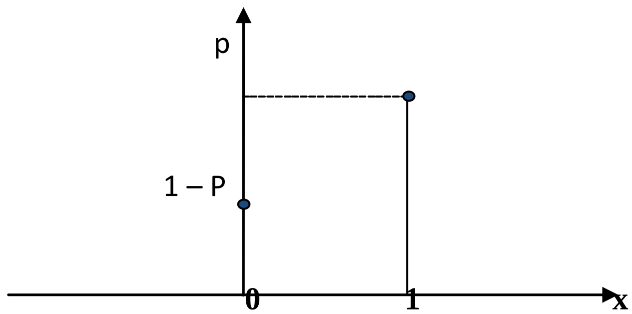
3.1.2 Important Random Variables

In communication, the most commonly used random variables are the following;

Bernoulli random variable:

- This is discrete random variable taking two values, 1 and 0, with probabilities p and 1 – p.
- A Bernoulli random variable is a good model for a binary-data generator.

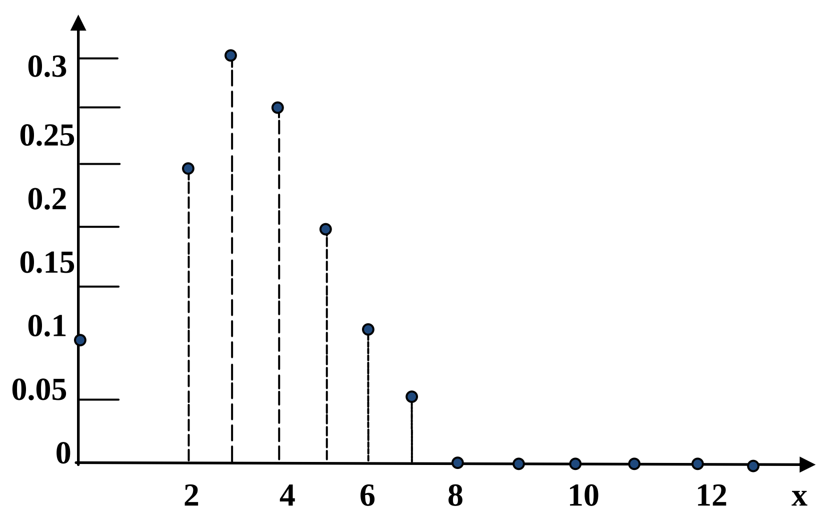
- When binary data is transmitted over a communication channel, some bits are received in error.
- We can model an error by modulo – 2 addition of a 1 to the input bit; thus, we change a 0 into a 1 and a 1 into a 0.
- Therefore a Bernoulli random variable can be employed to model the channel errors.
- This random variable models, for example, the total number of bits received is error when a sequence of a bits is transmitted over a channel with a bit-error probability of P.



The PMF for the Bernoulli random variable

Binomial random variable: This is discrete random variable giving the number of 1's in a sequence of n-independent Bernoulli trials. The PMF is given by

$$P(X = k) = \begin{cases} \binom{n}{k} P^k (1 - P)^{n-k}, 0 \leq k \leq n \\ 0, \qquad\qquad\qquad \text{otherwise} \end{cases}$$



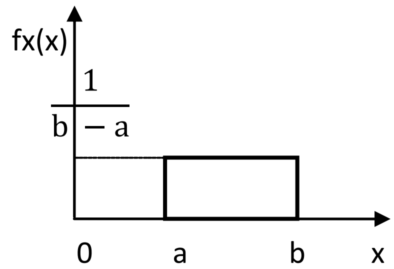
The PMF for the binomial random variable

Uniform random variable:

This is a continuous random variable taking values between a and b with equal probabilities for intervals of equal length.

The density function is given by

$$f_x(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$



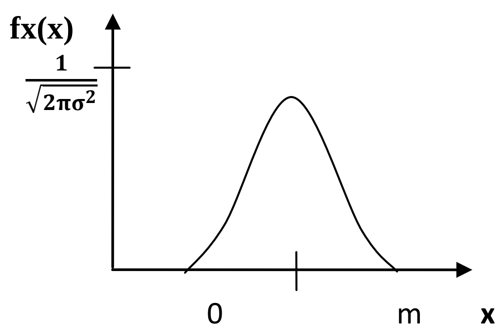
PDF of Uniform Random Variable

Normal or Gaussian random variable

This is a continuous random variable defined by density function

$$f(x) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-m)^2}{2\sigma^2}} , -\infty < x < \infty$$

mean -m ,Variance -σ^2



PDF of Gaussian Random Variable

- Important in communication
- Thermal noise is the major source of noise in communication systems has a Gaussian distribution.
- Also it is involved in central limit theorem.

****

3.2 Central Limit Theorem

2. State and Explain Central Limit Theorem. **[Apr - 2019]**

Central Limit Theorem

As Poisson and binomial distribution approaches the Gaussian distribution in the limiting case.

This suggests that the sum of independentidentically distributed random variable also approaches normal distribution.

Statement of Theorem:

It states that the probability density of sum of N independent identically distributed (iid)random variables tends to approach the normal density as the number N increases.

The mean and variance of normal densities are the sums of mean and variance of N independent random variables

If X1, X2 - - - - - - - - Xn be a sequence of independent identically distributed random variable with mean. E(xi) = μ& variance Var(xi) = σ^2 , where i = 1, 2 - - - - - - - - and

if S_N = X1 + X2 - - - - - Xn, then under certain conditions, S_N follows a normal distribution with mean nμ& variance = n σ^2 as n tends to ∞.

The Central Limit Theorem states that probability distribution of S_N approaches the normalized Gaussian distribution N(0,1) in the limit as the number of random variables , N approaches ∞.

According to Central Limit Theorem, the instantaneous value of noise gives normal distribution.

Applications of CLT:

- In signal processing, communication channel modeling, Random processes.
- In communication and signal processing, Gaussian noise is the most frequently used model for noise.
- The noise components in the channel is justified by Central limit theorem.

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3.3 Random Process

Random Process

A random process is an extension of random variable.
A random variable as a function of time is called a random process.
Depending upon the random variable 'X ' and time function 't' ,there are 4 types

They are:

i. **Continuous Random process:** Random variable 'X' & time function 't' are both continuous.

   e.g. maximum temperature at a place in the time interval(0,t)

ii. **Continuous Random sequence:** : Random variable 'X' is continuous and time function 't' ' is discrete.

e.g. Temperature at the end of a nth hour of a day. Since the temperature can take any value, it is continuous

iii. **Discrete Random Process:** Random variable 'X' is discrete and time function 't' is continuous.

e.g. Number of calls in a PCO in the time interval (0,t)

iv. **Discrete Random Sequence:** Random variable 'X' and time function 't' are both discrete.

e.g. Number of incoming calls at the nth hour of a day.

Random process can also be classified as

Deterministic Process

A process is called deterministic process if the future values of any sample function can be predicted exactly from past values.

Non Deterministic Process

A process is called Non – deterministic process if the future values of any sample function cannot be predicted exactly from past values.

3. Define Random process. Explain the various types of Random process.

List the different types of random process and give the definition. May 2012

When is a random process said to be Strict Sense Stationary (SSS), Wide Sense Stationary (WSS) and Ergodic process? Dec2011/May 2016/Nov 2016

Definition:

Random process is a collection of random variables x(s,t) that are the functions of a real variable n time t.

S $\rightarrow$ Sample space

t$\rightarrow$ time

E.g. Daily stock price, signal received by a mobile phone over time.

Classification of Random Process

i. Stationary Random Process
ii. SSS Process
iii. WSS Process
iv. Ergodic Process
v. Multiple Random Process
vi. Band pass Random Process

i. **Stationary Random Process:**

If the averages of the random variables does not depend on time, then it is known as stationary Random Process.

The statistical property of a process doesn't change with time is called Stationary Process.

First order stationary random process

If the first order density function does not change with the change in time , then it is first order stationary Random Process.

f (x,t) = f(x,t + Δ) for all t and any real no Δ.

E[x(t)] = constant

Note: The Random Process that are not stationary are called as **evolutionary process**.\

Second order stationary random process

If the second order joint density function depends only on the time difference i.e., $t_2 - t_1$ and not on the individual times t_1 and t_2.

Moreover second order stationary process have second order statistics that are invariant to a time shift of the process.

Example:

$$R_X(t_1, t_2) = R_X(t_1 - t_1)$$

nth order stationary process

Extending to higher order joint density function, a process is stationary of order n, if both the process, X(t) and X(t+τ) have the same nth order joint density function.

Note: [If E[x(t)] ≠ constant then it is not a first order Stationary Random Process].

ii. **Strictly sense stationary process (SSS Process)**

A Random Process is said to be SSS Process, if its statistical properties are independent of time.

In order words, a shift in the time origin does not change the statistical properties of the process.

If a random process is stationary to all order then it is said to be SSS process

E[x(t)] = E[x(t + τ)]

iii. **WSS Process [Wide $_S$ense Stationary Process]**

If mean is constant& auto correlation depends only on the time differentτ, (τ = t1 – t2), then it is a WSS process.

E[x(t)] = constant

E[x(t) x(t +τ)] = $R_{xx}(\tau)$

Putting τ = 0, we get $E[x^2(t)] = R_{xx}(0)$

All the strict sense stationary random processes are wide sense stationary process provided that the mean and autocorrelation function exist.

But the converse is not true i.e. A WSS process does not necessarily need to be stationary in strict sense.

A process which is not WSS is referred to as Non stationary.

iv. **Ergodic Process:**

For a stationary process, if the ensemble average is equal to the time average then the process is known as ergodic process.

The **ensemble average** of a random process X(t) are averages **"across the process".**

Eg:- The mean of a random process X(t) at some fixed times t_k is the expectation of the random variable $X(t_k)$

The **time average** of a random process X(t) are averages **"along the process".**

Multiple Random process

Two random process are jointly WSS if each of it is a WSS process and their cross correlation depends on the time difference.

$$R_{xy}(t, t+\tau) = E[x(t)x(t+\tau)] = R_{xy}(\tau)$$

Band pass Random process

A band pass (or) band limited random process x(t) can be expressed in terms of the in phase and quadrature phase components.

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3.4 Mean, correlation and Covariance Function [Nov 2018]

3.4.1 Characteristics of Random Variable

- Mean
- Variance
- Standard deviation

Mean :

- It indicates average value or expected value E(X).
- The mean or average of any random variable is expressed by the sum of the random variables weighted by its probabilities

Mean of discrete random variable

Let random variable [X] = {x1 x2, x3 ….. xj}

$$\text{Mean} = \mu_x = E(X) = \sum_{j=1}^{m} x_j f(x_j)$$

$$E(X^2) = \sum_{j=1}^{m} x_j^2 f(x_j)$$

Where **f(xj) -**probability mass function

Mean of Continuous random variable

Let X be a continuous random variable, then expectation or mean is defined as

$$\mu x = E(X) = \int_{-\infty}^{\infty} xf(x)\, dx$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

Where f(x) - probability density function (pdf)

Standard deviation:

- Standard deviation of a random variable is the **measure of the probability density function**.
- The larger the value of **σ**, wider the pdf.
- Standard deviation of a random variable x is defined as the positive **square root of variance**.

Standard deviation = $\sigma_x = \sqrt{\text{Variance}(x)}$

Importance of SD:

- It indicates the deviation in the value of the random variable from the mean value.
- For larger values the deviation is more.

Variance:

- It indicates how widely the values of random variables spread.
- It is a non – negative value.

$$\mathbf{Var}(\mathbf{x}) = \sigma^2 = E[x^2] - [E(x)]^2$$

= mean square – square of the mean

E[x] – mean

3.4.2 Correlation

Correlation is measuring the **similarity between amplitude of random Process** at two different instant of time.

4. Define auto correlation function.Discuss the properties of auto correlation

Give the properties of autocorrelation function **[April 2018]** **Dec2011/Dec2012**

Autocorrelation$\mathbf{R_{xx}(\tau)}$

Definition:

The autocorrelation function of the process X(t) is the expectation of the product of two random variable $X(t_1)$ & $X(t_2)$ obtained by observing the process X(t) at times t_1 & t_2.

$$R_x(t_1, t_2) = E[X(t_1).X(t_2)]$$

$$R_x(\tau) = E[X(t+\tau)X(t)]$$

Properties of Autocorrelation function:

The autocorrelation function of a stationary process X(t) is

$$R_x(\tau) = E[X(t+\tau)X(t).] \qquad \rightarrow (1)$$

Property1:

The mean square value of the random process may be obtained by putting $\tau = 0$ in eqn(1)

$$R_x(0) = E[X(t).X(t+\tau)]$$
$$= E[X^2(t)]$$

Property2:

The autocorrelation function $R_x(\tau)$ is an even function of τ

$R_x(\tau) = R_x(-\tau)$

This property can be defined directly from eqn (1)

The autocorrelation function

$R_x(\tau) = E[X(t).X(t-\tau)]$

Property3:

The autocorrelation function $R_x(\tau)$ has its maximum magnitude at $\tau = 0. |R_x(\tau)| \leq R_x(0)$
To prove this property consider the non – negative property
$E[x(t+\tau) \pm X(t)]^2 \geq 0$
Expanding and taking individual expectations
$E[X^2(t+\tau) \pm 2E[X(t+\tau)X(t)] + E[X^2(t)] \geq 0$

$E[X^2(t+\tau) + E[X^2(t)] \pm 2E[X(t+\tau)X(t)] \geq 0$

We know that ,

$R_x(\tau) = E[X(t+\tau)X(t)]$

$R_x(0) = E[X^2(t)] = E[X^2(t+\tau)]$

$R_x(0) + R_x(0) \pm 2R_x(\tau) \geq 0$

$2R_x(0) \pm 2R_x(\tau) \geq 0$

Equivalently,

$$-R_x(0) \leq R_x(\tau) \leq R_x(0)$$
$$\therefore |R_x(\tau)| \leq R_x(0)$$

Importance of auto correlation function

- It describes the interdependence of two random variables obtained by observing a random process X(t) at times τ seconds apart.

- If random process X(t) changes more rapidly with time then $R_x(\tau)$decreases from its maximum $R_x(0)$ as τ increases

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3.4.3 Covariance

Auto covariance x(t_1). x(t_2)

Cxx(t_1,t_2) is defined as the **covariance between the two time samples**

Cxx(t1,t2) = E[x(t1) - μ(t1)] (x(t2) - μ(t2)]

Auto Covariance Function

$C_X(t_1, t_2)$is defined as the covariance between the two time samples X(t_1) and X(t_2).

$C_X(t_1, t_2) = E[\{X_{t1} - m_x(t_1)\}\{X_{t2} - m_x(t_2)\}]$

$= R_x(t_1, t_2) - m_x(t_1)m_x(t_2)$

Variance of X(t) can be obtained as

$$\mathrm{Var}(X(t)) = E[\{X^2(t)] - \{m_x(t)\}^2]$$

$$= C_X(t, t)$$

$\mathrm{Var}(X(t))$is a function of time and is non – negative.

Cross Correlation Functions

Consider two random process X(t) and Y(t) with autocorrelation functions R_x(t,u) and R_y(t,u) respectively.

The two cross-correlation functions of X(t) and Y(t) can be defined as

$$R_{XY}(t, u) = E[X(t)Y(u)]$$

and

$$R_{YX}(t, u) = E[Y(t)X(u)]$$

Where t and u denotes two valuesof time at which the processes are observed.
Correlation properties of two random process X(t) and Y(t) can be expressed in matrix form as

$$R(t, u) = \begin{bmatrix} R_x(t, u) & R_{xy}(t, u) \\ R_{yx}(t, u) & R_y(t, u) \end{bmatrix}$$

$R(t, u)$ is called the correlation matrix of the random process X(t) and Y(t).
IF the random process X(t) and Y(t) are jointly stationary then the matrix becomes

$$R(\tau) = \begin{bmatrix} R_x(\tau) & R_{xy}(\tau) \\ R_{yx}(\tau) & R_y(\tau) \end{bmatrix}$$

where τ = t – u

The cross correlation function is not an even function and does not have a maximum at the origin
Relationship of cross correlation function is

$$R_{xy}(\tau) = R_{yx}(-\tau)$$

3.5 Power Spectral Density (PSD)

- PSD describes how average power is distributed across the frequency.
- PSD is the statistical description of random signal in frequency domain.

Let X(t) be a stationary process(SSS or WSS) with auto correlation function Rxx(τ), then the fourier transform of Rxx(τ) is called the Power Spectral Density .

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(f)\,df \qquad (\text{or})\ P = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\,d\omega$$

S(f) -Power spectral density

Average power is obtained by multiplying PSD with BW and integrating over the entire bandwidth
S(f) is the average power per unit BW and is known as PSD.

Relationship between input PSD and output PSD:

$$\xrightarrow{\mathbf{S_i(f)}} \boxed{\mathbf{H(f)(or)\,H(\omega)}} \xrightarrow{\mathbf{S_0(f)}}$$

$H(f)$ - transfer function of the LTI system

$$\boxed{\mathbf{S_0(f) = |H(f)|^2 S_i(f)}}$$

Weiner – Khintchine Relations

The power spectral density and autocorrelation function $R_x(\tau)$ of stationary random process x(t) forma Fourier transform pair.

Power spectral density

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau} d\tau = FT[R_x(\tau)] \qquad \rightarrow (1)$$

Autocorrelation

$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(f)\, e^{\,j2\pi ft} df = IFT[S_x(f)] \qquad \rightarrow (2)$$

These two relation together called as wiener – khintchine relations. This relation shows that if either the autocorrelation function or PSD of a random process is known, the other can be found exactly.

5. Define Power spectral density. Explain the properties of PSD. **[Apr - 2019]**

Power Spectral Density

If {X(t)} is a stationary process (either in strict sense or wide sense) with autocorrelation function $R_{xx}(\tau)$, then the Fourier transform of $R_{xx}(\tau)$ is called the power spectral density function of {X(t)}.

It is denoted as $S_{xx}(\omega)$ or $S(\omega)$ or $S_x(\omega)$

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau$$

Where $\omega = 2\pi f$

If ω is replaced with $2\pi f$ i.e. frequency variable then the power spectral density function will be a function of 'f.

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$$

This equation is referred to as Wiener – Khintchine relation

Inverse Fourier transform of $S_{xx}(\omega)$.

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega$$

(or)

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(f) e^{j2\pi f\tau} df$$

Properties of Power Spectral Density Function

Property 1

Statement: The zero frequency response of the power spectral density function of a stationary process is equal to the total area under the graph of the autocorrelation function.

Proof

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$$

Substitute f = 0, therefore $S_{xx}(f)$ becomes

$$S_{xx}(0) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{0} d\tau$$

We know that $e^0 = 1$

$$S_{xx}(0) = \int_{-\infty}^{\infty} R_{xx}(\tau) d\tau$$

Property 2

Statement: The power spectral density function of a real valued random process is an even function of frequency

. i.e. $S_{xx}(-f) = S_{xx}(f)$ if {X(t)} is real

Proof

We know that

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}d\tau$$

$S_{xx}(-f)$ is obtained by substituting –f for f

$$S_{xx}(-f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{\,j2\pi f\tau}d\tau$$

Next substitute -τ for τ, and $R_{xx}(-\tau) = R_{xx}(\tau)$

$R_{xx}(\tau)$ is an even function of τ

$$S_{xx}(-f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-\,j2\pi f\tau}d\tau$$

$$S_{xx}(-f) = S_{xx}(f)$$

Property 3

Statement: The mean square value of a stationary process equals the total area under the graph of spectral density

Proof

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(f)e^{j2\pi f\tau}df$$

$$E[X^2(t)]=\int_{-\infty}^{\infty} S_{xx}(f)\, df$$

This property is obtained by putting $\tau = 0$ and $Rxx(0)=E[X^2(t)]$

Property 4

Statement: The power spectral density of a stationary process is always non negative.

i.e., $S_{xx}(f) \geq 0$ for all f.

This property follows that the mean square value $E[X^2(t)]$ must always be non negative.

Property 5

Statement: The power spectral density, appropriately normalized has the propertiesusually associated with a probability density function.

The normalization is with respect to the total area under the graph of PSD (i.e. mean square value of the process). Consider the function

$$P_{xx}(f) = \frac{S_{xx}(f)}{\int_{-8}^{\infty} S_{xx}(f).\,df}$$

$P_{xx}(f) \geq 0$ for all f .The total area under the curve is$P_{xx}(f)$is unity .Hence the normalized form of power spectral density behaves similarto a probability density function.

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3.6 Ergodic Process

Definition:

For a stationary process, if the ensemble average is equal to the time average then the process is known as ergodic process.

The **ensemble average** of a random process X(t) are averages **"across the process".**

Eg:- The mean of a random process X(t) at some fixed times t_k is the expectation of the random variable $X(t_k)$

The **time average** of a random process X(t) are averages **"along the process".**

Consider the sample function x(t) of a stationary process X(t) with the observation interval $-T \leq t \leq T$. The DC value of x(t) is defined by the time average.

$$\mu_x(T) = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

Since the process x(t) is assumed to be stationary, the mean of time average $\mu_x(T)$ is given by

$$E[\mu_x(T)] = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

$$\frac{1}{2T}\int_{-T}^{T} \mu_x\,dt$$

$$= \frac{\mu_x}{2T}[t]_{-T}^{T}$$

$$= \frac{\mu_x}{2T}[2T]$$

$$\boxed{E[\mu_x(T)] = \mu_x}$$

Where μ_x is the mean of the process X(t).

Conditions for the random process X(t) to be Ergodic in mean.

(i) The time average $\mu_x(T)$ approaches the ensemble average μ_x when observation interval T approaches infinity

$$\underset{T \to \infty}{\mathrm{Lt}}\, \mu_x(T) = \mu_x$$

(ii) The variance of $\mu_x(T)$ approaches zero when the observation interval T approaches infinity.

$$\underset{T \to \infty}{\mathrm{Lt}}\, \mathrm{Var}[\mu_x(T)] = 0$$

Time average autocorrelation function:

$$R_x(\tau, T) = \frac{1}{2T} \int_{-T}^{T} x(t+\tau)\, x(t) dt$$

Conditions for the random process X(t) to be ergodic in autocorrelation

$$\underset{T \to \infty}{\text{Lt}} R_x(\tau, T) = R_x(\tau)$$

$$\underset{T \to \infty}{\text{Lt}} \text{Var}[R_x(\tau, T)] = 0$$

3.6 Gaussian Process

6. **Define Gaussian process. Explain the properties of Gaussian process.** **Nov 2016**

State and prove four properties of Gaussian process. **[April 2018]** **May2011**

Gaussian Process

Consider a Random Process x(t) for the interval t = 0 to t = T. Let us weight the random process x(t) by some function g(t) and then integrate the product g(t) x(t) over this observation interval to obtain a Random Variable y defined by

$$\mathbf{Y = \int_0^T g(t)\, x(t) dt}$$

Where y $\rightarrow$ linear functional of x(t)

The weighting function g(t) is such that the mean square value of the random variable Y is finite and if the random variable Y is a Gaussian distributed random variable for every g(t), then x(t) is a Gaussian process.

[The process x(t) is a Gaussian process if every linear function of x(t) is a Gaussian random variable.]

Random variable Y has a Gaussian distribution if its probability density function

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma y^2}}\, e^{\frac{-(y-\mu)^2}{2\sigma y^2}}$$

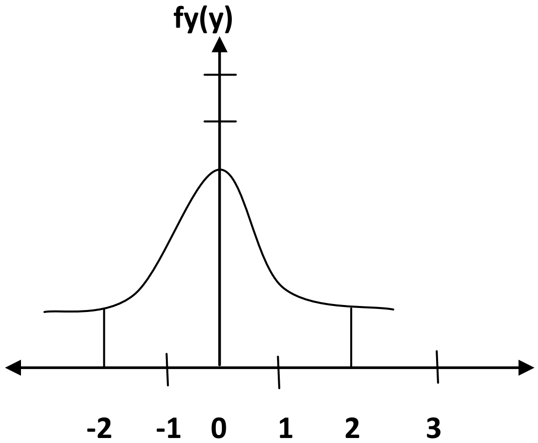
where$\mu_y \rightarrow$ mean

$\sigma_y{}^2 \rightarrow$ variance

Assume the random variable is normalized, to have mean$(\mu) = 0$ *and* $\sigma y^2 = 1$

$$f_y(y) = \frac{1}{\sqrt{2\pi}} e^{-(y^2/2)}$$

Such a normalized Gaussian distribution is commonly written as N(0,1). The Pdf of normalized Gaussian distribution is shown below.



Properties of Gaussian process

Property1:

If a Gaussian process x(t) is applied to a stable linear filter, then the random process y(t)developed at the output of the filter is also Gaussian.

Proof

Let us consider a LTI system

with h(t) $\rightarrow$ impulse response

x(t) $\rightarrow$ input

y(t) $\rightarrow$output

Assume x(t) is a Gaussian

x(t) &y(t) are related by the convolution integral

$$Y(t) = \int_{0}^{T} h(t-\tau)\, X(\tau) d\tau \qquad 0 \le t < \infty$$

To prove that Y(t) is Gaussian, we must show that any linear functional of it is a Gaussian random variable. We define a random variable 'z' as

$$Z = \int_{0}^{\infty} g(t) \int_{0}^{T} X(\tau)\, h(t-\tau) d\tau dt$$

Z must be a Gaussian random variable for every function of g(t) such that the mean square value of Z is finite.

Interchanging the order of integration

$$Z = \int_{0}^{T} X(\tau) \int_{-\infty}^{\infty} g(t)\, h(t-\tau) dt d\tau$$

$$= \int_{0}^{T} g(\tau)X(\tau)\,dt$$

where

$$g(\tau) = \int_{-\infty}^{\infty} g(t)h(t-\tau)d\tau$$

Since X(t) is the Gaussian process, 'Z' must be a Gaussian random variable.

We have thus shown that the input X(t) to a linear filter is a Gaussian process then the output Y(t) is also a Gaussian process.

Property2:

Consider a set of random variables (or samples) $X(t_1)$, $X(t_2)$, - - - - - - - - - - X(tn) obtained by observing the random process X(t) at times t_1, t_2, - - - - - - - - - - t_n

If the process X(t) is Gaussian for any 'n', then 'n' fold joint pdf is determined by

i. Set of means $\mu_{X\,(ti)} = E[x(ti)]$, I = 1,2, - - - - - - - - n

ii. Set of covariance functions

$Cx\,(t_k, t_i) = E[(X(t_k) - \mu_{x(tk)})\,(X(t_i) - \mu_{x(ti)}]$, k =1, 2, - - - - - - - - - n.

Property 3:

If a Gaussian process is stationary, then the process is also strictly stationary.

Property4:

If the random variables, X**(t_1), X(t_2), - - - - - - - - - - X(t_n)** obtained by sampling a Gaussian process X(t) at times **t_1, t_2, - - - - - - - - - - t_n** are uncorrelated

i.e. $E[(x(t_k) - \mu_{x(tk)})\,(x(t_i) - \mu_{x(ti)})] = 0$, $i \neq k$, then these random variables are statistically independent.

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3.7Transmission of a random process through a linear time invariant (LTI) filter.

7. Explain in detail about the transmission of a random process through a linear time invariant filter. **May 2016/Nov 2016**

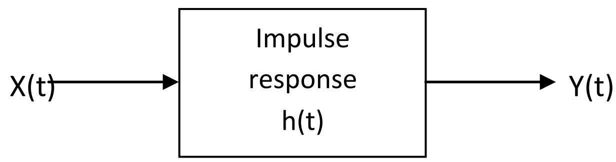
Derive the input and output relationship of a random process applied through a LTI filter.

[Nov 2018] Dec2017

Transmission of Random Process through a linear time Invariant filter

Suppose that a random process X(t) is applied as an input to a linear time invariant filter of impulse response h(t), producing a new random process Y(t) at the filter output .

Assume X(t) is a stationary process



Transmission of random process though a linear-time invariant filter

The output random process Y(t) in terms of the input random process X(t) is given by convolution integral

$$Y(t) = \int_{-\infty}^{\infty} h(\tau_1)\, X(t-\tau_1) d\tau_1$$

whereτ_1 $-$ integration variable

The mean of Y(t) is

$$\mu_Y(t) = E[Y(t)]$$
$$= E\left[\int_{-\infty}^{\infty} h(\tau_1)\, X(t-\tau_1) d\tau_1\right]$$

If $E[X(t)]$is finite for all t and the system is stable, we may interchange the order of expectation and integration

$$= \int_{-\infty}^{\infty} h(\tau_1)\, E[X(t-\tau_1)] d\tau_1$$

$$= \int_{-\infty}^{\infty} h(\tau_1)\, \mu_x(t-\tau_1) d\tau_1$$

When the input random process X(t) is stationary process, the mean $\boldsymbol{\mu_x(t)}$ is a constant $\boldsymbol{\mu_x}$**,** so we may simplify the equation as

$$\mu_y = \mu_x \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1$$

$$= \mu_x H(0)$$

Where H(0) is the zero frequency (DC) response of the system.

It states that the mean of the random process Y(t) produced at the output of an LTI system in response to X(t) , is equal to the mean of X(t) multiplied by the DC response of the system.

Consider the autocorrelation function of the output Random Process Y(t).

$$R_Y(t,u) = E[Y(t)Y(u)]$$

where t and u denote two values of the time at which the output process is observed.

$$\therefore R_Y(t,u) = E\left[\int_{-\infty}^{\infty} h(\tau_1)\, X(t-\tau_1)d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\, X(u-\tau_1)d\tau_2\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h\,(\tau_2)E[X(t-\tau_1)X(u-\tau_2)]d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h\,(\tau_2)R_x\,(t-\tau_1-u+\tau_2)d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h\,(\tau_2)R_x\,(t-\tau_1+\tau_2)d\tau_1 d\tau_2$$

where τ = t – u

Thus if the input to a stable LTI system is stationary then the output is also a stationary process.

Since $R_y(t) = E[Y^2(t)]$, it follows that the mean square values of output process Y(t) is obtained by putting τ = 0

$$E[Y^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h\,(\tau_2)(t_2-\tau_1)d\tau_1 d\tau_2$$

which is a constant

****

Solved problems

Sample problem 1:

If $X(t) = \cos(\omega t + \theta)$, θ is a random variable with probability density function $f(\theta) = \frac{1}{2\pi}$. Check whether the random process is stationary or not if - π<θ<π.

Given:

$$X(t) = \cos(\omega t + \theta)$$

$f(\theta) = \frac{1}{2\pi}$,- π<θ<π.

Solution:

$$E[x(t)] = \int_{-\infty}^{\infty} xf(t)\ dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(\omega t + \theta)\, d\theta$$

$$= \frac{1}{2\pi}[\sin(\omega t + \theta)]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}[\sin(\omega t + \pi) - \sin(\omega t + \pi)]$$

$$= \frac{1}{2\pi}[\sin \omega t - \sin \omega t]$$

$E[X(t)] = 0$ (Constant)

$\therefore$ It is a stationary process

Sample problem 2:

Given a random process X(t) = A cos(ωt + θ) where A, ω are constants and θ is a uniform random variable . Show that X(t) is ergodic in both mean and auto correlation. May 2010/May 2016

(or)

If X(t) = A cos(ωt + θ) where ϕ is uniform distributed in the interval $(-\pi, \pi)$. Find whether it is an ergodic process or not. [Apr -2019]

Given:

$X(t) = A\ \cos(\omega t + \phi)$

ϕis uniform distributed in the interval $(-\pi, \pi)$

Solution:

We know that

Uniform distribution function

$$f(x) = \frac{1}{b-a} \qquad a = -\pi \ and\ b = \pi$$

$$f(x) = \frac{1}{2\pi}$$

Ensemble Average

$$E[X(t)] = \frac{1}{2\pi}\int_{0}^{2\pi} A\ \cos(\omega t + \ \phi)\, d\phi$$

$$= \frac{1}{2\pi}\int_{0}^{2\pi} A\ \cos(\omega t + \ \phi)\, d\phi$$

$$= \frac{A}{2\pi}\int_{0}^{2\pi} A\ \cos(\omega t + \ \phi)\, d\phi$$

$$= \frac{A}{2\pi}\int_{0}^{2\pi} [\sin(\omega t + \ \phi)]_{0}^{2\pi}$$

$$= \frac{A}{2\pi}[\sin(\omega t + \ 2\pi) - \sin(\omega t + \ 0)]$$

$$= \frac{A}{2\pi}[\sin \omega t - \sin \omega t]$$

$E[X(t)] = 0$ (constant)

The process is stationary

Time average

$$\lim_{T\to\infty} \mu_x(T) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A\ \cos(\omega t + \ \phi)\, dt$$

$$= \lim_{T\to\infty} \frac{A}{2T}\left[\frac{\sin(\omega t + \ \phi)}{\omega}\right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{A}{2T}\left[\frac{\sin(\omega T + \ \phi)}{\omega} - \frac{\sin(\omega T + \ \phi)}{\omega}\right]$$

$$= \lim_{T\to\infty} \frac{A}{2\omega T}[\sin(\omega T + \ \phi)]$$

= 0

Time averages = Ensemble averages.

∴ The given process is Ergodic process

To prove $x(t)$ is correlation ergodic

$$Lt_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}x(t)\ x(t+\tau)dt = R_{xx}(\tau)$$

Ensemble Average

$$R_{xx}(\tau) = E[x(t+\tau)x(t)]$$

$$= E[A\ \cos\omega[(t+\tau)+\theta]\,A\cos(\omega t+\theta)]$$

$$= \frac{A^2}{2}E[\cos(\omega t+\ \omega\tau+\theta)\cos(\omega t+\ \theta)]$$

$$= \frac{A^2}{2}\ E[\cos(2\omega t+\ \omega\tau+2\theta)+\cos\omega\tau]$$

$$= \frac{A^2}{2}\{E[\cos(2\omega t+\ \omega\tau+2\theta)+E\cos(\omega\tau)]$$

$$= \frac{A^2}{2}\left\{E\left[\cos(2\omega t+\ \omega\tau+2\theta)+\frac{A^2}{2}\ \cos(\omega\tau)\right]\right.$$

$$E[\cos(2\omega t+\ \omega\tau+2\theta)] = \frac{1}{2\Pi}\int_{0}^{2\Pi}\cos(2\omega t+\ \omega\tau+2\theta)\,d\theta = \frac{1}{2\Pi}\left[\frac{\sin(2\omega t+\ \omega\tau+2\theta)}{2}\right]_0^{2\Pi}$$

$$= \frac{1}{4\Pi}[\sin(2\omega t+\ \omega\tau+4\Pi)-\sin(2\omega t+\ \omega\tau)]$$

$$= \frac{1}{4\Pi}[\sin(2\omega t+\ \omega\tau)]-[\sin(2\omega t+\ \omega\tau)]$$

= 0

sub in eqn (1)

$R_{xx}(\tau) = \frac{A^2}{2}(0) + \frac{A^2}{2}\cos\omega\tau = \frac{A^2}{2}\cos\omega\tau$____________(I)

Time average

$$= \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-T}^{T}x(t)\ x(t+\tau)dt$$

$$= \underset{T\to\infty}{Lt}\frac{1}{2T}\int_{-T}^{T}A\cos(\omega t+\ \omega\tau+\theta)\,A\cos(\omega t+\ \theta)dt$$

$$= \underset{T\to\infty}{Lt}\frac{A^2}{2T}\int_{-T}^{T}[\cos(2\omega t+\ \omega\tau+\theta)+\cos\omega\tau]\,dt$$

$$= \underset{T\to\infty}{Lt}\frac{A^2}{4T}\int_{-T}^{T}\cos(2\omega t+\ \omega\tau+\theta)\,dt+\underset{T\to\infty}{Lt}\frac{A^2}{4T}\int_{-T}^{T}\cos\omega\tau dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T}\left[\frac{\sin(2\omega t + \omega\tau + 2\theta)}{2\omega}\right]_{-T}^{T} + \underset{T\to\infty}{Lt} \frac{A^2}{4T}\cos\omega\tau[t]_{-T}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4T2\omega}[(\sin 2\omega T + \omega\tau + 2\theta) - \sin(-2\omega T + \omega\tau + 2\theta)] + \underset{T\to\infty}{Lt} \frac{A^2}{4T}\cos\omega\tau\,[2T]$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{8\omega T}\left\{2\cos\left[\frac{2(\omega\tau + 2\theta)}{2}\right]\sin\left(\frac{4\omega T}{2}\right)\right] + \frac{A^2}{2}\cos\omega\tau\right\}$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{4\omega T}\cos\left[\frac{2(\omega\tau + 2\theta)}{2}\right]\sin 2\omega T + \frac{A^2}{2}\cos\omega\tau$$

$$= \frac{A^2}{4}\cos\left[\frac{2(\omega\tau + 2\theta)}{2}\right]\underset{T\to\infty}{Lt}\sin\frac{2\omega T}{\omega T} + \frac{A^2}{2}\cos\omega\tau$$

$= 0 + \frac{A^2}{2}\cos\omega\tau$ __________ (II)

From eqn (1) & (II) Ensemble average = Time average

$\therefore$x(t) is correlation ergodic

****

Sample Problem 3:

Let X and Y be real random variable with finite second moments. Prove the Cauchy Schwartz inequality$\mathbf{E\{XY\}^2 \le E[X^2]E[Y^2]}$. May 2015

Proof:

For any two random variables X and Y we have $E\{XY\}^2 \le E[X^2]E[Y^2]$ where inequality holds if and if X=aY for some constant a$\in$R

Assume $W = (X - aY)^2$ where W is a nonnegative random variable for any value of a$\in$R. Thus we obtain $E[W] \ge 0$

$$E[W] = E\{X - aY\}^2 = E[X^2 - 2aXY + X^2Y^2]$$
$$= E[X^2] - 2aE[XY] + a^2E[Y^2]$$

Let f(a) = 0 for some a, then we have $E[W] = E(X - aY) = 0$ which means $X = aY$ with probability one.

To prove Cauchy – Schwartz inequality, choose $a = E[XY]E[Y^2]$

$$E[X^2] - 2aE[XY] + a^2E[Y^2] \ge 0$$
$$E[X^2] - 2E[XY]E[Y^2] + E\{XY\}^2\{E[Y^2]\}^2 \ge 0$$

$$E[X^2] - E\{XY\}^2E\{Y\}^2 \ge 0$$

$$E\{XY\}^2 \le \left[E[X^2]\right]E[Y^2]$$ Hence proved.

****

Sample Problem 4:

Let $\mathbf{x(t)}$ and $\mathbf{y(t)}$ are both zero mean and WSS random process. Consider the random process $\mathbf{Z(t) = X(t) + Y(t)}$. Determine the auto correlation and power spectrum of $\mathbf{x(t)}$ if $\mathbf{X(t)}$ and $\mathbf{Y(t)}$ are jointly WSS. May2015

Solution:

Auto correlation function of $W(t) = R_{WW}(t, t+\tau)$

$$= E[\{X(t) + Y(t)\}\{X(t+\tau) + Y(t+\tau)\}]$$

$$= E[\{X(t)\, X(t+\tau)\} + E\{Y(t)\, X(t+\tau)\}] + E\{X(t)Y(t+\tau)\}$$

$$+ \qquad E[Y(t)Y(t+\tau)]\}$$

$$R_{WW}(t, t+\tau) = R_{xx}(t, t+\tau) + R_{yx}(t, t+\tau) + R_{yy}(t, t+\tau)$$

This is the auto correlation function when X(t)and Y(t) are correlated.

$$\text{Power Specturm } S_x(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j\omega\tau} d\tau$$

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Sample Problem 5:

Let $\mathbf{X(t) = A\cos(\omega t + \phi)}$and $\mathbf{Y(t) = A\sin(\omega t + \phi)}$,where A and ω are constants and ϕ is a uniform random variable (0,2Π). Find the cross correlation of $\mathbf{x(t)}$and $\mathbf{y(t)}$. May 2015/ May 2016

Given: $X(t) = A\cos(\omega t + \phi)$and $Y(t) = A\sin(\omega t + \phi)$ ϕ is uniform random variable $(0,2\Pi)$

Solution: $R_{xy}(t, t+\tau) = E[x(t)y(t+\tau)] R_{xy}$ - is the cross correlation.

$$= A^2 E[\cos(\omega t + \theta)\sin(\omega t + \omega\tau + \theta)]$$

$$= \frac{A^2 E}{2}[2\sin(\omega t + \omega\tau + \theta)\, cos(\omega t + \theta)]$$

$$= \frac{A^2 E}{2}[\sin(2\omega t + \omega\tau + 2\theta) + \sin\omega t]$$

$$= \frac{A^2 E}{2}[\sin\omega\tau] + \frac{A^2}{2} E[\sin(2\omega t + \omega\tau + 2\theta)]$$

$$R_{xy}(t, t+\tau) = \sin\omega\tau + \frac{A^2}{4\Pi}\int_0^{2\Pi} \sin(2\omega t + \omega\tau + 2\theta)\, d\theta\tau$$

$$= \frac{A^2}{2}\sin\omega\tau + \frac{A^2}{4\Pi}(0)$$

$$R_{xy}(\tau) = \frac{A^2}{2}\sin\omega\tau$$

Exercise Problem:

1. The amplitude modulated signal is defined as X AM(t)=Am(t)Cos(ωct+θ) where m(t) is the base band signal and A Cos(ωct+θ) is the carrier. The base band signal m(t) is modeled as zero mean stationary random process with the auto correlation function Rxx(τ) and PSD Gx (f) and the . The carrier amplitude A and the frequency ωc are assumed to be constant and the initial carrier phase θ is assumed to be a random uniformly distributed in the interval (-π , π) . Furthermore, m(t) and θ are assumed to be independent.

i. Show that XAM(t) is wide sense stationary.

ii. Find PSD of XAM(t). **May 2017**

2. Consider a random process defined as X(t)=A cos ω t , where ω is a constant and A is random uniformly distributed over [0,1]. Find the autocorrelation and auto covariance of X (t).

Dec 2017

Given:

$X(t) = A\ \cos \omega t$

Φ is uniform distributed in the interval $(0,1)$

Solution:

We know that

Uniform distribution function

$f(x) = \frac{1}{b-a}$ $\quad a = 0\ and\ b = 1$

$f(x) = \frac{1}{1}$ =1

$$R_{xx}(\tau) = E[x(t+\tau)x(t)]$$

$$= E[A\ \cos\omega[(t+\tau)]\,[A\cos(\omega t)]$$

$$= E[\,A^2\ \cos(\omega t +\ \omega\tau)\ \cos\omega t]$$

$$= \int_0^1 \cos(\omega t +\ \omega\tau)\ \cos\omega t\quad A^2\, dA$$

$$= \cos(\omega t +\ \omega\tau)\ \cos\omega t\ \ \left[\frac{A^3}{3}\right]$$

$$= \left[\frac{1}{3}\right]\ \cos(\omega t +\ \omega\tau)\ \cos\omega t$$

UNIT- III

Random process

1. ***What is a Random Experiment?***
   An experiment whose outcome cannot be predicted exactly is called a random experiment.

2. ***Define Sample space.***
   All possible outcomes of a random experiment is called a sample space.

3. ***Define Sample.***
   A particular outcome of a random experiment is called a sample point or sample.

4. ***What is an Event?***
   The sub-collection of a sample space under a definite rule or law is called an event.
   It is the subset of sample space or collection of outcomes.

5. ***Define probability.***

   The probability of occurrence of an event A is defined as,

   $$P(A) = \frac{\text{number of favorable outcomes}}{\text{Total number of outcomes}}$$

   $P(A) \leq 1$

6. ***What are mutually exclusive events?***

   Two possible outcomes of an experiment are mutually exclusive if the occurrence of one outcome does not affect the occurrence of the other.

7. ***Define probability density function.***

   Probability density functions are important as they represent the uncertainty of a signal in a close form of mathematical equation. This helps in estimating the message content (or) error
   Probability density function fx(x) is defined in terms of cumulative distribution function Fx (x) as
   fx(x) = d Fx (x) / dx

8. ***State Baye's theorem.***
   State Baye's rule. **Dec 2015**
   If the events $\{E_i\}_{i=1}^{n}$ are disjoint and their union makes the sample space, then they make a partition of the sample space Ω. Then, if for an event A, we have the conditional probabilities$\{P(A|E_i)\}_{i=1}^{n}$. $P(A)$ can be obtained by applying the total probability theorem states as

   $$P(A) = \sum_{i=1}^{n} P(E_i)\, P(A|E_i)$$

   Baye's rule gives the conditional probabilities $P(E_i|A)$ by the following relation

   $$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_j)P(A|E_j)}$$

9. Define random variable. ***May 2012 /Dec 2012/Dec 2015***

Random variable is defined as a function which maps the outcome of a random experiment to a number. It is also known as stochastic variable

Random variables are denoted by upper case letter X,Y etc., Values assumed by RV are denoted by lower case letters with subscripts. x1, x2, y1, y2, etc.,]

10. What is a discrete random variable?

A Random variable that takes on a finite number of values is known as discrete random variable

11. What is a continuous random variable?

A Random Variable that takes on an infinite number of values is called continuous Random Variable

12. Define mean of a random process.

Mean of a random process is defined as the expectation of the random variable E(X) obtained by observing the process at time t.

Mean of Discrete random variable

Let X be a discrete random variable [X] = {x1 x2, x3 ….. xj} ,then

$$\text{Mean} = \mu_x = E(X) = \sum_{j=1}^{m} x_j f(x_j)$$

*Where **f(xj) -**probability mass function*

Mean of Continuous random variable

Let X be a continuous random variable, then expectation or mean is defined as

$$\mu x = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Where f(x) - probability density function (pdf)

13. What is meant by Variance of random variable?

Variance indicates how widely the values of random variables spread.

It is a non – negative value.

$$Var(x) = \sigma^2 = E(x^2) - [E(x)]^2$$

Where $E(x) = \mu = \text{mean}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

14. What is Standard deviation?

Standard deviation of a random variable is the measure of the probability density function. The larger the value of σ*, wider the pdf*

Standard deviation of a random variable x is defined as the positive square root of variance.

Standard deviation = $\boldsymbol{\sigma x = \sqrt{Var(x)}}$

15. Define autocorrelation function. ***May 2016***

Auto correlation of a random process is the expectation of product of two random variables $X(t_1)\, X(t_2)$

The random variables are obtained by observing the process X(t) at two different times t_1, t_2 respectively i.e., $R_{xx}(t) = E[X(t_1) X(t_2)]$ where $t=t_1-t_2$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f(x_1, x_2) dx_1 dx_2$$

16. What are the properties of the Auto Correlation Function?

Auto correlation function of a stationary process x(t) is defined as

$R_{xx}(\tau) = E[x(t+\tau)x(t)]$ *for all t.*

This auto correlation function has several important properties.

- *The mean – square value of the process is obtained by putting $\tau = 0$.*

$$\therefore R_{xx}(0) = E[x^2(t)]$$

- *The auto correlation function $R_{xx}(\tau)$ is an even function of τ, i.e.*

$$R_{xx}(\tau) = R_{xx}(-\tau)$$
$$R_{xx}(-\tau) = E[x(t-\tau)x(t)] = R_{xx}(\tau)$$

- *The auto correlation function $R_{xx}(\tau)$ has its maximum magnitude at $\tau = 0$, i.e.*

$$|R_{xx}(\tau)| \leq R_x(0)$$

17. State Central limit theorem.(CLT) **Dec 2008/May 2016/Dec 2016/ Dec2017**

The Central Limit Theorem states the probability distribution of S_N approaches the normalized Gaussian distribution N(0,1) in the limit as the random variable "N" approaches infinity i.e., $N \rightarrow \infty$.

According to CLT ,instantaneous value of noise will have normal distribution.

18. Give the applications of Central Limit Theorem.

The applications of CLT are

It provides simple method for computing approximate probabilities of sum of independent random variables.

In communication and signal processing, Gaussian noise is the most frequently used model for noise.

19. Define Random process. [April 2018]

A Random process X (s,t) is a function that maps each element of a sample space into a time function called sample function. Random process is a collection of time functions

E.g., *Daily stock price, signal received by a mobile phone over time*

20. What are the classification of random process? **May 2006**

The Classification of Random Process are

- *Stationary Random Process*
- *Strict Sense Stationary (SSS) Process*
- *Wide Sense Stationary (WSS) Process*
- *Ergodic Process*

- *Multiple Random Process*
- *Band pass Random Process*

21. When is a random process called deterministic? **[Nov 2018]** ***May 2010 /Dec 2011***

A random process is called deterministic, if the future values of any sample function can be predicted from past values.

22. When is a random process called non deterministic?

A random process is called deterministic, if the future values of any sample function cannot be predicted from past values.

23. What is a stationary process?

The statistical property of a random process does not change with time called stationary random process.

If the first order density function of a random process X(t) is independent of time for all t , then the process is first order stationary process.

24. What is an evolutionary process?

The Random Process that are not stationary are called as ***evolutionary process****.*

25. What is meant by Strict sense stationary process (SSS Process)? **[Apr - 2019]**

A Random Process is said to be SSS Process, if its statistical properties are independent of time.

$$E[x(t)] = E[x(t+\tau)]$$

26. What is meant by WSS Process [Wide sense Stationary Process]?

If mean is constant & auto correlation depends only on the time difference τ, ($\tau = t_1 - t_2$), then it is a WSS process.

$$E[x(t)] = \text{constant}$$

$$E[x(t)\, x(t+\tau)] = R_{xx}(\tau)$$

Putting $\tau = 0$, we get $E[x^2(t)] = R_{xx}(0)$

27. What is meant by Ergodic Process? ***Dec2017***

For a stationary process, ***if the ensemble average is equal to the time average*** *then the process is known as ergodic process.*

Ergodic in mean

$\mu_x(T) = E[x(t)]$ *E[X(t)]-Ensemble average* $\mu_x(T)$ *-Time average*

Ergodic in auto correlation

$$R_x(\tau,T) = \frac{1}{2T}\int_{-T}^{T} x(t+\tau)\, x(t)dt = E[x(t)\, x(t+\tau)] = Rxx(\tau)$$

$E[x(t)\,x(t+\tau)]$ - Ensemble average
$R_x(\tau, T)$- Time average

28. What is meant by ensemble average and time average?

Ensemble average

*The **ensemble average** of a random process X(t) are averages **"across the process".***

The Ensemble average of a Random process {X(t)} is the expected value of the random variable X at time t.

Ensemble average- E[X(t)]

Time average

*The **time average** of a random process X(t) are averages **"along the process".***

The time average of a random process {X(t)} is defined as

$$\mu_x(T) = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

29. What are the conditions for random process to be ergodic in mean and ergodic in auto correlation?

Conditions for the random process X(t) to be Ergodic in mean.

$$\underset{T\to\infty}{Lt}\ \mu_x(T) = \mu_x$$

$$\underset{T\to\infty}{Lt}\ Var[\mu_x(T)] = 0$$

Conditions for the random process X(t) to be ergodic in autocorrelation

$$\underset{T\to\infty}{Lt}\ R_x(\tau, T) = R_x(\tau)$$

$$\underset{T\to\infty}{Lt}\ Var[R_x(\tau, T)] = 0$$

30. Define Power Spectral Density.

If X(t) is a stationary process(SSS or WSS) with auto correlation function $Rxx(\tau)$ then the fourier transform of $R_{xx}(\tau)$ is called the Power Spectral Density function of X(t)

.

$$Sx(f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\Pi f\tau}\, d\tau = FT\,[Rx\,(\tau)]$$

31. State Weiner – Khintchine Relation. **Nov 2016**

The power spectral density & auto correlation function $R\,xx(\tau)$ of Stationary random process X(t) form a Fourier transform pair.

$$Sx(f) = \int_{-\infty}^{\infty} Rx(\tau)\, e^{-j2\Pi f\tau}\, d\tau = FT\,[Rx\,(\tau)]$$

$$Rx(\tau) = \int_{-\infty}^{\infty} Sx(f)\, e^{j2\Pi f\tau}\, df = IFT[\,Sx(f)]$$

32. What are the properties of Power Spectral Density ?

The properties of Power Spectral Density are

- *The value of the spectral density function at zero frequency is equal to the total area under the graph of the autocorrelation function.*
- *PSD of a real random process is an even function. i.e,* $S_{xx}(-\omega) = S_{xx}(\omega)$
- *PSD of a stationary process is always non negative.*
- *PSD approximately normalized has the properties associated with a probability density function.*

33. Give the difference between Random Process and Random Variable. **[Apr - 2019]** ***Dec2017***

S.No	**Random Process**	**Random Variable**
1	*It is a Waveform.*	*It is a set of numbers.*
2	*The outcome of a random experiment is mapped into waveform that is a function of time.*	*The outcome of a random experiment is mapped into a number.*
3	*It can be Stationary or Ergodic.*	*It may not be further classified.*
4	*Ensemble as well as time averages can be calculated.*	*Only ensemble averages can be calculated.*

34. Define Gaussian Process. ***Dec 2003***

Consider a Random Process x(t) for the interval t = 0 to t = T. Let us weight the random process x(t) by some function g(t) and then integrate the product g(t) x(t) over this observation interval to obtain a Random Variable y defined by

$$y = \int_0^T g(t)\, x(t)\, dt$$

Where y $\rightarrow$ linear function of x(t)

In other words, the weighting function g(t) is such that the mean-square value of the random variable Y is finite and if the random variable Y is an Gaussian distributed random variable for every g(t), then the process X(t) is said to be Gaussian process.

35. What are the properties of Gaussian process?

Properties of Gaussian process:

If a Gaussian process x(t) is applied to a stable linear filter, then the random process y(t)developed at the output of the filter is also Gaussian.

If a Gaussian process is stationary, then the process is also strictly stationary.

If the random variables, $X(t_1), X(t_2), ---------- X(t_n)$ obtained by sampling a Gaussian process X(t) at times $t_1, t_2, ---------- t_n$ are uncorrelate, then these random variables are statistically independent.

36. What is an LTI system?

A system which obeys both the linearity and time shifting property is called an LTI system. Such systems are characterized by its impulse response.

37. How the output of LTI system is obtained?

If the input and impulse response of a LTI system is given the output can be obtained by convolution.

38. *What is the* input and output relationship of a random process applied through a LTI filter?

The mean of the random process Y(t) produced at the output of an LTI system in response to X(t) , input process is equal to the mean of X(t) multiplied by the DC response of the system.

If the input to a stable LTI system is stationary then the output is also a stationary process.

39. Define Heterodying. [Nov 2018]

Heterodying is a signal processing technique. It is used to shift one frequency range into another and is also involved in the processess of modulation and demodulation.

40. What is narrow band noise? [April 2018]

Narrow-band noise (NBN) is a type of noise stimulus that is centered around a small range of frequencies. It is produced by filtering a 1/3 octave range from a broad-band noise stimulus.

UNIT-IV
NOISE CHARACTERIZATION

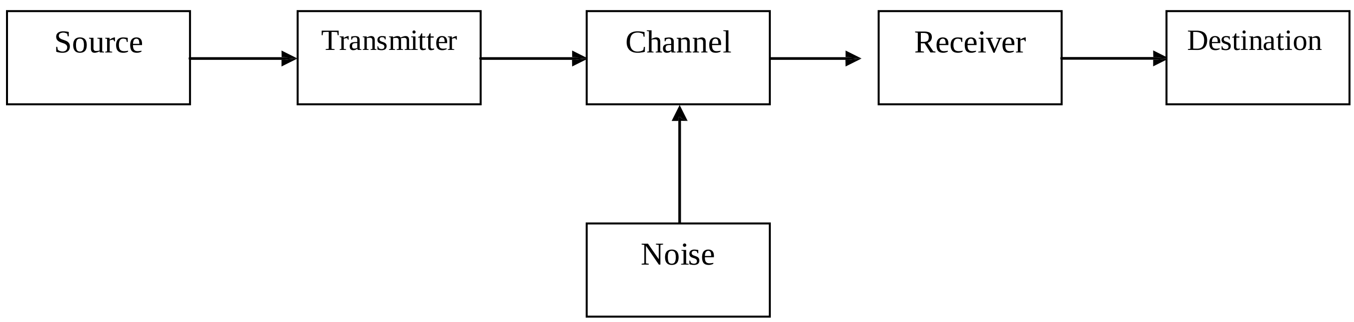
Noise sources – Noise figure, noise temperature and noise bandwidth – Noise in cascaded systems. Representation of Narrow band noise –In-phase and quadrature, Envelope and Phase – Noise performance analysis in AM & FM systems – Threshold effect, Pre-emphasis and de-emphasis for FM.

Introduction

Noise:

Noise is an unwanted signal that disturbs the transmission and processing of signals in communication systems.

Block diagram of communication system



4.1 Noise source and types

1. **What are the different types of noise? Explain.** **[Nov 2018] [April 2018]**
   Classify the different noise sources and its effect in real time scenario. **May 2017**

   Noise is an unwanted signal that disturbs the transmission and reception of signal.

Predictable Noise

- This noise can be estimated and eliminated by proper engineering design.

Examples:

Power supply hum, spurious oscillations in amplifiers, fluorescent lighting.

Random Noise (Unpredictable Noise)

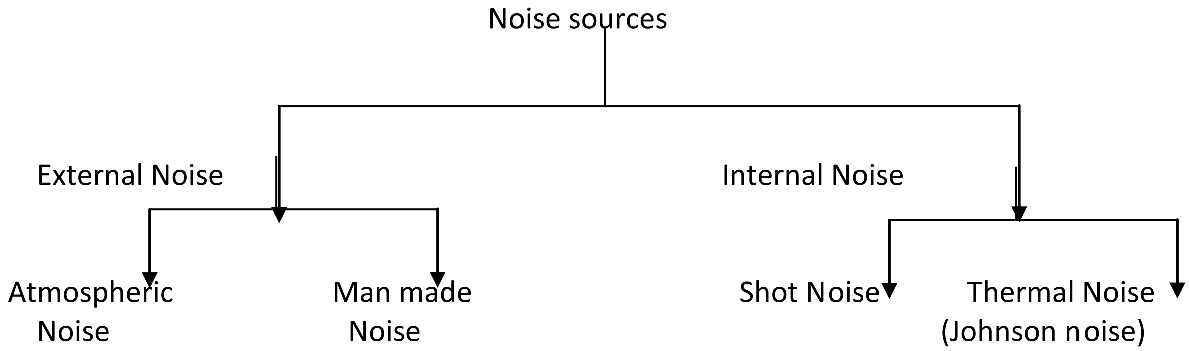
- This noise varies randomly with time
- The presence of this noise complicates the communication system.
- The amount of noise power in the received signal decides the minimum power level of the desired message signal at the transmitter.

Examples:

Atmospheric noise, manmade noise, Thermal and shot noise

Sources of noise

There are various sources of random noise



External Noise

This type of noise is created outside the communication system.
i.e. External noise is added to the signal during the transmission from the transmitter to receiver.

Atmospheric Noise or Static noise

- This noise is unpredictable and **caused by lightning discharges and the electrical disturbances that occur in the atmosphere.**
- It is in the form of impulses and spread on the radio frequency spectrum used for broad casting.
- It is less severe above 30MHz.

Man made Noise

- This type of noise is predicable and under human control.
- This **noise is caused by undesired pick-ups from electrical appliances such as motors, automobiles, aircraft ignition which produces spark**.
- This noise is effective in the range of 1MHz to 500MHz.
- It is avoided by proper shielding of electrical appliances.

Internal Noise (fluctuation noise)

- This **noise is created by the active and passive components** present in the communication circuits itself.

Thermal Noise or (Johnson noise)

- The random motion of electrons within a conductor such as resistoras thermal noise is called [also called resistor noise]
- The intensity of random motion is proportional to thermal energy supplied. So this noise is called thermal noise.
- The mean square value of the thermal noise voltage V_{TN} across the terminals of a resistor is

  $\mathbf{V_{TN}^2 = 4KTRB}$

  $V_{TN} = \sqrt{4KTBR}$

  Where K $\rightarrow$ Boltzmann's constant = 1.38 x 10^{-23} Joules / degree Kelvin

  T $\rightarrow$ Temperature in degree Kelvin

  R $\rightarrow$ Resistance in ohms

  B $\rightarrow$ Band Width in Hertz.

Shot noise

- Shot noise in electronic devices such as diodes and transistors is due to the discrete nature of current flow in these devices.
- This type of noise also occurs due to random generation and recombination of electron and hole pairs.
- In photo defector, a current pulse is generated when an electron is emitted by the cathode (due to incident light from a source of constant intensity).

****

2. Discuss on Thermal noise. **May 2014/Dec 2014**

Define noise and write short notes on Thermal noise. **Nov 2016**

Noise:

Noise is an unwanted signal that disturbs the transmission and processing of signals in communication systems.

Thermal Noise or (Johnson noise)

- The random motion of electrons within a conductor such as resistoras thermal noise is called [also called resistor noise]
- The intensity of random motion is proportional to thermal energy supplied. So this noise is called thermal noise.
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  Where K $\rightarrow$ Boltzmann's constant = 1.38 x 10^{-23} Joules / degree Kelvin

  T $\rightarrow$ Temperature in degree Kelvin

  R $\rightarrow$ Resistance in ohms

  B $\rightarrow$ Band Width in Hertz.

If the resistors are connected in series and maintained at same temperature

$$V_{TN} = \sqrt{4KTB(R1 + R2)}$$

If the resistors are connected in parallel and maintained at same temperature

$$V_{TN} = \sqrt{4KTB\left(\frac{R1R2}{R1 + R2}\right)}$$

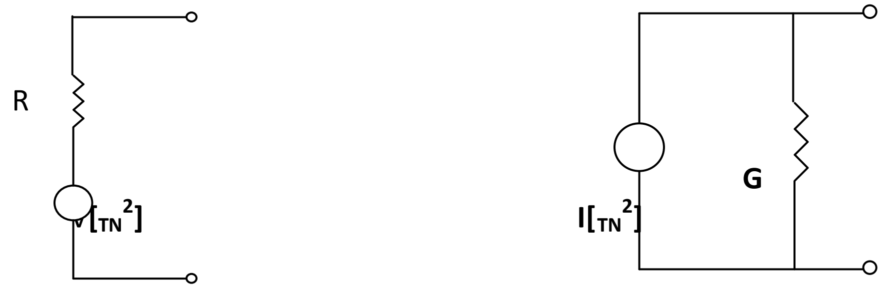
If two resistors are connected in series and maintained at different temperature

$$V_{TN} = \sqrt{4KB(T1R1 + T2R2)}$$

Noise Model

We can model a noisy resistor by the Thevenin and Norton equivalent circuit .

Thevenin equivalent circuit consisting of a noise voltage generator in series with a noiseless resistor.



Thevenin equivalent circuitNorton equivalent circuit

Norton equivalent circuit consisting of a noise current generator in parallel with a noiseless conductance.
The mean square value of noise current generator is

$$[I_N]^2 = 4KTGB$$
$$[I_N] = \sqrt{4KTB}G \text{ amps}$$

where G = 1/R = Conductance
According to H.B. Johnson, the noise power generated in the resistor is proportional to the temperature and BW

$P_n \alpha\ TB$

P_n = KTB

where

K- proportionality constant

$P_n \rightarrow$ noise power

Power Spectral Density:
The power spectral density is average noise power across the BW.

Sn = Pn/B

Sn = KTB/B

S_n= KT

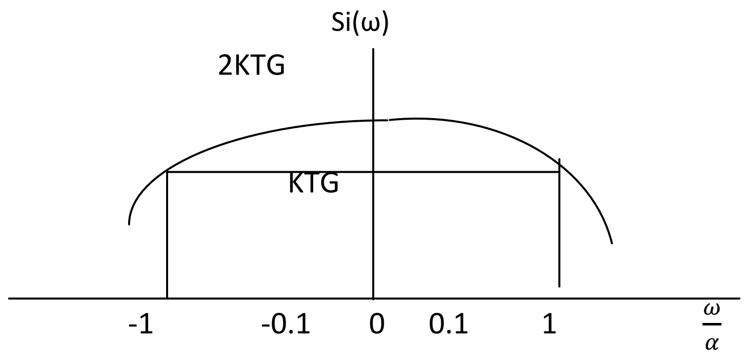
Power Spectral density (Power Density Spectrum)of resistor noise:

- If the random motion between the free electrons which contributes thermal noise are assumed to be independent.
- The thermal noise is Gaussian distributed with zero mean.
- The power density spectrum of the thermal noise is given by,

$$Si(\omega) = \frac{2KTG}{1+(\frac{\omega}{\alpha})^2}$$

Where α - average number of collisions per second per electron

- The variation of Power density spectrum with frequency is shown in the fig.



PSD of the resistor noise current

- When $\frac{\omega}{\alpha} \leq 0.1$,

  $$Si(\omega) = 2KTG$$

- Then the spectrum obtained is considered to be flat.

****

3. Write a short note on shot noise and also explain about power spectral density of shot noise. May 2014/ Nov 2016

Shot noise

- Shot noise in electronic devices such as diodes and transistors is due to the discrete nature of current flow in these devices.
- This type of noise also occurs due to random generation and recombination of electron and hole pairs.
- In photo defector, a current pulse is generated when an electron is emitted by the cathode (due to incident light from a source of constant intensity).
- These electrons are random in nature and emitted at time denoted by T_K.
  Where $-\infty < K < \infty$
  Let us assume the random emission of electron for a long period of time
- The total current flowing through photo detector is infinite sum of current pulses. Therefore

  $$X(t) = \sum_{K=-\infty}^{\infty} h(t - T_K)$$

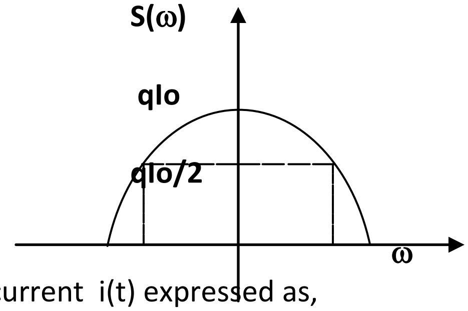
  Where $h(t - T_K) \rightarrow$ current pulse generated at time T_K.
- The process x(t) is a stationary process called shot noise.
- The number of electrons N(t) emitted in the time interval [0,t] constitute a discrete stochastic process.
- The value of N(t) increases by one each time when an electron is emitted.
- Let the mean value of N(t), emitted between times t and $(t + t_0)$ is defined by $E(v) = \tau$ to where $\tau \rightarrow$ rate of the process and its value constant.
- The total number of electrons emitted in the interval (t, t + t0) is given by

  $$V = N(t + t0) – N(t)]$$

  This follows a Poisson distribution with a mean value equal to $\tau$to.
  The probability that K electrons are emitted in the interval (t, t + t0) is defined by

$$P(V=K) = \frac{(\lambda t_o)^K e^{-\lambda t_o}}{K!} \qquad K = 0, 1 \text{ - - - - - - - - - -}$$

Probability spectral density function of shot noise



- The total current i(t) expressed as,

$$\mathbf{I(t) = I_0 + i_n(t)}$$

I_0 – constant current

$i_n(t)$ -- shot noise current

- $i_n(t)$ is an deterministic function which can be expressed as a function of time.
- But it can be expressed with its power density spectrum.
- According to central limit theorem, shot noise is Gaussian- distributed with zero mean.
- The power density spectrum of shot noise current is,

$$S_i(\omega) = q\, I_0$$

Where, q-electron charge (1.59×10^{-19} coulombs)

I_0 -mean value of the current in amperes

PSD of shot noise is independent of frequency.

****

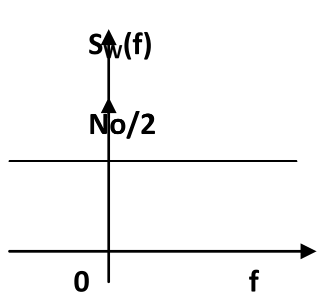
4. Write short notes onWhite noise and mention its characteristics. **Nov 2016**

White noise

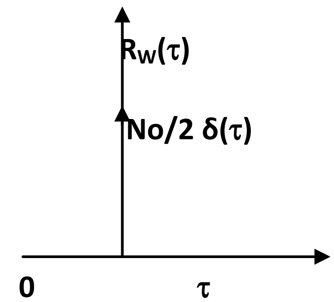
- White noise is an **idealized form of noise** used in the noise analysis of communication system. **PSD of white noise is independent of operating frequency**.
- White light contains equal amount of all frequencies within the visible band of electromagnetic radiation. i.e. white light contains all frequencies in equal amount.
- When the probability of occurrence of white noise is specified by Gaussian distribution, thenit is called **AWGN – Additive white Gaussian Noise**.
- White noise has zero mean and PSD $S_W(f) = N0/2$
- The dimension of No is in watts /Hertz and it refers the input stage of the receiver of a communication system.

Characteristics Of white noise

PSD of white noise



Auto correlation of white noise



$S_W(f)$ = No/2

No ∞ Te

No = K Te

Where Te $\rightarrow$ Equivalent noise temperature

K $\rightarrow$ Boltzmann's constant=1.38×10^{-23} J/^{0}K

- **S**ince PSD of white noise is independent of frequency, it can be treated as white Gaussian noise for practical purposes
- The auto correlation function is the inverse Fourier transform of the power spectral density.

  $R_W(\tau)$ = IFT $S_W(f)$

  = IFT[No/2]

  = No/2 $\delta(\tau)$
- Autocorrelation function of white noise consists of a delta function weighted by the factor No/2 and occurring at $\tau = 0$.

****

4.2 Noise Figure and Noise temperature

Signal to noise (SNR)

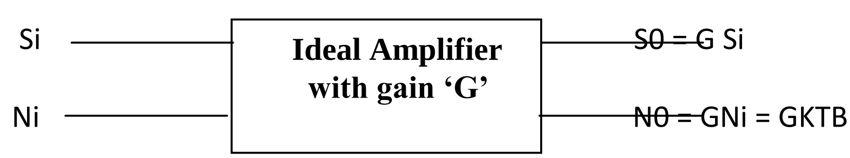
SNR is the ratio of signal power to noise power.

$$S/N = \frac{\text{Signal power}}{\text{Noise power}}$$

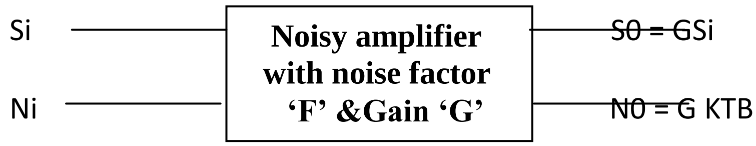
$$SNR = \frac{S_S(\omega)}{S_N(\omega)} = \frac{\text{PSD of signal voltage}}{\text{PSD of noise voltage}}$$

Noise Figure:

Consider a 2 port Network



Practical Amplifier



Noise factor is defined as the ratio of signal to noise ratio at the input to the signal to noise ratio at the output.

$$F = S_i/N_i \times N_0/S_0$$

$$F = N_0/GN_i$$

$$\mathbf{N_0 = GFN_i}$$

- For an ideal amplifier, the signal to noise ratio at the input and the output are the same.

Therefore F=1

- But for practical amplifier, the network introduces noise and S/N is reduced
- Comparison of S/N ratio at the input & output provides the noise present in the network.
- F is the factor by which the amplifier increases the output noise.
- Noise figure is always larger than unity and it is expressed in dB is

$$\boxed{\text{Noise figure (dB)} = 10 \log_{10} F}$$

- Noise factor is unity for noiseless ideal amplifier.
- Closer the noise factor to unity, better the amplifiers of noise.

Equivalent noise temperature

It is the temperature at which a noisy resistor has to be maintained such that by connecting the resistor to the input of noiseless system.

It produces the same available noise power at the output of the system as that produced by all the source of noise in the actual system.

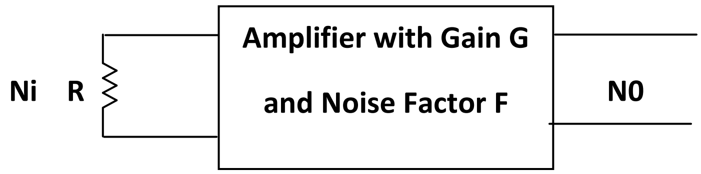
4.3 Noise in cascaded System

Noise figure of cascaded stages of amplifier is given by Friss's formula.

5. Derive the noise figure of amplifiers connected in cascade.
Derive Friss formula relating the noise figure with the gain of thecascaded amplifiers.
Explain with derivation the effect of noise in cascaded amplifier circuit. May 2015
Discuss the effect of noise in cascaded system. May 2017

Noise figure of cascaded stages of amplifier(Friss's formula)

Let us consider an amplifier with gain G and noise factor 'F'.



Let Ni $\rightarrow$ input noise power generated by the resistance

N0 $\rightarrow$ output noise power generated by the resistance

$$N0 = GNi + Na \qquad \text{------------------ (1)}$$

where Na$\rightarrow$ amplifier's noise

We know that $F = \frac{N0}{GNi}$ ------------------ (2)

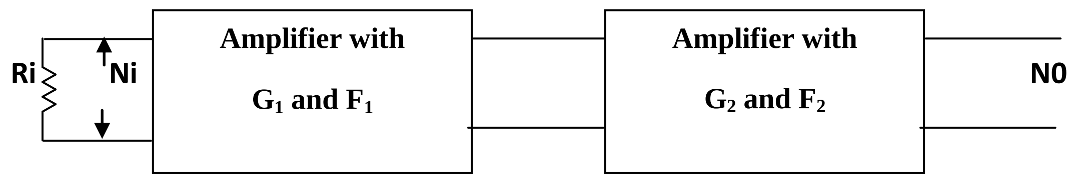
Dividing equation (1) by afactor GNi

$\frac{N0}{GNi} = \frac{GNi}{GNi} + \frac{Na}{GNi}$

$F = 1 + \frac{Na}{GNi}$

$F - 1 = \frac{Na}{GNi}$

Na = (F – 1) GNi



- Let us consider two stages of cascaded amplifier. The noise produced by the first stage is amplified by the later stage.
- Therefore overall noise figure depends upon the noise figure of individual stages.
  G1 and F1 $\rightarrow$ Gain & Noise figure of 1st stage
  G2 and F2 $\rightarrow$ Gain & Noise figure of 2nd stage
  Ni $\rightarrow$ noise power generated by resistance 'R' at the input of the first stage.
- Noise power at the final output due to Ni is
  N_{01} = G1 G2 Ni (Assume noise contributed by amplifier stages is zero)
- First stage introduces its own internal noise. The total noise at the output of the 1st stage is
  N_{o2}=G1(F1 – 1)Ni
- This noise is amplified at second stage and appears as **N_{03} = G1 G2(F1 – 1)Ni**
- Similarly the noise contributed by the 2nd stage at the final output is
  N_{03} = G2(F2 – 1)N

- Total noise power at the output is
  N0 = N01 + N02 + N03

$= Ni\, G1\, G2 + G1\, G2Ni(F1 - 1) + G2Ni(F2 - 1)$

$\frac{N0}{Ni} = G1\, G2[1 + F1 - 1 + \frac{F2-1}{G1}]$

$\frac{N0}{Ni G1\, G2} = F1 + \frac{F2-1}{G1}$

$\boxed{F = F1 + \frac{F2-1}{G1}}$ $\rightarrow$ noise figure for 2 cascaded stages.

For 'n' number of cascaded stages of amplifier

$$F = F1 + \frac{F2 - 1}{G1} + \frac{F3 - 1}{G1\,G2} + \frac{F4 - 1}{G1\,G2\,G3} + \ldots\ldots\ldots\ldots$$

Equivalent noise temperature of cascaded stages

Let us assume

Te1 $\rightarrow$ noise temperature of first stage

Te2 $\rightarrow$ noise temperature of second stage

- Te indicates reduction in SNR as signal propagates through the receiver.
- Lower the value of Te, better the quality of receiver
- Noise source can also be represented by noise equivalent temperature.

  Noise power due to amplifier = (F – 1) GNi

  = (F – 1) G KTB

  = (F – 1) KTB, if G = 1 -----------------(1)

If Te represents equivalent noise temperature representing the noise source.

K Te B --------------- (2)

(1) = (2)

(F – 1)KTB = K Te B

(F – 1) T = Te

$\frac{Te}{T}$ = F – 1

$$\boxed{\mathbf{F} = \frac{Te}{T} + \mathbf{1}} \qquad \text{------------ (3)}$$

Sub equation (3) inFriss formula

$$\frac{Te}{T} + 1 = \frac{Te1}{T} + 1 + \frac{Te2}{T} + 1 - 1$$

$$Te = Te1 + \frac{Te2}{G1} + \frac{Te3}{G1G2} + 1$$

****

Equivalent noise resistance of cascaded amplifier stages (Req)

It is due to the noise produced by the input noise resistance of 1st stage, equivalent noise resistance of succeeding stages.

$$Req = R1 + \frac{R2}{G1^2} + \frac{R3}{G1^2 G2^2} + \text{- - - - - - - - - - - -}$$

Noise Figure interns of noise resistance

$$\boldsymbol{F = 1 + \frac{Req}{Rs}}$$

Where, Rs ---- source resistance of amplifier

4.4 Narrow band noise

6. Explain narrowband noise n(t). show that how n(t) is represented in terms of inphaseand quadrature phase component. **[Nov 2018]** **May 2016**

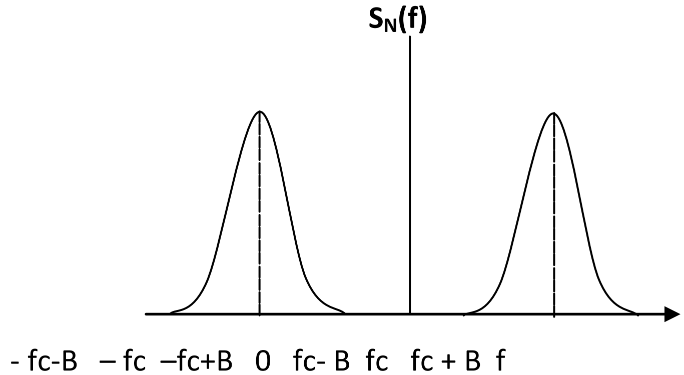
What is meant by narrow band noise ? Explain the characteristics of narrow band noise.

Dec2014

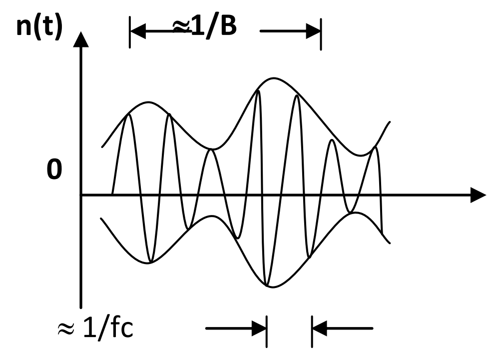
Narrow Band Noise

- In communication system the message signal intermixed with noise is allowed to pass through a frequency selective filter.
- These signals are usually passed through the filter and then given to the receiver. Such filters are called **narrow band filter.**
- This filter will have narrow bandwidth compared to center frequency.
- The narrowband filter bandwidth is large enough to pass modulated component of the received signal undistorted but not so large as to admit excessive noise through the receiver.
- The noise appearing at the output of narrowband filter is known as **narrow band noise.**

PSD of Narrow Band Noise



Sample function of Narrow band Noise



To analyze the effects of Narrow band noise on the performance of communication system, we need a mathematical representation.

There are 2 ways of representation of narrowband noise.

(i) **Narrowband noise n(t) is defined in terms of in phase and quadrature phase components**

$n(t) = n_I(t)\cos(2\pi f_c t) - n_q(t)\sin(2\pi f_c t)$ canonical form of n(t) .

Where $n_I(t)$ – In phase component of n(t)

$n_q(t)$ – quardrature component of n(t)

(ii) **Narrowband noise n(t) is defined in terms of envelope and phase components**

$n(t) = r(t)\cos[2\pi f_c t + \varphi(t)]$

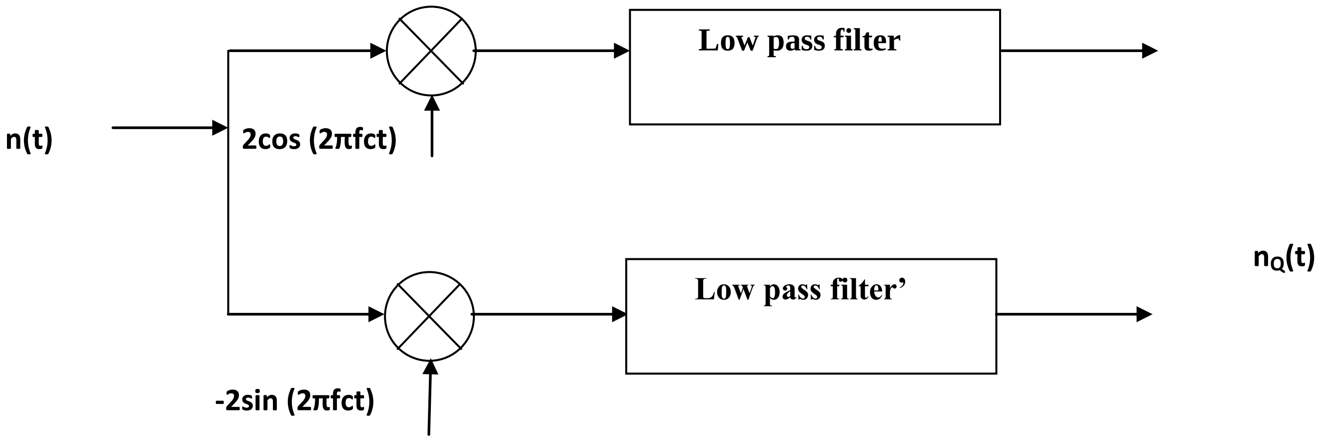
where $r(t) \rightarrow$ envelope of n(t) i.e. amplitude or magnitude

$r(t) = \sqrt{n_I^2(t) + n_q^2(t)}$

$\varphi(t)$ - phase component of n(t)

$\varphi(t) = \tan^{-1}\frac{nq(t)}{nI(t)}$

Extraction of in phase component ,nI(t) and quadrature phase nq(t) if n(t) is known



- Given the narrowband noise n(t), we can extract its inphasenI(t) and quadrature phase components nq(t) .
- It is assumed that the two low-pass filters used in this scheme are ideal with bandwidth 'B' (i.e. one half of the bandwidth of narrowband noise n(t))

We know $n(t) = n_I(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)$

After multiplication by the multiplier

$$n(t) = n_I(t) 2\cos^2(2\pi f_c t) - n_q(t)\, 2\sin 2\pi f_c t \cos 2\pi f_c t$$

$$= 2\, n_I(t)\left(\frac{1+\cos 4\pi f_c t}{2}\right) - n_q(t) \sin 4\pi f_c t$$

$$= n_I(t) + n_I(t)\cos 4\pi f_c t - n_q(t) \sin 4\pi f_c t$$

After LPF, the high frequency components are filtered

$$n(t) = n_I(t)$$

After multiplication by $- 2 \sin(2\pi f_c t)$

$$= -2n_I(t)\cos(2\pi f_c t) \sin(2\pi f_c t) + 2n_q(t) \sin^2(2\pi f_c t)$$

$$= -2n_I(t)\sin 4\pi f_c t + 2n_q(t)\left(\frac{1-\cos 4\pi f_c t}{2}\right)$$

After LPF,

$$n(t) = n_q(t)$$

Generation of narrow band noise from Inphaseand Quadrature phase component

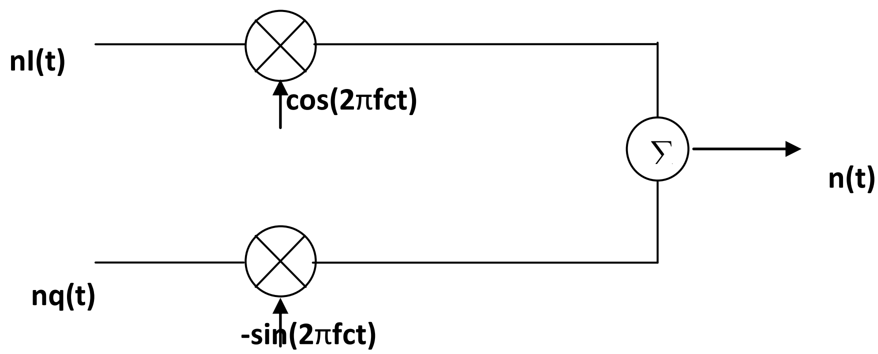
n(t) - Narrow band noise

$n_I(t)$ -In phase component of narrow band noise

$n_q(t)$ -Quadrature phase component of narrow band noise

We know that

$n(t) = n_I(t)\cos 2\pi f_c t - n_q(t) \sin 2\pi f_c t.$



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7. Discuss on the properties of narrow band noise. **[Apr - 2019]**

Properties of narrowband noise n(t):

1. The in phase and quadrature phase component of n(t) have zero mean.
2. If narrowband noise n(t) is Gaussian, then in phase component and quadrature phase component are jointly Gaussian.
3. If n(t) is stationary, then n I(t) &n q(t) are jointly stationary.
4. Both n I(t) &n q(t) have same power spectral density (PSD) which is related to the PSD of n(t) as

   $$S_{NI}(f) = S_{NQ}(f) = \begin{cases} S_N(f - fc) + S_N(f + fc), & -B \le f \le B \\ 0, & \text{otherwise} \end{cases}$$

   Where it is assumed that $S_N(f)$ occupies the frequency interval

   $fc - B \le |f| \le fc + B$ & $fc > B$
5. The n I(t) &n q(t) have the same variance as n(t).
6. The cross-spectral density of the n I(t) &n q(t) of n(t) is purely imaginary as given by

   $$S_{NINQ}(f) = -S_{NQNI}(f)$$
   $$= \begin{cases} j[S_N(f+fc) - S_N(f - fc)], & -B \le f \le B \\ 0, & \text{otherwise} \end{cases}$$
7. If n(t) is Gaussian and its PSD $S_N(f)$ is symmetric about the mid-band frequency fc, then n I(t) and n q(t) are statistically independent.

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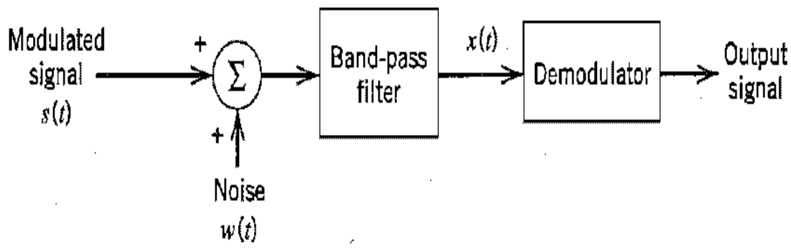
4.5 Noise performance of AM systems

Noisy Receiver Model



S(t) -modulated signal
W(t)-Additive White Gaussian Noise
X(t)= S(t)+n(t)

To study the effects of channel noise in the received CW modulated signal, two assumptions are made

- The channel is distortion less and disturbed by Additive white Gaussian noise – **channel model**.
- BPF is an ideal filter followed by an ideal detector **– receiver model**.

Characteristics of BPF

- BPF is used to minimize the effect of channel noise.
- It allows only the narrowband signal about carrier frequency fc and rejects the other frequencies.
- BPF is an ideal filter (narrow band pass filter) having BW equal to transmission BW (Bτ) of modulated signal and mid band frequency equal to fc.

Detector: Performance of detector depends upon the type of modulation used

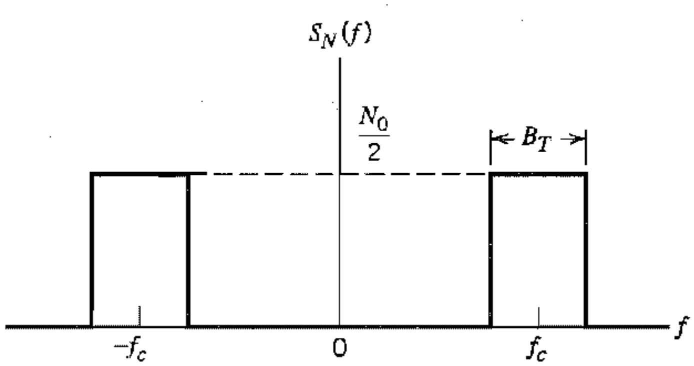
PSD of NB noise $=\frac{No}{2}$

$\therefore$ Average noise power = PSD / BW

$= \frac{No}{2}$ x 2W

= NoW

PSD of Narrow band noise ,$S_N(f)$



SNR and Figure of merit

A more useful measure of noise performance is the output signal to noise ratio.

Output signal to noise ratio is

$$(SNR)_o = \frac{\text{Average power of the demodulated signal } m(t)}{\text{Average noise power}}$$

Both theparameters are measured at the receiver output

Channel signal to noise ratio is

$$(SNR)_c = \frac{\text{Average power of the modulated signal } S(t)}{\text{Average power of the channel noise } n(t) \text{ in the message BW}}$$

Both parameters are measured at the receiver input

For the purpose of comparing different CW modulation systems, we normalize the receiver performance by dividing (SNR)O by (SNR)C. This ratio is called figure of merit for the receiver and is defined as

Figure of merit (γ) = $\frac{(SNR)_o}{(SNR)_c}$

If '**γ**' is high, better is the noise performance of the receiver

If γ = 1, DSBSC (no improvement)

γ< 1. AM noise dominates

γ> 1, FM (less noise)

8. Derive an expression for SNR at input (SNR c) and output of (SNR o) of a coherent detector.

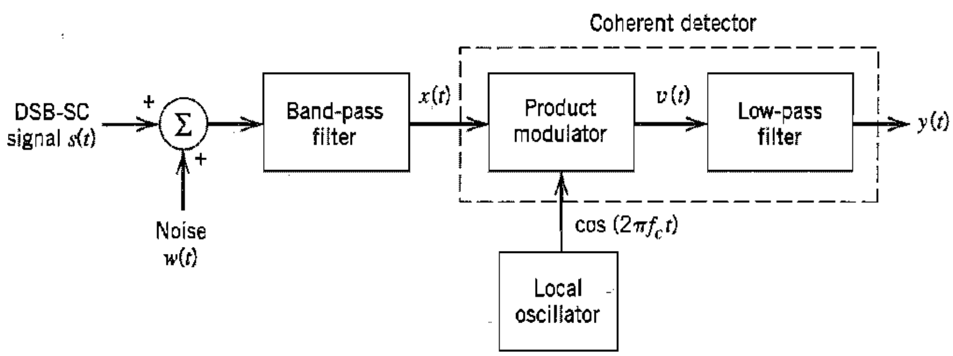
Dec2012/May2012/Dec2011/may2011/May2010

Derive the SNR performance of DSB System. **May2015**

Explain the noise in DSB-SC receiver using synchronous or coherent detection and calculate the figure of merit for a DSB-SC system. **May 2016**

To study the noise performance characteristics, we have to calculate signal power and noise power

Noise model of DSB SC receiver



Let the DSB SCmodulated signal

$S(t) = A_C \cos 2\pi f_c t\, m(t)$
where $m(t) \rightarrow$ message signal
$A_C \cos 2\pi f_c t \rightarrow$ carrier signal
The noise signal n(t) (white Gaussian noise) is
$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$

The DSBSC signal is combined with noise and it is passed into the band pass filter.

$$\text{Figure of merit} = \frac{(SNR)_O}{(SNR)_C}$$

$$\mathbf{(SNR)o} = \frac{\text{Average power of demodulated signal}}{\text{Average Noise power}}$$

$$\mathbf{(SNR)c} = \frac{\text{Average power of the modulated signal}}{\text{Average power of channel noise in message bandwidth}}$$

Channel signal to noise ratio (SNR)c

Averagepower of the modulated signal

$$= \frac{Ac^2 P}{2}$$

where $P \rightarrow$ Average power of the message signal ,m(t)

Average noise power = PSD of noise x BW
$= N_O/2 \times 2w = N_O w$

$$(SNR)c \text{or baseband} = \frac{\text{Average power of the modulated signal}}{\text{Average power channel noise in message bandwidth}}$$

$$(SNR)_C = \frac{A_C^2 P/2}{WN_O} = \frac{A_C^2 P}{2WN_O}$$

$$\boxed{\mathbf{(SNR)_C = \frac{A_C^2 P}{2WN_O}}}$$

Output signal to noise ratio (SNR)o
The output of the BPF is
$x(t) = S(t) + n(t)$
$= A_C m(t) \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$

The output of the product modulator is

$V(t) = x(t) \cos 2\pi f_c t$

$= [A_C m(t) \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t] \cos 2\pi f_c t$

$= [A_C m(t) + n_I(t)] \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \cos 2\pi f_c t$

$= [A_C m(t) + n_I(t)] \cos^2 2\pi f_c t - n_Q(t) \sin 4\pi f_c t/2$

$= [A_C m(t) + n_I(t)] \frac{1+\cos(4\pi f_c t)}{2} - n_Q(t) \sin 4\pi f_c t/2$

The output of the LPF is

$y(t) = \frac{A_c m(t)}{2} + \frac{n_I(t)}{2}$

= signal component+ noise component

Quadrature component of the narrow band noise is completely eliminated (or) filtered by coherent detector.

Average power of the demodulated signal y(t)

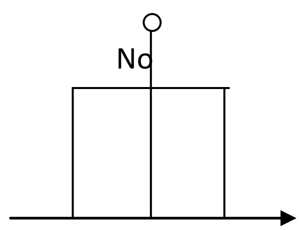
$= \frac{A_c^2 P}{4}$

Where P is the average power of m(t)

Average power of the filtered noise is $2WN_O$

= PSD x BW = No x 2w = $2wN_O$

$S_{N_I}(f) = S_{N_Q}(f)$



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PSD of In phase components and quadrature phase components

Noise power at the receiver output is $= (1/2)^2 \times 2wN_o = wN_o/2$

[Since noise component is $n_I(t)/2$]

$$(SNR)_o = \frac{\textbf{Average power of demodulated signal}}{\textbf{Average Noise power}}$$

$$(SNR)_o = \frac{A_C^2 P/4}{N_o w/2}$$

$$\boxed{(SNR)_O = \frac{A_C^2 P}{2N_O w}}$$

Figure of merit $(\gamma) = \frac{(SNR)_O}{(SNR)_C}$

$= \frac{A_C^2 P/2N_o w}{A_C^2 P/2N_o w}$

∴ γ= 1

Figure of merit for DSB – SC AM using coherent detection is unity.

∴ No noise improvement in the output.

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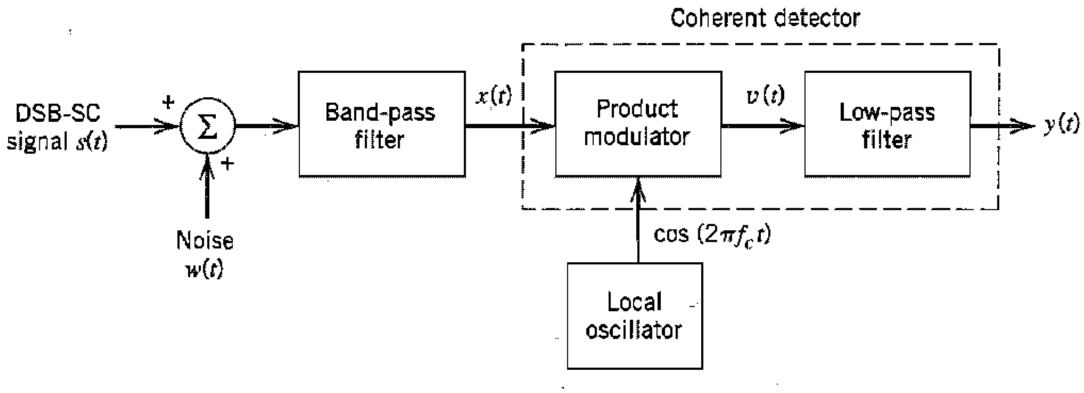
9. Derive the SNR performance of SSB SC System.

Effect of noise on SSB AM



S(t) → SSB SC Modulated Signal

$S(t) = A_C \cos 2\Pi f_C t\, m(t) \pm A_C \hat{m}(t) \sin 2\Pi f_C\, t$

$\hat{m}(t)$ ⇒ Hilbert Transform m(t)

$$\left({}^{S}/_{N}\right)_{\text{base band(or)c}} = \frac{\text{Average power ofSSB SC Modulated Signal}}{\text{Average Noise Power}}$$

$$\text{Average power of SSBSC} = \frac{{A_C}^2 P}{2} + \frac{{A_C}^2 P}{2} = {A_C}^2 P$$

$$\text{Average Noise power} = \frac{N_o}{2} x2\omega = N_o \omega$$

$$\left({}^{S}/_{N}\right)_b = \frac{{A_C}^2 P}{N_o \omega} = \left[\left(\frac{P_R}{N_o \omega}\right) \text{where } P_R = {A_C}^2 P\right]$$

$x(t) = S(t) + n(t)$

$$= A_C \cos 2\Pi f_C t\, m(t) \pm A_C \hat{m}(t) \sin 2\Pi f_C\, t + n_I(t) \cos 2\Pi f_C t - n_Q(t) \sin 2\Pi f_C\, t$$

$V(t) = x(t) \cos 2\Pi f_C t$

$$= \left[A_C \cos 2\Pi f_C t\, m(t) + A_C \hat{m}(t) \sin 2\Pi f_C\, t + n_I(t) \cos 2\Pi f_C t - n_Q(t) \sin 2\Pi f_C\, t\right] \cos 2\Pi f_C t$$

$$= A_C m(t) \cos 2\Pi f_C t + A_C \hat{m}(t) \cos 2\Pi f_C\, t \sin 2\Pi f_C\, t + n_I(t) \cos 2\Pi f_C t - n_Q(t) \sin 2\Pi f_C\, t \cos 2\Pi f_C t$$

$$= A_C m(t) \left[\frac{1 + \cos 4\Pi f_C t}{2}\right] + \frac{A_C \hat{m}(t)}{2} \sin 4\Pi f_C\, t + n_I(t) \left[\frac{1 + \cos 4\Pi f_C t}{2}\right] - \frac{n_Q(t)}{2} \sin 4\Pi f_C\, t$$

$$= \frac{A_C m(t)}{2} + \frac{A_C m(t) \cos 4\Pi f_C t}{2} + \frac{A_C m(t)}{2} \sin 4\Pi f_C\, t + \frac{n_I(t)}{2} + \frac{n_I(t) \cos 4\Pi f_C t}{2} - \frac{n_Q(t)}{2} \sin 4\Pi f_C\, t$$

$$y(t) = \frac{A_C m(t)}{2} + \frac{n_I(t)}{2}$$

$$\left(S/_{N}\right)_o = \frac{\text{Average power of Modulating Signal}}{\text{Aversge Noise Power}}$$

$$\text{Average power of Modulateing Signal} = \frac{A_C{}^2 P}{4}$$

$$\text{Aversge Noise Power} = \text{PSO of } n_I(t)\text{xBW} = N_o x\omega = N_o\omega$$

$$\text{Aversge Noise Power} = \frac{1}{4} N_o\, x\omega = \frac{1}{4}\, N_o\omega$$

$$\left(S/_{N}\right)_o = \frac{A_C{}^2 P}{4} x \frac{4}{N_o\omega} = \frac{A_C{}^2 P}{N_o\omega}$$

$$\left(S/_{N}\right)_o = \left(S/_{N}\right)_{\text{base band}}$$

Figure of merit=1

****

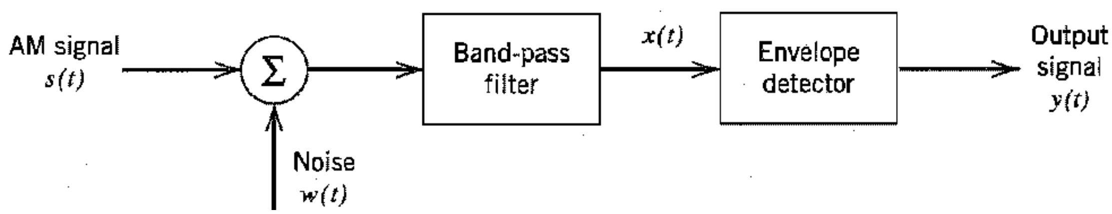
10. Derive the expression for figure of merit of a AM receiver using envelope detection. What do you infer from the expression? **Dec2006/May 2013/Nov 2016/ Dec 2017**

Derive the SNR performance of AM system. Also prove that the output SNR in AM is at least 3dB worse than that of DSB system. **May 2015**

Derive an expression for signal to noise ratio for an AM signal with the assumption that the noise added in the channel is AWGN. Compare its performance with FM system. **May 2017**

Noise performance of AM Receiver using envelope detection

Noisy model of AM receiver



In AM signal, both sidebands and the carrier signalare transmitted

$S(t) = A_C[1 + Ka\, m(t)] \cos(2\pi fct)$

Where $m_a \rightarrow$ modulation index

$m(t) \rightarrow$ message signal

$A_C \cos 2\pi fct \rightarrow$ carrier signal

Channel signal to Noise ratio (SNR)c

$$(SNR)C = \frac{\text{Average power of modulated signal}}{\text{Average noise power}}$$

Average power of the modulated signal

$= A_C^2/2[1 + Ka^2 m(t)]$
$= A_C^2/2[1 + Ka^2 P]$

Average Noise power = PSD x BW $= N_0/2$ x2w = Now

$$\boxed{(SNR)_C = \frac{A_C^2[1 + Ka^2 P]}{2wNo}}$$

Output Signal to NoiseRatio (SNR)o
The filtered signal x(t) applied to the envelope detector is

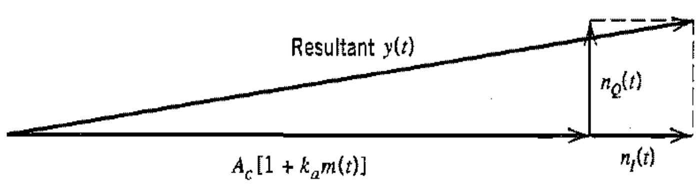
$x(t) = S(t) + n(t)$
$= A_C[1 + Ka\, m(t)] \cos 2\Pi fc\, t + n\, I(t) \cos 2\Pi fc\, t - n_Q(t) \sin 2\Pi fc\, t$
$x(t) = [A_C + A_C K a\, m(t) + n\, I(t)] \cos(2\Pi fc\, t - n_Q(t) \sin 2\Pi fc\, t$

Receiver output y(t) = envelope of x(t)

$= \sqrt{(A_C + A_C Ka\, m(t) + n_I(t)]^2 + n_Q^2 (t)}$

When signal power is large compared to noise power, $n_I(t)$ and $n_Q(t)$ will be very small.

<u>Phasor diagram of AM for high carrier to noise ratio</u>



$\therefore y(t) = A_C + A_C Ka\, m(t) + n_I(t)$

Here A_C (the carrier amplitude) is constant i.e dc and has no relationship with m(t) and it is removed with the help of blocking capacitor after envelope detector.

$\therefore y(t) \approx A_C Ka\, m(t) + n_I(t)$

Average power of demodulated signal

$= A_C^2 Ka^2 P$
Since average power of m(t) is P

Average noise power =PSD of $n_I(t)$x BW

=Nox2w

$\therefore (SNR)_O = \frac{\text{Average power of demodulated signal}}{\text{Average noise power}}$

$$\boxed{(SNR)_O = \frac{A_C^2\, Ka^2\, P}{2Now}}$$

Figure of merit (γ) = $\frac{(SNR)_O}{(SNR)_C}$

$$= \frac{\dfrac{A_C^2\, Ka^2\, P}{2Now}}{\dfrac{A_C^2\,(1 + Ka^2 P)}{2N_Ow}}$$

$$\boxed{\left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} \simeq \frac{k_a^2 P}{1 + k_a^2 P}}$$

γ<1 noises dominate

Note:

- Figure of merit of an AM receiver using envelope detection is always less than unity.
- It is always inferior to that of a DSB – SC receiver, due to wastage of transmitting power in the carrier.

Let $m(t) = Am \sin \omega mt$

power $= Am^2/2$

$$\left.\frac{(SNR)_O}{(SNR)_C}\right|_{AM} = \frac{\frac{1}{2}k_a^2 A_m^2}{1 + \frac{1}{2}k_a^2 A_m^2}$$

Let Ka Am = μ = ma = modulation index

$ma^2/(2+ma^2)$

For 100% modulation ma=1

$\gamma_{AM} = 1/3$

$\therefore \gamma_{AM} < 1$

$\therefore$ AM detection using the envelope detector must transmit 3 times as much as average power as DSB system using coherent detection to achieve the same quality of noise performance.

****

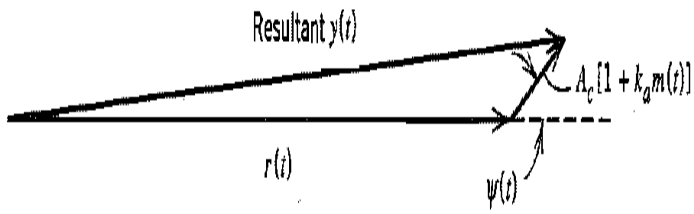
11. Discuss the threshold effect in AM

Threshold effect in AM

When carrier to noise ratio is small compared to unity, the noise dominates(low carrier to noise ratio). The loss of information when carrier to noise ratio is low is called threshold effect in AM.

$$y(t) = \sqrt{[A_C[1 + Kam(t)] + nI(t) + n_Q(t)]^2}$$
$$= \sqrt{[A_C^2[1 + Kam(t) + nI(t)]^2 + 2A_C[(1 + Kam(t))\, nI(t)] + n_Q^2(t)]}$$

<u>Phasor diagram for low carrier to noise ratio i.e. (noise dominates)</u>



Assume noise dominates, then

$$y(t) = \sqrt{n_I^2(t) + n_Q^2(t) + 2A_C[1 + Ka\, m(t)]n_I(t)}$$
$$= \sqrt{r^2(t) + 2A_C[1 + K_a m(t)]n_I(t)}$$
$$= \sqrt{r^2(t)}\, 1\left(+ \frac{2A_C[1 + K_a m(t)]n_I(t)}{r^2(t)}\right)$$
$$= r(t)\sqrt{1}\left(+ \frac{2A_C(1 + K_a m(t)\, n_I(t)}{r(t)\, .\, r(t)}\right)$$
$$= r(t)\sqrt{1}\left(+ \frac{2A_C(1 + K_a m(t)\cos\Psi(t)}{r(t)}\right) \qquad \text{since } \cos\Psi(t) = nI(t)/r(t)$$
$$= r(t)\sqrt{1 + \frac{2A_C(1 + K_a m(t)\cos\Psi(t)}{r(t)}}$$
$$= r(t)\, 1\left(+ \frac{A_C(1 + K_a m(t)\cos\Psi(t)}{r(t)}\right)$$
$$== r(t)\, r\left(\frac{(t) + A_C(1 + K_a m(t)\cos\Psi(t)}{r(t)}\right)$$

$$y(t) \simeq r(t) + A_c\cos[\psi(t)] + A_c k_a m(t)\cos[\psi(t)]$$

∴ Detector output has no component strictly proportional to the message signal m(t).

- The last term in the detector outputy(t) contains the message signal m(t) multiplied by noise in the form of cos Ψ(t).
- The phase Ψ (t) of the narrow band noise is uniformly distributed over the range of 2Π radians.
- It is not possible to separate the message signal ,therefore information is lost.

Threshold effect : The loss of signal (Information) in an envelope detector that operates at low carrier to noise ratio is called as threshold effect.

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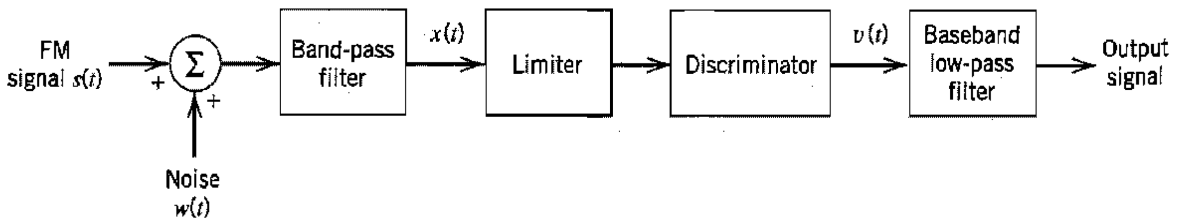
4.6 Noise performance in FM systems

12. Derive the figure of merit for FM Receiver. **[April 2018]** **Dec2011/ May 2013**
Discuss the noise performance of FM receiver.

Explain the noise in FM receiver and calculate the figure of merit for a FM system. Nov 2016

Noise performance in FM Receivers using FSD

Noise Model of an FM receiver



- Noise **ω(t)** is modeled as white Gaussian noise of zero mean and PSD = No/2
- Amplitude variations due to noise and interference is removed by amplitude limiter followed by BPF.
- The output of the limiter is then fed to the discriminator which consists of 2 components

1. A slope circuit or differentiator with a purely imaginary frequency respect that varies linearly with frequency.
2. An envelope detector which recovers the amplitude variations and thus reproduce the message signal.

- The output of discriminator is fed to the base band LPF which is used to remove the out of band components of the noise of the discriminator output.

The incoming FM signal S(t) is given by

$$S(t) = A_C \cos[2\Pi f_c t + 2\Pi k_f \int_0^t m(t)\, d\tau]$$

where $A_C \rightarrow$ carrier amplitude

$f_C \rightarrow$ carrier frequency

$K_f \rightarrow$ frequency sensitivity

$m(t) \rightarrow$ message signal

In simple form S(t) can be expressed as

$$S(t) = A_C \cos[\omega_c t + \phi(t)]$$

$$\text{where } \phi(t) = 2\Pi k_f \int_0^t m(\tau)\, d\tau$$

Channel signal to noise ratio(SNR)c

Averagepower of modulated signal at the channel = $A_C^2/2$

Average noise power at the channel is No/2 x 2w = Now

$$(SNR)c = \frac{\text{Average power of modulated signal}}{\text{Average noise power}}$$

$$\therefore (SNR)_C = \frac{A_C^2/2}{2Now}$$

Output signal to noise ratio(SNR)o

The noisy signal at the output ofBPF is x(t) = S(t) + n(t)

where $n(t) = n_I(t) \cos 2\Pi fct - n_Q(t) \sin 2\Pi fct$

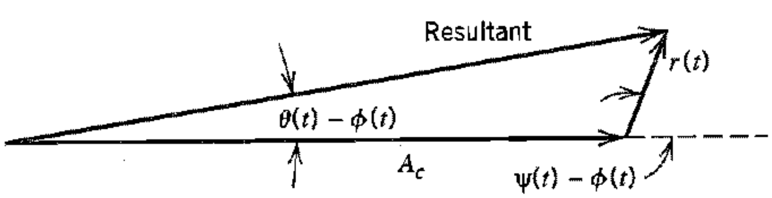
Here, $n_I(t)$ & $n_Q(t)$ are in phase and quadrature phase components

n(t) can be expressed in terms of envelope and phase component.

i.e. $n(t) = r(t) \cos[2\Pi fct + \Psi(t)]$
where envelope $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$
& phase $\Psi(t) = \tan^{-1} [n_Q(t) / n_I(t)]$

The envelope r(t) is Rayleigh distributed and the phase Ψ (t) is uniformly distributed over 2Π radians.

$x(t) = S(t) + n(t)$
$\therefore x(t) = A_C \cos[2\Pi fct + \theta(t)] + r(t)\ \mathrm{con}[2\Pi fct + \Psi(t)]$



[θ(t) → phase of the resultant]

Assume high carrier to noise ratio,
$$\theta(t) = \varphi(t) + \frac{r(t)}{Ac} \sin[\psi(t) - \varphi(t)] \qquad (1)$$

where $\phi(t) = 2\Pi k_f \int_0^t m(\tau)\, d\tau$
kf → frequency sensitivity
m(t) → message signal

sub. $\varphi(t)$ in equation(1)

$$\theta(t) = 2\Pi k_f \int_0^t m(\tau)\, d\tau + \frac{r(t)}{Ac} \sin[\psi(t) - \varphi(t)]$$

Discriminator output V(t):

$$V(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = K_f m(t) + n_d(t)$$

$V(t) = k_f m(t) + n_d(t)$

where $n_d(t) = 1/2\Pi\{d/dt\ [\ \frac{r(t)}{Ac} \sin[\psi(t) - \varphi(t)]\ \} \rightarrow$ additive noise component.

- If the phase $\Psi(t)$ of the narrow band noise is uniformly distributed over 2Π radians. then the phase difference $\psi(t) - \varphi(t)$ is also uniformly distributed over 2Π radians.
- In that case, the noise $n_d(t)$ at the discriminator output would be independent of the modulating signal.

$$\therefore n_d(t) = \frac{1}{2\pi Ac}\frac{d}{dt}[r(t)\sin\psi(t)]$$

$$n_d(t) = \frac{1}{2\pi Ac}\frac{d}{dt}[n_q(t)]$$

where $r(t) \sin \Psi(t) = n_q(t)$

$S_{Nd}(f) = 1/2\Pi A_C$ FT $[d/dt[n_Q(t)]$

Since FT $(d/dt) = j2\Pi f$

$$\frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$$

$$S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

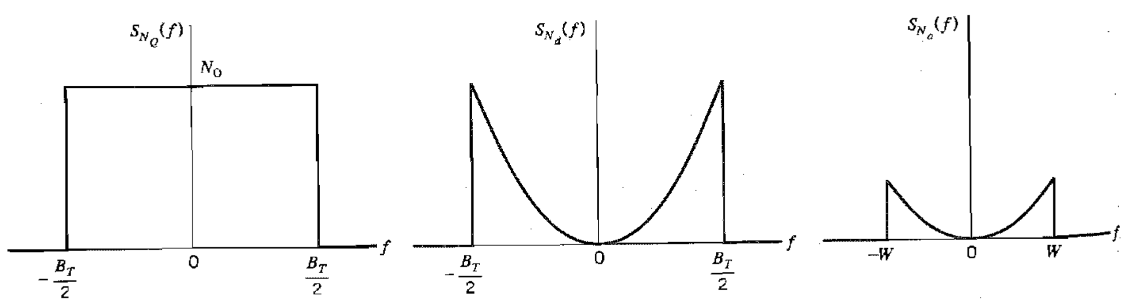
Where $S_{Nd}(f)$ - power spectral density of the noise $n_d(t)$

$S_{NQ}(f)$- power spectral density of $n_Q(t)$

We know $S_{NQ}(f) = No \quad - B_T/2 \le f \le B_T/2$

PSD of $n_q(t)$

$S_{Nd}(f) = \frac{N0\, f^2}{Ac^2}, \quad - B_T/2 \le f \le B_T/2$



(a) PSD of $n_Q(t)$ (b) PSD of $n_d(t)$ at the (c) PSD of noise $n_O(t)$

discriminator output at the receiver output

After Passing though LPF

$$S_{No}(f) = \begin{cases} \frac{N0\, f^2}{Ac^2}, & -W \le f \le W \\ 0, & \text{otherwise} \end{cases}$$

Where W$\rightarrow$ BW of LPF

Average noise power at the output of Receiver

$$\int_{-W}^{W} \frac{N0 f^2}{Ac^2} df$$

$$= \frac{N0}{Ac^2} \int_{-W}^{W} f^2 df \quad = \frac{N0}{Ac^2} \quad [f^3/3]$$

$$= \frac{2N0\, W^3}{3Ac^2}$$

Average signal power at the output

$V(t) = k_f m(t) + n_d(t)$

Low pass filter output is $k_f m(t)$

$\therefore$ The average signal power = $k_f^2 P$

Output signal to noise ratio

$$(SNR)_O = \frac{\text{Average signal power}}{\text{Average noise power}} = \frac{k_f^2 P}{\frac{2N0\, W^3}{3Ac^2}}$$

$$\boxed{(SNR)_O = \frac{3kf^2 P A_C^2}{2N_O W^3}}$$

Figure of merit

$$\gamma_{FM} = \frac{(SNR)_O}{(SNR)_C} = \frac{3k_f^2 P A_C^2}{2N_O W^3} \frac{2N_O W}{A_C^2}$$

$$\boxed{\gamma_{FM} = \frac{3k_f^2 P}{W^2}}$$

$$\gamma_{FM} = (3/2)\, \beta^2$$

Transmission BW increases as βincreases and γ of FM increases

****

4.7 Pre – emphasis and De – emphasis

To improve output signal to noise ratio in FM communication system.

13. Discuss about Pre – emphasis and De – emphasis used in FM receiver.
Dec2006/Dec2009/ May2010/ Dec2011/May2011/ Dec2012/May 2016

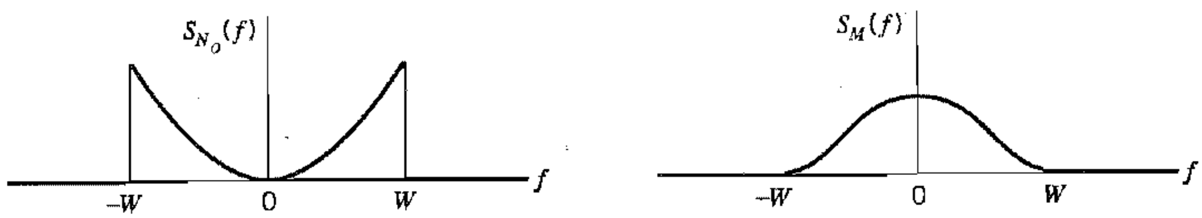
Explain the operation of Pre – emphasis and De – emphasis in the FM communication system.
[Apr - 2019] [Nov 2018] Dec 2017

Pre emphasis and De emphasis

- Used to improve output signal to noise ratio
- Mainly used in commercial FM radio transmission & reception

Importance of Pre emphasis

- Power spectral density of the message usually falls at higher frequencies
- But PSD of the output noise increases rapidly with frequency.

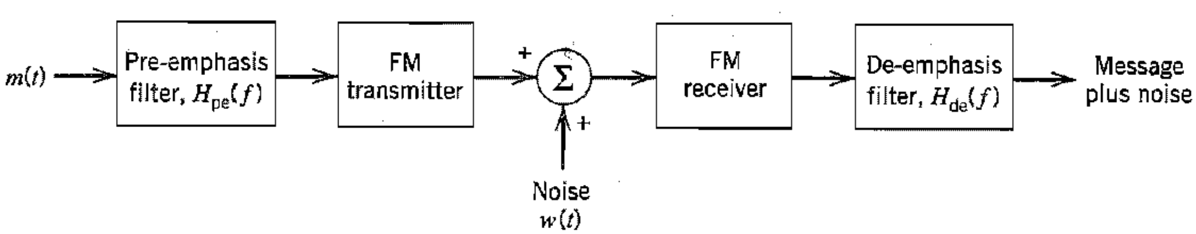


(a) PSD of noise at FM receiver output(b) PSD of a typical message signal

Around f = $\pm$ W, PSD of message is low, and noise is high

This means the message is not using the frequency band allotted to it in an efficient manner.
There are 2 ways in which output signal to noise can be increased.

1) To reject a large amount of noise power by losing only a small amount of message power by reducing the BW of the post detection LPF.
2) By using Pre emphasis in the transmitter and De emphasis in the receiver,the allowed frequency bandiss efficiently used



Pre – emphasis

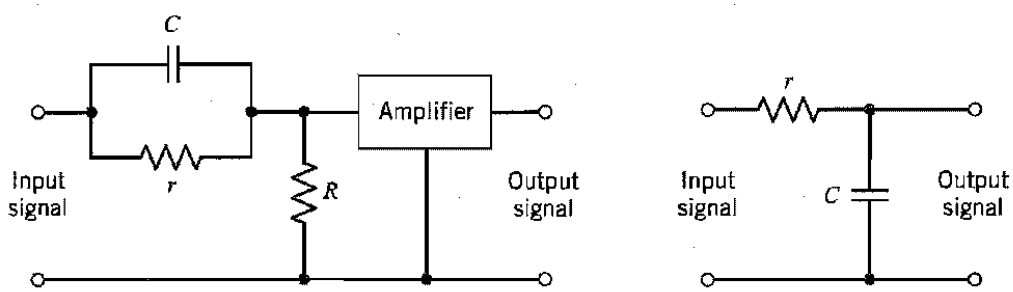
The message signal is not effectively using the allotted frequency band. So the amplitude of high frequency components are artificially boosted up prior to modulation before noise is introduced.

This process is called Pre emphasis and done by Pre – emphasis circuit.

De – emphasis

At the receiver, the artificial boostingof the high frequency components are nullified thereby removing the high frequency noise. This process is called De emphasis .

- The low frequency and high frequency portion of PSD of message are equalized in such a way that the message fully occupies the allowed frequency band.
- This is done by Pre emphasis filter. (HPF)
- In the receiver, at the discriminator inverse operation is performed to remove the high frequency component and to restore the original signal power distribution by the message.
- This is done by De emphasis filter (LPF).



(a) Pre emphasis filter (b)De emphasis filter

To produce undistorted message at the receiver, the Pre emphasis circuit at the transmitter and De emphasis circuit at the receiver must have frequency response that is inverse to each other.

i.e. Hpe(f) = 1/Hde(f) , $-W \le f \le W$.

- PSD of noise at the discriminator output assuming high carrier to noise ratio is

  $S_{Nd}(f) = \frac{No\ f^2}{Ac^2} f \le B_T/2$
- The average noise power with de- emphasis filter at the receiver output is equal to

$$|H_{de}(f)|^2 S_{Nd}(f) = \begin{cases} \int_{-W}^{W} No f^2/A_C^2 |H_{de}(f)|^2 df, & |f| \le B_T/2 \\ 0, & \text{otherwise} \end{cases}$$

- Average power of the modified noise at the receiver output is

$$N_O/A_C^2 \int_{-w}^{w} f^2 ||H_{de}(f)|^2 df$$

- Improvement in output signal to noise ratio produced by Pre emphasis and De emphasis is given by the improvement factor I.

$$I = \frac{\text{average output noise power without Pre emphasis and De emphasis}}{\text{average output noise power with Pre emphasis and De emphasis}}$$

$$I = \frac{2N_O w^3 / 3A_C^2}{N_O / A_C^2 \int_{-w}^{w} f^2 |H_{de}(f)|^2 df}$$

$$I = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df}$$

- Frequency response of Pre emphasis filter is

$$H_{pe}(f) = 1 + \frac{jf}{f_0}$$

- Frequency response of De emphasis filter is

$$H_{de}(f) = \frac{1}{1 + jf/f_0}$$

$$|H_{de}(f)| = \frac{1}{\sqrt{1+(\frac{f}{f0})^2}}$$

sub in the equation for I

$$I = \frac{2W^3}{3\int_{-W}^{W} \frac{f^2 \, df}{1 + (f/f_0)^2}}$$

$$= \frac{(W/f_0)^3}{3[(W/f_0) - \tan^{-1}(W/f_0)]}$$

In commercial FM broadcasting, for fo = 2.1 KHz & w = 15KHz. We get I = 22, and increase in output signal to noise ratio of the receiver is 13dB.

[The output signal to noise ratio without pre emphasis and de emphasis is 40 – 50 dB]

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4.8 Capture effect and Threshold effect

Capture effect

- The ability of FM system to minimize the effects of unwanted signal.
- Interference is produced by another FM signal whose frequency is close to the carrier frequency of the desired FM.
- Interference suppression in an FM receiver works well only when the interference is weaker than the desired FM input.
- When the frequency of interference is stronger than the desired FM, the receiver locks on to the stronger signal thereby suppressing the desired FM input.
- But when the two frequencies are of equal strength, the receiver fluctuates back and forth between them.
- This phenomenon is known as the capture effect.

  A receiver should have ability to avoid this capture effect.

14. Discuss about FM Threshold effect. **Dec2006/Dec2008/May2013**

FM Threshold effect

When the input noise power is increased, (carrier to noise ratio is decreased) the FM receiver breaks and clicks are heard in the receiver output.

As the carrier to noise ratio decreases further, the individual clicks rapidly merge and produce crackling or sputtering sound. This phenomenon is known as the threshold effect.

Threshold

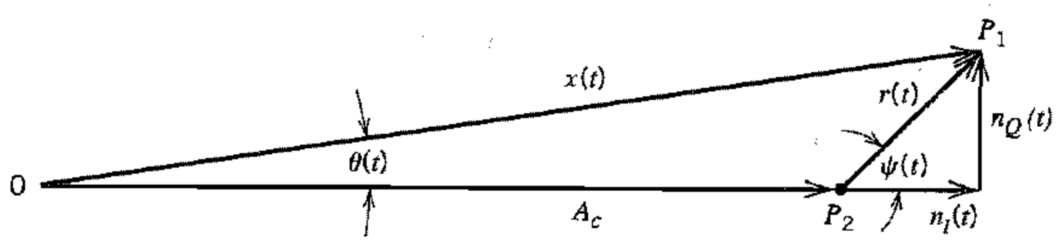
Threshold is defined as the minimum carrier to noise ratio for which the FM noise improvement is not significantly deteriorated from the value predicted by the usual carrier to signal noise assuming a small noise power.

- Threshold effect is more severe in FM than AM
- let us consider that there is no message signal present. So that the carrier wave is un modulated

$$x(t) = A_C \cos 2\Pi f_c t + n_I(t) \cos 2\Pi f_c t - n_Q(t) \sin 2\Pi f_c t$$
$$= [A_C + n_I(t)] \cos 2\Pi f_c t - n_Q(t) \sin 2\Pi f_c t$$

- The phasor diagram shows the phase relations between the various components of x(t)

Phasor representation of FM noise



P_1

- When the carrier to noise ratio is large, $n_I(t)$ &$n_Q(t)$ are much smaller than A_C. So P_1 is near P_2& Angle $\theta(t) \approx n_Q(t)/A_C$.
- When the carrier to noise ratio is low, P_1 sweeps around the origin and(comes closer to the origin) and $\theta(t)$ increases or decreases by 2Π radians. The discriminator output produce impulse like components.

i.e. $V(t) = \theta^1(t)/2\Pi = 1/2\Pi\ [d/dt\theta(t)]$

- These impulse like components have different heights depending on how close the wandering point P_1 comes to the origin but all have areas nearly equal to $\pm 2\Pi$ radians.
- When the discriminator output is fed to the post detection low pass filter, wider impulse like components are excited in the receiver output and heard as clicks.
- The clicks are produced only when $\theta(t)$ changes by $\pm 2\Pi$ radians

Conditions for clicks to occur

Positive going clicks occurs when

$r(t) > A_C$
$\Psi(t) < \Pi \leq \Psi(t) + d\Psi(t)$
$d\Psi(t) / dt > 0$

Negative going clicks occurs when

$r(t) > A_C$
$\Psi(t) < -\Pi > \Psi(t) + d\Psi(t)$
$d\Psi(t) / dt < 0$

Here $\theta(t)$ changes by -2Π radians during the H_{me} increment dt.
Carrier to noise ratio $\rho = A_C^2/2B_TN_O$

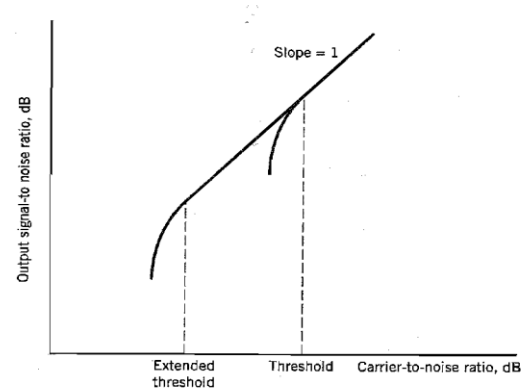
As ρ is decreased, average number of clicks per unit time increases. When this number becomes appreciably large threshold is said to occur.

- When the message signal is present, the resulting modulation of the carrier tends to increase the average number of clicks/second. Clicks are heard in the receiver output at a carrier to noise ratio of 1db.
- Average number of clicks/sec increases, output SNR falls off more sharply just below the threshold level in the presence of modulation.
- Threshold effects can be avoided if $\rho \geq 20$.
  i.e.[$A_C^2/2B_TN_O$]$\geq$ 20.
- Average transmitter power satisfies the condition $\mathbf{A_C^2/2 \geq 20\ B_TN_O}$.
  where $B_{T\rightarrow}$transmission BW of FM wave.

****

Threshold improvement (or) Threshold Extension

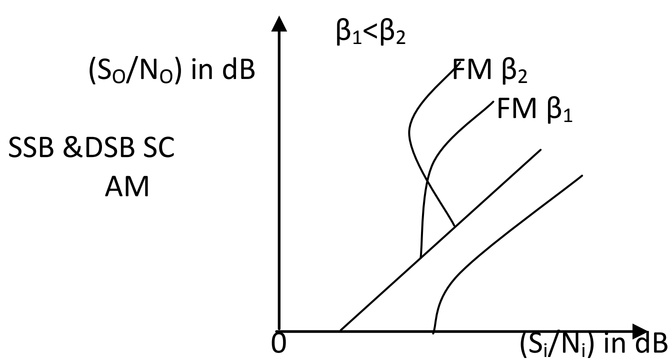
- Threshold is defined as the minimum carrier to noise ratio below which the noise performance of the receiver deteriorates much more rapidly than the usual signal to noise ratio (assuming a small noise power).
- Threshold effect is more severe in FM than AM because the carrier to noise ratio at which threshold occurs is higher.
- The threshold is reduced in FM receiver to operate receiver with the minimum signal power possible and hence it is necessary to lower the threshold level.
- The process of lowering the threshold is called threshold improvement (or) threshold extension.
  There are 2 methods of improvement of threshold
  (i) Pre – emphasis & De – emphasis
  (ii) FM FB [Frequency modulator with feedback]



4.7.1 Comparison of noise performance of AM & FM systems

Comparison of noise performance of AM & FM systems

- No threshold effect in AM – SC systems (curve is linear)
- Threshold at lower (S_i/N_i) is better because the threshold can be avoided with comparatively weaker signals.
- AM Threshold effect occurs for low (S_i/N_i)
- Noise performance of AM SC using coherent detection is superior to envelope detection.



<u>**Comparison between AM& FM**</u>

- Above threshold, FM is superior to AM since FM provides more (S_O/N_O) for a n equivalent (S_i/N_i)
- Below threshold, FM is inferior because threshold occur at higher (S_i/N_i). also (S_O/N_O) falls more rapids, than AM.
- Therefore in larger noise case, suppressed carrier AM using coherent detection is most suitable.

15. Compare important characteristics of FM systems **May 2014**

Comparison between FM systems of Different Bandwidth

- Above threshold, FM with large modulation indexβ_2 is (large BW) is superior than small modulation index β_1.(lower BW)
- Below threshold, the system with large BW exhibits faster decay in (S_O/N_O) and threshold occurs at higher (S_i/N_i). Hence it is inferior than lower BW FM in this case.

Conclusion

- For large noise systems (below threshold), DSB SC is most suitable.
- For small noise systems (above threshold) wideband FM is most appropriate.

Common channel interference

- FM is better than AM because of capture effect.
- The interfering weak signals received from the neighboring channels are almost completely suppressed in FM, but they produce distortion in AM.

Externally generated noise pulses

- FM receiver has limiter circuit which limits the amplitude of noise pulses.
- But if the FM receiver is slightly mistuned, its ability to suppress noise pulses is highly reduced. However in AM, such exact tuning is not essential.

Comparison of Figure of merit

- Figure of merit for FM is $\gamma_{FM} = 3/2\ \beta^2$
- Figure of merit for AM is $\gamma_{FM} = 1/3$
  Noise performance of AM is very poor

Noise performance of FM is improved by increasing the BW.

****

16. Compare various AM systems and FM system.

Comparison of noise performance of AM,DSB SC , SSB SC and FM systems

S.No	**Parameter**	**AM**	**DSB SC**	**SSB SC**	**FM**
1.	Figure of merit	$\gamma < 1$	$\gamma = 1$	$\gamma = 1$	$\gamma > 1$
2.	Bandwidth	$2f_m$	$2f_m$	f_m	$2f_m \rightarrow$ NBFM $2(\Delta f + f_m) \rightarrow$ WBFM
3.	Threshold effect	Exhibits threshold effect for low (S_i/N_i)	No threshold effect	No threshold effect	Threshold effect is more severe in FM than AM
4.	Noise performance	Poor	Better than AM	Better than AM	FM is superior to AM above threshold

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UNIT- IV

Noise Characterization

1. ***Define noise.***

   Noise is defined as an unwanted signal which disturbs the transmission and reception of wanted signal.

2. ***Give the classification of noise.***

   Noise is broadly classified into two types. They are External noise and internal noise.

   ***External noise** can be classified into*

   Atmospheric noise, Extraterrestrial noises, Manmade noises or industrial noises.

   ***Internal noise** can be classified into*

   Thermal noise, Shot noise.

3. ***What is meant by External noise?***

   Noise that is present external to the communication system is called as External noise.

   It disturbs the signal during the transmission from transmitter to receiver.

4. ***List the external sources of noise.***

   ***The external noise** sources of noise are*

   - *Atmospheric noise,*
   - *Extraterrestrial noises,*
   - *Manmade noises or industrial noises.*

5. ***What is meant by Internal noise?***

   Noise that is present within the communication system is called as Internal noise.

   Noise due to the active and passive components used within the communication system.

6. ***List the internal sources of noise.*** **[Nov 2018]**

   ***The internal noise** sources of noise are*

   - *Thermal noise.*
   - *Shot noise.*
   - *Flicker noise.*

7. ***What is Atmospheric Noise ?***

   - *Atmospheric noise is unpredictable and caused by lighting discharges and the electrical disturbances that occur in the atmosphere.*
   - *This type of noise is in the form of impulses and spread on the radio frequency spectrum used for road casting.*
   - *It is less severe above 30MHz.*

8. What is Man made Noise?

- *Man made noise is caused by undesired pick-ups from electrical appliances such as motor, automobile, aircraft ignition which produces spark.*
- *This type of noise is predicable and under human control.*
- *This noise is effective in the range of 1MHz to 500MHz.*
- *It is avoided by proper shielding of electrical appliances.*

9. Define thermal noise. **Dec 2006 /Dec 2009**

Write the equation for the mean square value of thermal noise voltage in a resistor. **May 2015**

Thermal noise is an electrical noise arising from the random motion of electrons in a conductor

The mean –square value of thermal noise voltage is given by

$V_{Tn}^{\ 2} = 4KTBR$

$V_{Tn} = \sqrt{4KTBR}$

K – Boltzmann constant-1.38 x10^{-23} Joules/degree Kelvin

R – Resistance

T – Absolute temperature in kelvin

B – Bandwidth

10. Give the expression for noise voltage when several sources are cascaded.

The expression for noise voltage when several sources are cascaded is

$V_{Tn} = \sqrt{4KTB(R1\ +\ R2\ +\ \ldots..)}$

Where R1 , R2 --- are the resistances of the noise resistors.

K – Boltz man constant

T – absolute temperature

B – Bandwidth

11. What is shot noise? **May 2004/ Dec 2007/May2009**

Shot noise is a noise that arises in electronic devices such as transistors ,diodes, because of discrete nature of current flow in these devices.

12. Define signal to noise ratio.

Signal to noise ratio is defined as the ratio of signal power to the noise power at the same point in a system.

SNR= Signal power / Noise power

13. Define Noise Factor

It is defined as the ratio of input signal to noise ratio to output signal to noise ratio

Nose factor $F = (SNR)_I/(SNR)o$

SNR = Signal power / Noise Power

14. Define noise figure. **[Apr -2019]**

Dec 2003/ May 2007/Dec2013 / Dec 2006 /May 2013/May 2014/Dec 2015

Nose factor F = (SNR) $_i$/(SNR)o

Noise factor expressed in decibels is as called Noise Figure

Noise figure in dB= 10 log F

F-noise factor

15. Define White Noise. Mention the characteristics

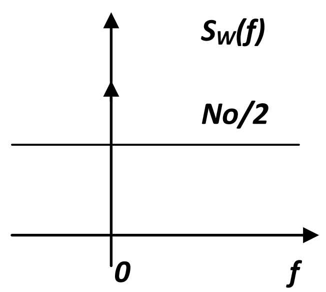
May2006 /May 2007 /Dec 2007/ Dec 2009/May 2011/ May2013

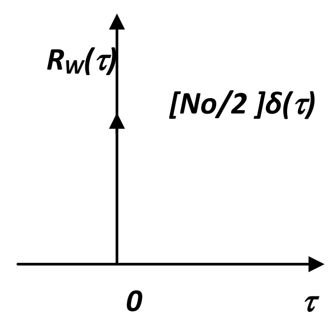
*White noise is an **idealized form of noise** used in the noise analysis of communication system.*
*PSD of white noise is **independent of operating frequency**.*
The spectrum has all frequency components in equal portion, and is therefore called white noise.
White light contains equal amount of all frequencies within the visible band of electromagnetic radiation. i.e. white light contains all frequency in equal amount.

Characteristics of white noise

Sw(f)=No/2 and Rw(τ)= [No/2] δ(τ) No=K Te

PSD of white noise ***Auto correlation function of white noise***





16. Define noise equivalent temperature **[Apr - 2019]** ***May 2004/May2012/ Dec 2014/May 2016/Dec 2017***

It is the temperature at which a noisy resistor has to be maintained such that by connecting the resistor at the input of noiseless system, it produces the same available noise power at the output of the system as that produced by all the source of noise in the actual system.

17. Give the expression for equivalent noise temperature in terms of hypothetical temperature.

The expression for equivalent noise temperature in terms of hypothetical temperature is Te = (F- 1) T0

Where, F is the noise figure and T0 absolute temperature.

18. Give the Friss formula in terms of noise temperature.

The Friss formula in terms of noise equivalent temperature is

Te = T1 + T2 / G1 + T3 / G1 G2 + ………

T1,T2….. Noise temperature of cascaded stages

G1,G2 ,… Gain of amplifiers of cascaded stages

19. *Define noise equivalent bandwidth.*

It is the bandwidth of the ideal band pass system which produces the same noise power as the actual system.

$$B = \frac{\int_{0} |H(f)|^{2}\, df}{H^{2}(0)}$$

20. *What is narrowband noise?* **[Apr-2018]**

The preprocessing of the signal at the receiver takes place in the form of a narrowband filter whose bandwidth is large enough to pass modulated component of the received signal undistorted but not so large as to admit excessive noise through the receiver. The noise appearing at the output of such filter is called narrow band noise.

21. *Give the ways of representation of Narrow band noise.*

Two ways of representation of n(t)

Narrow band noise in terms of inphase $n_i(t)$ and quadrature phase $n_Q(t)$component

$$n(t) = nI(t) \cos(2\pi fct) - nq(t) \sin(2\pi fct)$$

Narrow band noise in terms of envelope r(t) and phase φ (t) component

$$r(t) = [n_I^{2}(t) + n_Q^{2}(t)]^{1/2}$$

22. *Give the representation of narrowband noise in terms of Inphase and quadrature phase components.*

How will you define the narrow band noise at the IF filter output in terms of Inphase and quadrature phase components? **Dec 2013**

Narrowband noise in terms of Inphase and quadrature phase component is

$n(t) = n_i(t) \cos 2\pi fct - n_Q(t) \sin 2\pi fct$

$n_i(t)$ -*Inphase component of n(t)*

$n_Q(t)$ -*quadrature phase component of n(t)*

23. *Give the representation of narrowband noise in terms of envelope and phase components.*

Narrowband noise in terms of envelope and phase components is

$n(t) = r(t) \cos(2\pi fct + \varphi(t))$

$r(t) = [n_I^{2}(t) + n_Q^{2}(t)]^{1/2}$

$\varphi(t) = \tan^{-1}(n_Q(t) / n_I(t))$

r (t) –envelope of narrow band noise

φ (t) - phase of narrow band noise.

24. Define figure of merit **Dec 2006**

$$\text{Figure of merit}, \gamma = \frac{(SNR)_O}{(SNR)_C}$$

$$(SNR)c = \frac{\textit{Average power of the modulated signal}}{\textit{Average power channel noise in message bandwidth}}$$

$$(SNR)o = \frac{\textit{Average power of demodulated signal}}{\textit{Average Noise power}}$$

25. Define output signal to noise ratio

The output signal to noise ratio is defined as the ratio of the average power of the demodulated message signal to the average power of the noise, both measured at the receiver output".

$$(SNR)o = \frac{\textit{Average power of demodulated signal}}{\textit{Average Noise power}}$$

26. Define channel signal-to-noise ratio.

The channel signal-to-noise ratio is defined as the ratio of the average power of the modulated signal into the average power of channel noise in the message bandwidth both measured at receiver input".

$$(SNR)c = \frac{\textit{Average power of the modulated signal}}{\textit{Average power channel noise in message bandwidth}}$$

27. Why the figure of merit of AM is always < 1?

The figure of merit of an AM receiver using envelope detection is always less than unity. This is due to the wastage of transmitting power due to transmitting the carrier that does not contain any information.

28. What is meant by Capture effect? ***Dec 2010***

What is capture effect in FM? ***May 2012/ May 2016***

The FM receiver receives only the stronger signal. When the interference signal is stronger than the FM signal it receives only the stronger interference signal

When the interference signal and FM input are of equal strength, the receiver fluctuates back and forth between them .This phenomenon is known as the capture effect.

29. Define threshold.

The minimum carrier-to-noise ratio for which the FM noise improvement is not significantly deteriorated from the value predicted by the usual signal-to-noise formula assuming a small noise power is called as threshold.

30. Define threshold effect in AM receiver. ***Dec 2011/May 2015***

Specify the cause of threshold effect in AM systems. ***May 2017***

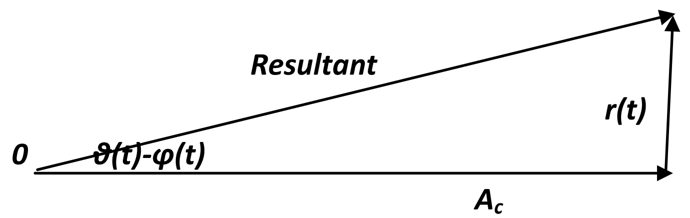
When carrier to noise ratio is small compared to unity, the noise dominates. The loss of information when carrier to noise ratio is low is called threshold effect in AM.

31. What is FM threshold effect?

May 2004/May 2006 / May 2007/Dec 2007/ May 2009/Dec 2009/Dec 2011

As the input noise power is increased i.e the carrier to noise ratio is decreased clicks are heard in the receiver output and as the carrier to noise ratio is reduced further individual clicks merge to produce crackling sound at the receiver output . This phenomenon is known as threshold effect.

32. Draw the phasor representation of FM noise.



33. How is threshold reduction achieved in FM system? **[Nov 2018]**

What are the methods to improve FM Threshold reduction? ***Dec 2003/May 2010/May 2011***

Threshold reduction is achieved in FM system by using an

- *FM demodulator with negative feedback or by using a phase locked loop demodulator.*
- *Pre-emphasis and de-emphasis.*

34. Define pre-emphasis. ***Dec 2007/Dec 2010/ Dec 2012/May 2015***

Pre-emphasis is a high pass circuit which boosts the signal amplitude of high frequency components in the message band at the transmitter before modulation.

The boosted signal at transmitter increases, the output signal to noise ratio,$(S/N)_0$ ratio at the detector output.

$(S/N)_0$ ratio becomes large enough to improve the threshold level over the entire message band.

35. Define de-emphasis. ***Dec 2007/Dec 2012***

De emphasis is a low pass circuit used to remove the artificially boosted higher modulating frequencies by pre-emphasis circuit to improve the noise immunity

The artificially boosted high frequency signals are brought to their original amplitude using the de-emphasis circuit.

36. What is the need for Pre-emphasis and de-emphasis? **April 2018** **May 2013/June 2014/May 2015**

Comment the role of pre -emphasis and de -emphasis circuit in SNR improvement. **May 2017**

Pre emphasis *is needed to boost up the signal amplitude of high frequency components in the message band at the transmitter before modulation.*

De emphasis *is needed to remove the higher modulating frequencies boosted by pre-emphasis circuit to improve the noise immunity*

- *To increase the output signal to noise ratio in FM receiver*
- *FM threshold reduction*

37. Define improvement factor.

The improvement factor in output signal-to-noise ratio produced by the use of pre-emphasis in the transmitter and de-emphasis in the receiver is defined by,

$$I = \frac{\text{average output noise power without pre-emphasis and de-emphasis}}{\text{average output noise power with pre-emphasis and de-emphasis.}}$$

38. Compare the noise performance of AM receiver with that of DSB-SC receiver **April 2018** **Dec 2012**

AM receiver

- *The figure of merit of AM receiver using envelope detection is always less than unity.*
- *Threshold effect occurs*

DSB SC receiver

- *The figure of merit of DSB-SC or SSB-SC receiver using coherent detection is always unity.*
- *Threshold effect occurs*

Therefore noise performance of AM receiver is always inferior to that of DSBSC due to the wastage of power for transmitting the carrier.

39. Compare the noise performance of DSB SC receiver with that of FM receiver. **Dec 2014**

- *The figure of merit of DSB-SC or SSB-SC receiver using coherent detection is always unity.*
- *The figure of merit of FM receiver is always greater than unity.*

Therefore noise performance of FM receiver is always superior to that of DSBSC .

Solved problems:

1. ***An amplifier operating at the frequency range 18MHz to 20MHz. calculate noise voltage at input if the ambient temperature is 26°C and resistor is 10KΩ.***

Given:

T = *26°C*

= 26 +273 = 299°K

B = 20 –18 MHz $\Rightarrow$ *2MHz*

Solution:

V_{TN} = $\sqrt{4KTBR}$

= $\sqrt{4}$ x 1.38 x 10^{-23} x 299 x 2 x 10^{6} x 10 x 10^{3}

= 18.16 μV

2. ***If two resistors 20KΩ & 50KΩ are connected at temperature 70℃ for a BW of 100KHz. Calculate the (i) noise voltage of each resistor and (ii)when two resistors are in series (iii) resistors in parallel.***

[Apr - 2019]

Given:

R1 = 20 KΩ. R2 = 50 KΩ.

T = 70℃ $\Rightarrow$ 70 + 273 = 343K

BW=100KHz

Solution:

(i) Noise voltage of each resistor

For R1 = 20 KΩ.

VTN = $\sqrt{4KTBR}$

V_{TN} = $\sqrt{4}$ x 1.38 x10^{-23} x343 x20x10^{3} x100 x 10^{3}

= 6.15μV

For R2 = 50 KΩ.

V_{TN} = $\sqrt{4}$ x 1.38 x10^{-23} x343 x 50 x 10^{3} x 100 x 10^{3}

= 9.729μV

(ii) When resistors are in series

$$V_{TN} = \sqrt{4KTB(R1+R2)}$$

V_{TN} = $\sqrt{4}$ x 1.38 x10^{-23} x343 x 100 x 10^{3} x (50 + 20) x 10^{3}

= 11.51μV

(iii) When resistors are in parallel

V_{TN} = $\sqrt{4}$ x 1.38 x10^{-23} x343 x 100 x 10^{3} Req

1/Req = 1/R1 + 1/R2 $\Rightarrow$ Req = $\frac{R1R2}{R1+R2}$ = $\frac{50 \times 20 \times 10^{6}}{70 \times 10^{6}}$

Req = 14.28KΩ

V_{TN} = $\sqrt{4}$ x 1.38 x10^{-23} x343 x 100 x 10^{3} x 14.28 x 10^{3}

V_{TN} = 5.200μV

3. ***Two resistors of 20 K΄Ω. 50 K΄Ω are at room temperature (290 K).for a bandwidth of 100 KHz Calculate the thermal noise voltages generated by the two resistors in series.*** ***Dec 2011***

Given data:

R1=20KΩ

R2=50KΩ

Room temperature T=290

Bandwidth = 100KHz

Solution :

When resistors are in series

V_{TN} = $\sqrt{4KTB(R1+R2)}$

V_{TN} = √4 x 1.38 x10-^{23} x343 x 100 x 10^{3} x (50 + 20) x 10^{3} $= 11.51\mu V$

4. ***Calculate noise figure and equivalent noise temperature for a receiver connected to an antenna whose resistance is 100 Ω and equivalent noise resistance is 50 Ω.*** ***Dec2008***

Given data:

Ro=100 ***Ω***

Req=50 ***Ω***

Solution :

F=1+ Req/Ro=1.5 dB

Teq=To(F-1)=174 K

5. ***Noise figure of the individual stages of 2 stage amplifier is 2.03 and 1.54 respectively. The available power gain is 62. Calculate the overall noise figure.***

Given data:

F1 = 2.03
F2 = 1.54
G1= 62

Solution:

F = F1 + (F2 – 1) /G1
= 2.03 + (1.54 – 1) /62
= 2.03 + 0.54/62
F = 2.03871

6. ***Calculate thermal noise voltage across the RC circuit with R=1KΩ and C= 1uf at T= 27^{0} C***

Dec 2012

Given data:

R=1KΩ , C= 1uf , T= 27^{0} C

Solution:

V TN ^{2}= 4KTRB, $B=\frac{1}{2\pi RC}$

Vn = $\sqrt{4KTBR}$

7. ***An amplifier has 3 stages with gain 5 dB , 20 dB and 12 dB. The noise figures of the stages are 7 dB, 13 dB and 12 dB respectively. Determine the overall noise figure and the noise equivalent temperature.*** ***Dec 2017***

UNIT V
SAMPLING & QUANTIZATION

Low pass sampling – Aliasing- Signal Reconstruction-Quantization - Uniform & non-uniform quantization - quantization noise - Logarithmic Companding – PAM, PPM, PWM, PCM – TDM, FDM.

5.1 Communication:

The purpose of a Communication System is to transport an information bearing signal from a source to a user destination via a communication channel.

Communication Systems are divided into 3 categories:

1. Analog Communication Systems are designed to transmit analog information using analog modulation methods.
2. Digital Communication Systems are designed for transmitting digital information using digital modulation schemes, and
3. Hybrid Systems that use digital modulation schemes for transmitting sampled and quantized values of an analog message signal.

5.2 Advantages and disadvantages of Digital Communication

1. List the advantages and disadvantages of digital communication. [Nov 2017]

Advantages of Digital Communication

1. The effect of distortion, noise and interference is less in a digital communication system. This is because the disturbance must be large enough to change the pulse from one state to the other.
2. Regenerative repeaters can be used at fixed distance along the link, to identify and regenerate a pulse before it is degraded to an ambiguous state.
3. Digitalcircuits are more reliableand cheaper compared to analog circuits.
4. The Hardware implementation is more flexible than analog hardware because of the use of microprocessors, VLSI chips etc.
5. Signal processing functions like encryption, compression can be employed to maintain the secrecy of the information.
6. Error detecting and Error correcting codes improve the system performance by reducing the probability of error.
7. Combining digital signals using TDM is simpler than combining analog signals using FDM. The different types of signals such as data, telephone, TV can be treated as identical signals in transmission and switching in a digital communication system.
8. We can avoid signal jamming using spread spectrum technique.

Disadvantages of Digital Communication:

1. Large System Bandwidth:- Digital transmission requires a large system bandwidth to communicate the same information in a digital format as compared to analog format.
2. System Synchronization:- Digital detection requires system synchronization whereas the analog signals generally have no such requirement.

5.3 Comparison of Analog and Digital communications:

2. Distinguish between Analog and Digital communications.

Sl. No.	Analog Communication	Digital Communication
1.	Transmitted signal is analog in nature	Transmitted signal is analog or digital in nature
2.	Amplitude, frequency or phase variations in the transmitted signal represent the information or message.	Amplitude, width or position of the transmitted pulses is constant. The information or message is transmitted I the form of code words.
3.	Noise immunity is poor.	Noise immunity is excellent.
4.	It is not possible to separate the noise and signal.	It is possible to separate the signal from the noise.
5.	Coding is not possible.	Coding techniques can be used to detect and correct the errors.
6.	Bandwidth required is low.	Bandwidth required is higher.
7.	FDM is used for multiplexing.	TDM is used for multiplexing.
8.	Not suitable for transmission of secret information in military applications.	Due to coding techniques, it is suitable for military applications.
9.	Analog Modulation systems are AM, FM, PM, PAM, PWM, PPM, etc.	DM systems are PCM, DM, ADM, DPCM, etc.

5.4 LOW PASS SAMPLING:

- A message signal maybegin from a digital or analog source.
- If the message signal is analog in nature, then it has to be converted into digital form before it can transmit by digital means.
- ***Sampling:***The process by which the continuous-time signal is converted into a discrete–time signal is called Sampling.
- Sampling operation is performed in accordance with the sampling theorem.

3. State and prove Nyquist sampling theorem. (Dec 2010) (or)
What is sampling? Explain and derive the expression of sampling. (Nov 2015) (or)
Describe the process of sampling and how the message signal is reconstructed from its samples. Also illustrate the effect of aliasing with neat sketch. (Dec 2015) (or)
State the low pass sampling theorem and explain reconstruction of the signal from its samples. [May 2016]

Sampling: The process by which the continuous-time signal is converted into a discrete–time signal is called Sampling.

5.4.1 Sampling Theorem For Low-Pass Signals:-

Sampling theorem:

The bandpass signal $g(t)$ whose maximum bandwidth is *2W* can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth.

Sampling Theorem statements:

We may now state the ***sampling theorem**** for band-limited signals of finite energy in two separate parts

1. *If a finite-energy signal $g(t)$ contains no frequencies higher than W hertz, it is completely described by specifying its ordinates at a sequence of points spaced 1/2W seconds apart.*
2. *If a finite-energy signal $g(t)$ contains no frequencies higher than W hertz, it may be completely recovered from its ordinates at a sequence of points spaced 1/2W seconds apart.*

Proof:-

- Consider an analog signal $g(t)$ that is Continuous in both time and Amplitude.
- Assume that $g(t)$ has infinite duration but finite energy.
- A segment of the signal $g(t)$ is depicted in Figure (1).
- Let the sample values of the signal $g(t)$ at times $t = 0, \pm T_s, \pm 2T_s, ...,$ be denoted by the series $\{g(nT_s), n = 0, \pm 1, \pm 2, ...\}$.
- We refer to T_s as the Sampling period and as the sampling rate.
- We define the discrete-time signal, $g_\delta(t)$, that results from the sampling process as,

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s) \qquad (1)$$

where, $\delta(t - nT_s)$ =**Dirac delta function** located at time $t = n(T_s)$

$g_\delta(t)$ = Sample values

$g(t)$ = Signal

- In equation (1) each delta function in the series is weighted by the corresponding sample value of the input signal $g(t)$.
- Figure. 1(b) illustrates the representation of $g_\delta(t)$ from sample values of $g(t)$.
- From the definition of a delta function, we have

$$g(nT_s)\delta(t - nT_s) = g(t)\delta(t - nT_s)$$

- Hence we may rewrite equation (1) in the equivalent form

$$g_\delta(t) = g(t)\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$= g(t)\delta_{T_s}(t) \qquad (2)$$

where $\delta_{T_s}(t)$ = Dirac comb (or) ideal sampling function

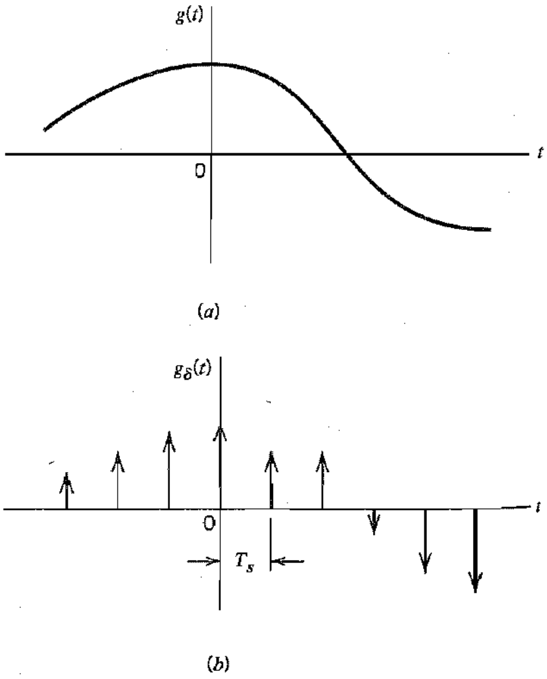


Figure 1. Sampling process

- From equation (2), the discrete-time signal $g_\delta(t)$ is the output of an ***impulse modulator***, which operates on $g(t)$ as the modulating wave and $\delta_{T_s}(t)$ as the carrier wave.
- This circuit-theoretic interpretation of $g_\delta(t)$ is depicted in Fig. (2)

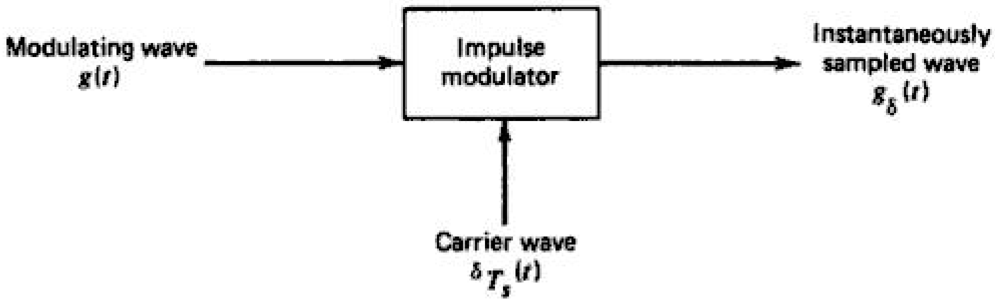


Figure 2: Circuit-theoretic interpretation of the ideal sampling process as impulse modulation

- From the properties of the F.T., the multiplication of two time functions, as in equation (2), is equivalent to the convolution of their respective Fourier transforms.

- Let $G(f)$ and $G_\delta(f)$ denote the Fourier transforms of $g(t)$ and $g_\delta(t)$, respectively.
- For the Fourier transform of $\delta_{T_s}(t)$, we have

$$F[\delta_{T_s}(t)] = f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s) \qquad (3)$$

where $F[\bullet]$ signifies the Fourier transform operation, and f_s is the sampling rate.

- Thus, transforming equation (2) into the frequency domain, we obtain

$$G_{\delta}(f) = G(f) * [f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)] \tag{4}$$

where $*$ denotes convolution.

- Interchanging the orders of summation and convolution yields.

$$G_{\delta}(f) = f_s \sum_{m=-\infty}^{\infty} G(f) * \delta(f - mf_s) \tag{5}$$

- From the properties of a delta function, we find that convolution of $G(f)$ and $\delta(f - mf_s)$ equals $G(f - mf_s)$.
- Hence, we may simplify equation (5) as follows:

$$G_{\delta}(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \tag{6}$$

- From equation (6) $G_{\delta}(f)$ represents a spectrum that is periodic in the frequency f with period f_s, but not necessarily continuous.
- In other words, the process of uniformly sampling a signal in the time domain resultsin a periodic spectrum in the frequency domain with a period equal to the sampling rate.
- Thus, $G_{\delta}(f)$ represents a periodic extension of the original spectrum $G(f)$.

- Another useful expression for the Fourier Transform $G_{\delta}(f)$ may be obtained by taking the Fourier Transform of both sides of Eq. (1) and noting that the F.T. of the Delta function $\delta(t - nt_s)$ is equal to $\exp(-2n\pi f T_s)$.
- We may thus write

$$G_{\delta}(f) = \sum_{m=-\infty}^{\infty} g(nT_s) \ \exp(-j2n\pi f T_s) \tag{7}$$

- This relation may be viewed as a complex F.S. representation of the periodic frequency function $G_{\delta}(f)$, with the sequence of samples $\{g(nT_s)\}$ defining the coefficients of the expansion.

- Note that in the F.S. defined by Eq. (7) the usual roles of time and frequency have been interchanged.

- These relations are applied to any continuous-time signal $g(t)$ of finite energy and infinite duration.
- Suppose, however that the signal is strictly band limited, with no frequency components higher than W hertz.

- That is the F.T. $G(f)$ of the signal $g(t)$ has the property that $G(f)$ is zero for $|f| \geq W$, as illustrated in Fig. 3(a)002E
- The shape of the spectrum shown in this figure is intended for the purpose of illustration only.
- The fact that the signal $g(t)$ has finite energy means that the area under the curve of the energy spectral density $|G(f)|^2$ is likewise finite.
- Suppose also that we choose the samplingperiod $T_s = 1/2W$.
- Then the corresponding spectrum $G_\delta(f)$ of the sampled signal $g_\delta(t)$ is as shown in Fig. 3 (b).

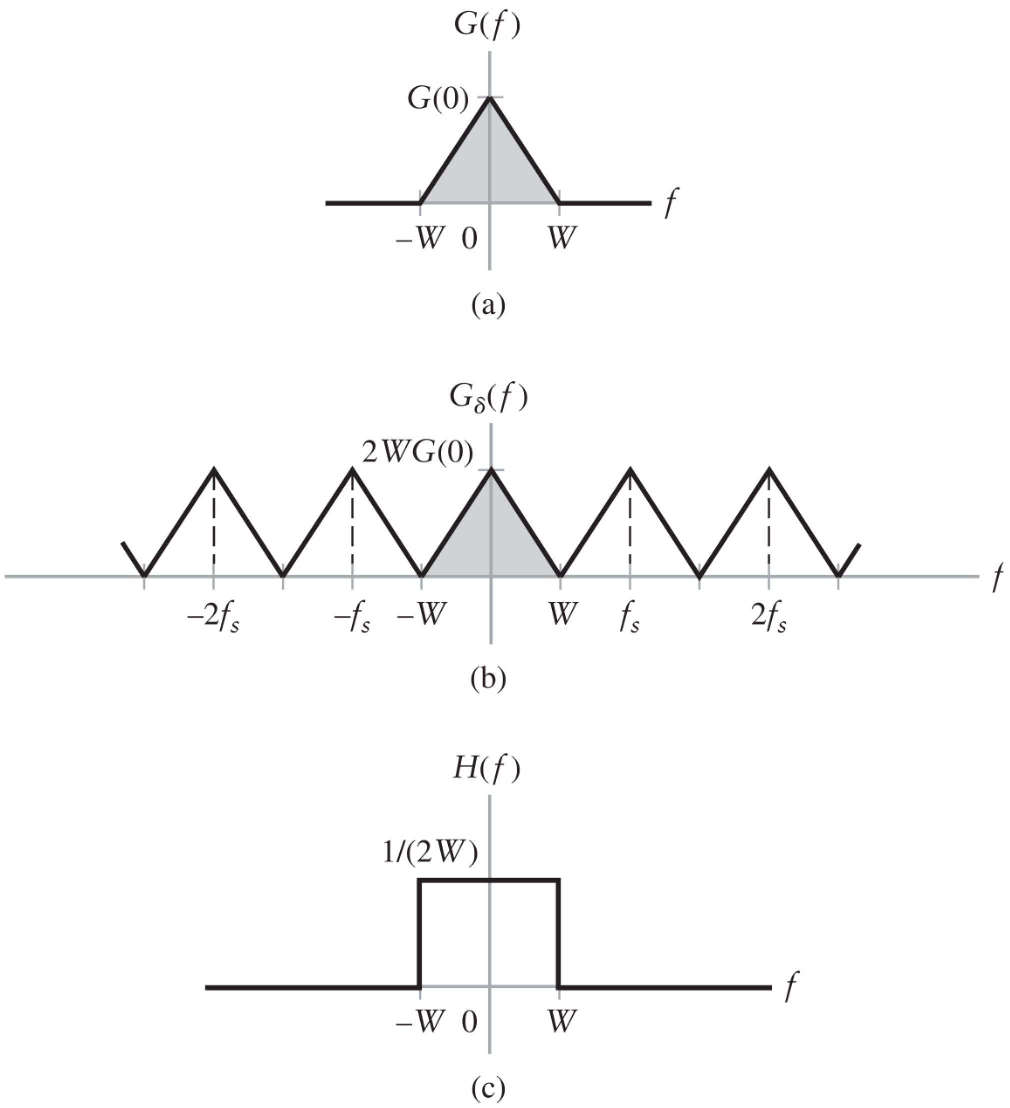


Figure 3: (a) Spectrum of signal $g(t)$. (b) Spectrum of sampled signal $g_\delta(t)$ for a sampling rate $f_s = 2W$. (c) Ideal amplitude response of reconstruction filter.

- substituting $T_s = 1/2W$ in Eq. (7) yields

$$G_\delta(f) = \sum_{m=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi nf}{W}\right) \qquad (08)$$

Putting $f_s=2W$ in Eq. (6), we have

$$G(f)=\frac{1}{2W}G_\delta(f),\ -W< f < W \qquad (09)$$

- It follows from Eq. (8) that we may also write

$$G(f)=\frac{1}{2W}\sum_{m=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right),\ -W< f < W \qquad (10)$$

- Therefore, if the sample values $g(n/2W)$ of the signal $g(t)$ are specified for all time, then the F.T. $G(f)$ of the signal is uniquely determined by using the F.S. of Eq. (10)
- Because $g(t)$ is related to $G(f)$ by the inverse F.T., it follows that the signal $g(t)$ is itself uniquely determined by the sample values $\{g(n/2W)\}$ for $-\infty \le n \le \infty$.
- In other words, the sequence $\{g(n/2W)\}$ contains all the information of $g(t)$.
- Consider next the problem in reconstructing the signal $g(t)$ from the sequence of sample values $\{g(n/2W)\}$.
- Substituting Eq. (10) in the formula for the inverse F.T. defining $g(t)$ in terms of $G(f)$, we get

$$g(t)=\int_{-\infty}^{\infty} G(f)\exp(j2\pi ft)df$$

$$=\int_{-W}^{W}\frac{1}{2W}\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\exp\left(-\frac{j\pi nf}{W}\right)\exp(j2\pi ft)df$$

- Interchanging the order of summation and integration:

$$g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{1}{2W}\int_{-W}^{W}\exp\left[j2\pi f\left(t-\frac{n}{2W}\right)\right]df \qquad (11)$$

- The integral term in Eq. (11) may be readily evaluated, yielding*

$$g(t)=\sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right)\frac{\sin(2\pi Wt-n\pi)}{(2\pi Wt-n\pi)} \qquad (12)$$

- We may simplify the notation in Eq. (12) by using the *sinc function*, defined as

$$\operatorname{sinc}x=\frac{\sin(\pi x)}{\pi x} \qquad (13)$$

wherexis an independent variable.

- The *sinc function* exhibits an important property known as the *interpolatory property*, which is describes as follows:

$$\text{sinc}x = \begin{cases} 1, x = 0 \\ 0, x = \pm 1, \pm 2, \ldots . \end{cases} \tag{14}$$

- Using the definition of the sinc function, we may rewrite Eq. (12) as follows:

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \text{sinc}\,(2\pi Wt - n) \tag{15}$$

- Eq. (15) provides an interpolation formula for reconstructing the original signal $g(t)$ from the sequence of sample values $\{g(n/2W)\}$,
- The sinc function *sinc(2Wt)*playing the role of an *interpolation function.*

- Each sample is multiplied by a delayed version of the interpolation function, and all the resulting waveforms are added to obtain.
- It is important that Eq. (15) represents ***the response of an ideal low-pass filter of bandwidthWand zero transmission delay***, which is produced by an input signal consisting of the sequence of samples $\{g(n/2W)\}$ for $-\infty \le n \le \infty$.

- From the spectrum in Fig. 3 (b), the *original signal* $g(t)$ *may be recovered* exactly from the sequence of samples $\{g(n/2W)\}$*by passing it through an ideal low-pass filter* of bandwidth *W*.
- This is illustrated in block diagrammatic form in Fig. (4).

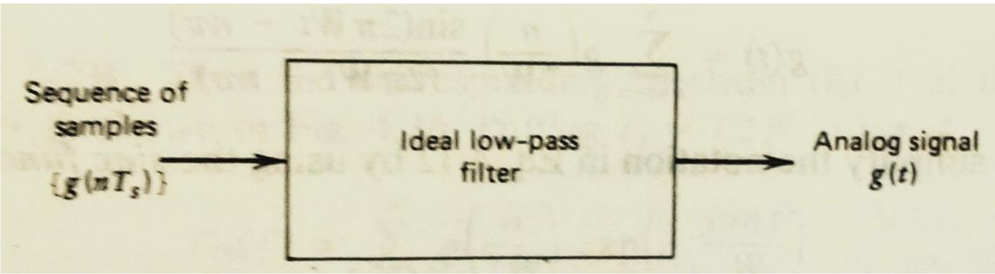


Figure 4: Reconstruction filter

- The ideal amplitude response of the reconstruction filter is shown in Fig. 3(c).

(1) Signal Space Interpolation

4. Explain in detail about signal space interpolation.

- We may develop another interpretation of Eq. (15) b using the property of the function *sinc*(2*Wt*– *n*), where *n* is an integer, is one of a family of shifted ***sinc*** functions that are mutually orthogonal.
- To prove this property, we use the formula

$$\int_{-\infty}^{\infty} g_1(t) g_2{}^*(t) dt = \int_{-\infty}^{\infty} G_1(t) G_2{}^*(t) df \qquad (16)$$

Where $g_1(t)$ and $G_1(f)$ form a F.T. pair; likewise for $g_2(t)$ and $G_2(f)$.

- This relation may be viewed as a generalization of *Rayleigh's energy theorem*.
- Put

$$g_1(t) = \text{sinc}\,(2Wt - n) = \text{sinc}\left[2W\left(t - \frac{n}{2W}\right)\right]$$

and

$$g_2(t) = \text{sinc}\,(2Wt - m) = \text{sinc}\left[2W\left(t - \frac{m}{2W}\right)\right]$$

- For a sinc pulse, *sinc(2Wt)*, we have the F.T pair:

$$\text{sinc}\,(2Wt) \Leftrightarrow \frac{1}{2W} rect\left(\frac{f}{2W}\right) \qquad (17)$$

where, on the right side, the definition of a rectangular function is used, namely

$$\text{rect}(x) = \begin{cases} 1, -\frac{1}{2} < x < \frac{1}{2} \\ 0, |x| > \frac{1}{2} \end{cases} \qquad (18)$$

- The functions of $g_1(t)$ and $g_2(t)$ are time-shifted versions of the sinc pulse *sinc(2Wt)*.
- Using the time shifting property of the F.T., we may express the F.T.s' of $g_1(t)$ and $g_2(t)$, as follows, respectively.

$$G_1(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right) \exp - \frac{j\pi nf}{W}$$

and

$$G_2(f) = \frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right) \exp - \frac{j\pi mf}{W}$$

- Hence, the use of these two F.T.s' inEq. (16) yields

$$\int_{-\infty}^{\infty} \text{sinc}\,(2Wt - n)\,\text{sinc}\,(2Wt - m)\,dt = \left(\frac{1}{2W}\right)^2 \int_{-W}^{W} \exp\left[-\frac{j\pi f}{W}(n - m)\right] df$$

$$= \frac{\sin[\pi(n-m)]}{2W\pi(n-m)}$$

$$= \frac{1}{2W} \text{sinc}\,(n - m)$$

- This result equals *1/2W*when $n = m$, and zero when $n \neq m$ (see Eq. (14)).
- We therefore have

$$\int_{-\infty}^{\infty} \operatorname{sinc}(2Wt-n)\operatorname{sinc}(2Wt-m)\,dt = \begin{cases} \frac{1}{2W}, n=m \\ 0, n \neq m \end{cases} \tag{19}$$

- Eq.(15) represents the expansion of the signal $g(t)$ as an infinite sum of orthogonal functions with the coefficients of the expansion, defined by

$$g\left(\frac{n}{2W}\right) = 2w\int_{-\infty}^{\infty} g(t)\operatorname{sinc}(2Wt-n)\,\mathrm{dt} \tag{20}$$

- The coefficients $g(n/2W)$ of this expansion are coordinates in an infinite-dimensional signal space.
- In this space each signal corresponds to precisely one point and each point to one signal.

(2) Statement of the Sampling Theorem:

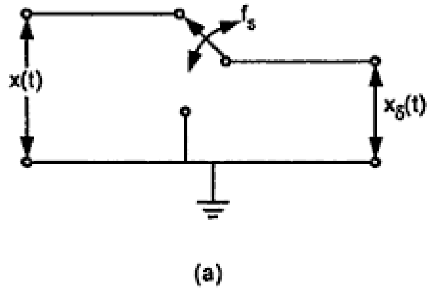
5. Give the statement of sampling theorem. (Nov 2013, Dec 2010, May 2012)

- The ***sampling theorem**** for band-limited signals of finite energy in two separate parts
  1. *If a finite-energy signal $g(t)$ contains no frequencies higher than W hertz, it is completely determined by specifying its ordinates at a sequence of points spaced 1/2W seconds apart.*
  2. *If a finite-energy signal $g(t)$ contains no frequencies higher than W hertz, it may be completely recovered from its ordinates at a sequence of points spaced 1/2W seconds apart.*
- Part 1 is a restatement of Eq. (10), and part 2 is restatement of Eq. (15).
- ***Nyquist rate:***The minimum sampling rate of *2W* samples per second, for a signal bandwidth of *W* hertz, iscalled the *Nyquist rate*.
- ***Nyquist interval*** **:**The reciprocal, *1/2W*, is called the *Nyquist interval*.
- The sampling theorem is the beginning for the ***interchangeability of analog signals and digital sequences***, which is so valuable in digital communication systems.

5.4.2 Types of sampling (Practical Sampling):

6. What are the types of sampling? (or)What is natural sampling and flat top sampling? (May 2010)

1. ***Ideal Sampling (or)Instantaneous sampling (or) Impulse sampling:***



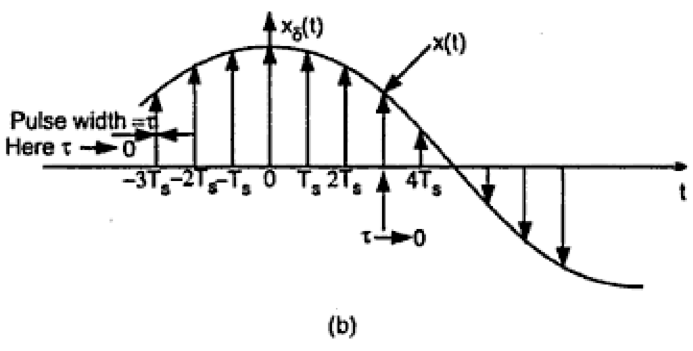


Fig 5(a) *Functional diagram of a* ***Fig 5(b)*** *Message $x(t)$ and sampled $x_\delta(t)$*

switching sampler *signals*

- Ideal sampling is same as instantaneous sampling.
- Fig.5(a)shows the switching sampler.
- If closing time't' of the switch approaches zero the output $x_{\delta}(t)$ gives only instantaneous value. The waveforms are shown in Fig. 5(b).
- Since the width of the pulse approaches zero, the instantaneous sampling gives train of impulsesin $x_{\delta}(t)$. The area of each impulse in the sampled version is equal to instantaneous value of input signal $x(t)$.

2. *Natural Sampling (or) Chopper Sampling:*

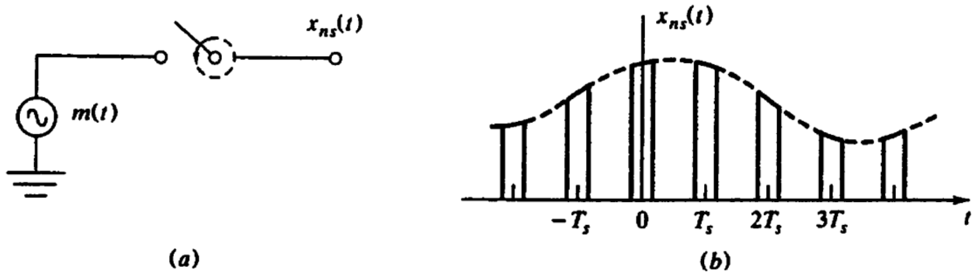


Figure6. Natural sampling

- Although instantaneous sampling is a convenient model, a more practical way of sampling a band-limited analog signal *m(t)* is performed by high-speed switching circuits.
- An equivalent circuit employing a mechanical switch and the resulting sampled signal are shown in Fig. 6(a) and (b),respectively.
- The sampled signal $x_{ns}(t)$ can be written as

$$x_{ns}(t) = m(t)x_p(t) \qquad \rightarrow (1)$$

Where $x_p(t)$ is the periodic train of rectangular pulses with period T_s, and each rectangular pulse in $x_p(t)$ has width *d* and unit amplitude.

- The sampling here is termed natural sampling, since the top of each pulse in $x_{ns}(t)$ retains the shape of its corresponding analog segment during the pulse interval.

3. *Flat-Top Sampling (or) Rectangular Pulse Shaping:*

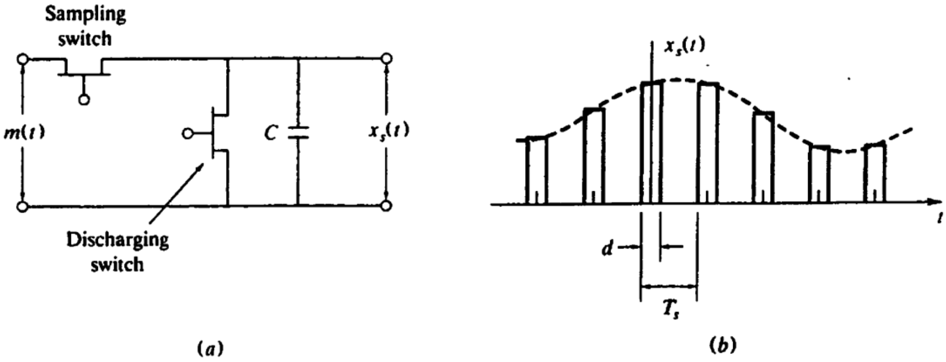
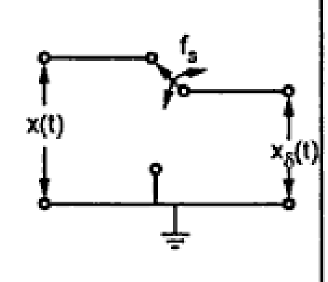
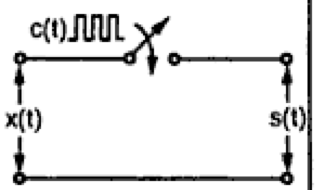
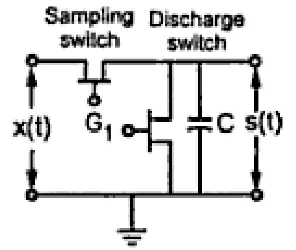
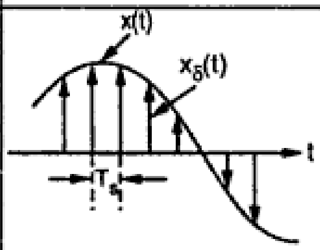
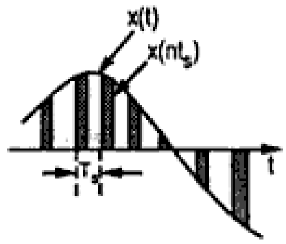
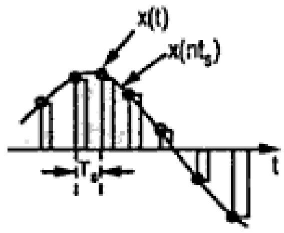


Figure 7. Flat-top Sampling

- ➢ The simplest and thus most popular practical sampling method is actually performed by a functional block termed the sample-and-hold (S/H) circuit [Fig. 7(a)].
- ➢ This circuit produces a flat-top sampled signal $x_s(t)$ [Fig. 7(b)].

5.4.3 Comparison of Various Sampling Techniques

7. Compare the types of sampling.

Sr. No.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1	Principle of sampling	It uses multiplication by an impulse function	It uses chopping principle	It uses sample and hold circuit
2	Circuit of sampler			
3	Waveforms			
4	Realizability	This is not practically possible method	This method is used practically	This method is used practically

5.5 Aliasing:

8. What is meant by aliasing effect, how it could be rectified?(Dec 2015)(or)
Write a detailed note on Aliasing and Signal Restoration. (06m) (April/May 2018) (or)
Explain in detail about aliasing.(or)
Explain in detail about the effects of undersampling.(Nov 2015)(or)
Write short notes on signal reconstruction (3m) (Nov 2017)

[Apr - 2019]

Aliasing Phenomenon

- Derivation of the sampling theorem, is based on the assumption that the signal $g(t)$ is strictly band-limited.
- In practice, the information-bearing signalfrom the source is not a strictly band-limited signal.
- So, it resultsin some degree of ***undersampling***.
- As a result, aliasing is produced by the sampling process.

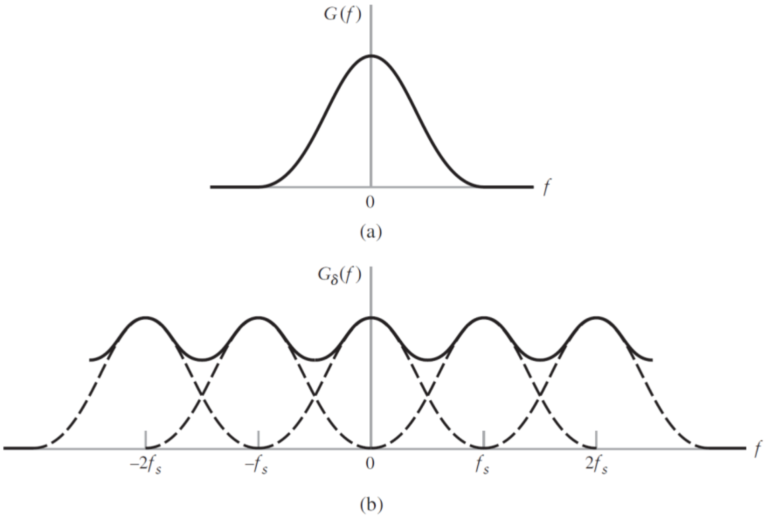


Figure 8. (a) Spectrum of a signal. (b) Spectrum of an undersampled version of the signal, exhibiting the aliasing phenomenon.

- Aliasing effect:
  - ✓ Aliasing refers to the phenomenon of ***a high-frequency component in the spectrum of the signal interferes and appears as lower frequency in the spectrum of its sampled version***, (as illustrated in Fig.)
- The aliased spectrum shown by the solid curve in Fig. 8(b) is related to an "undersampled" version of the message signal represented by the spectrumof Fig. (a).
- To reduce the effects of aliasing in practice, thereare two corrective measures:

1. Before sampling, a low-pass anti-alias filter is used to ***attenuate those high-frequencycomponents of the message signal*** that are not essential to the information being conveyedby the signal.
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.

- The use of a sampling rate higher than the Nyquist rate eases the design of the synthesis filter which is used to recover the original signal from its sampledversion.
- Consider the example of a message signal that has been ***anti-alias (low-pass) filtered***, resulting in the spectrum shown in Fig. 9(a).
- The spectrum of theinstantaneously sampled version of the signal is shown in Fig. 9(b), assuming a samplingrate higher than the Nyquist rate.
- Fromfig. 9(b), the design of a physically realizable reconstruction filter to recoverthe original signal from its uniformly sampled version may be achieved as follows (seeFig. 9(c)):
  - ✓ The reconstruction filter is of a low-pass kind with a passband extending from $-W$ to W, which is itself determined by the anti-alias filter.
  - ✓ The filter has a non-zero transition band extending (for positive frequencies) from W to $f_s - W$, where f_s is the sampling rate.

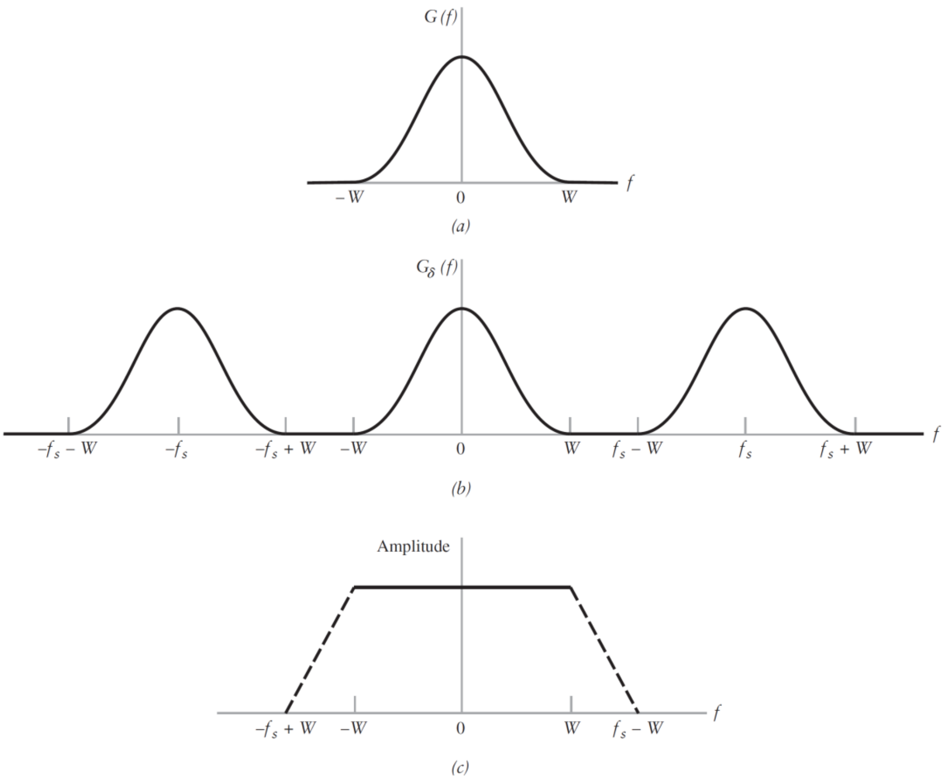


Fig 9 (a) Anti-alias filtered spectrum of an information-bearing signal. (b) Spectrumof instantaneously sampled version of the signal, assuming the use of a sampling rate greaterthan the Nyquist rate. (c)Idealized amplitude response of the reconstruction filter.

- The non-zero transition band of the filter assures physical realizability, it is shown as dashed linesto emphasize the arbitrary way of actually realizing it.

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5.6 Signal Reconstruction

Reconstruction of a message process from its samples

9. Explain in detail about the reconstruction message process from its samples. (or) Derive the mean square value of error in reconstruction process. (Dec 2015)

- This process completes the sampling process.
- Consider a wide-sense stationary message process $X(t)$ with autocorrelation function $R_X(\tau)$ and power spectral density $S_X(f)$.
- We assume that

$$S_x(f) = 0 \quad for|f| \geq W \tag{01}$$

- Consider an infinite sequence of samples taken at a uniform rate equal to $2W$, that is, twice the highest frequency component of the process.
- Using $X'(t)$ to denote the reconstructed process, based on this infinite sequence of samples, we may write

$$X'(t) = \sum_{n=-\infty}^{\infty} X\left(\frac{n}{2W}\right) sinc(2Wt - n) \tag{02}$$

where $X(n/2W)$ is the random variable obtained by sampling or observing the message process $X(t)$ at time $t = n/2W$.

- The mean-square value of the error between the original message process $X(t)$ and the reconstructed message process $X'(t)$ equals

$$\xi = E[(X(t) - X'(t))^2]$$

$$= E[(X^2(t)] - 2E[X(t)X'(t)] + E[(X'(t))^2] \tag{03}$$

- The first expectation term on the right side of Eq. (03) as the mean-square value of $X(t)$, which equals $R_X(0)$; thus

$$E[(X^2(t)] \quad = R_X(0) \tag{04}$$

- For second expectation term, use Eq. (02) and so write

$$E[X(t)X'(t)] \quad = E\left[X(t)\sum_{n=-\infty}^{\infty} X\left(\frac{n}{2W}\right) \text{sinc}(2Wt - n)\right]$$

- Interchanging the order of summation and expectation:

$$E[X(t)X'(t)] = \sum_{n=-\infty}^{\infty} E\left[X(t)X\left(\frac{n}{2W}\right)\right] \operatorname{sinc}(2Wt-n)$$

$$= \sum_{n=-\infty}^{\infty} R_X\left(t-\frac{n}{2W}\right) \operatorname{sinc}(2Wt-n) \qquad (05)$$

- For a stationary process, the expectation $E[X(t)X'(t)]$ is independent of time *t*.
- Hence, putting *t=0*in the right side of Eq. (05) and recognizing that

$$R_X\left(-\frac{n}{2w}\right) = R_X\left(\frac{n}{2W}\right)$$

- It may be written as

$$E[X(t)X'(t)] = \sum_{n=-\infty}^{\infty} R_X\left(\frac{n}{2W}\right) \operatorname{sinc}(-n) \qquad (06)$$

- The term $R_X\left(\frac{n}{2W}\right)$represents sample of the autocorrelation function $R_X(\tau)$ taken at $\tau = n/2W$.
- Now, since the power spectral density $S_X(f)$ or equivalently the F.T. of $R_X(\tau)$ is zero for $|f| > W$, we may represent $R_X(\tau)$ in terms of its samples taken at $\tau = n/2W$ as follows

$$R_X(\tau) = \sum_{n=-\infty}^{\infty} R_X\left(\frac{n}{2W}\right) \operatorname{sinc}(2W\tau-n) \qquad (07)$$

If $\tau = 0 \Rightarrow R_X(0) = \sum_{n=-\infty}^{\infty} R_X\left(\frac{n}{2W}\right) \operatorname{sinc}(-n)$

Accordingly, we deduce from Eqs. (06) and (07) that

$$(06) \Rightarrow E[X(t)X'(t)] = R_X(0) \qquad (08)$$

- For third and final expectation term on the right side of Eq. (03), we again use Eq. (02) and so write

$$E[(X'(t))^2] = \mathrm{E}\left[\sum_{n=-\infty}^{\infty} X\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt-n) \sum_{k=-\infty}^{\infty} X\left(\frac{k}{2W}\right) \operatorname{sinc}(2Wt-k)\right]$$

$$= \mathrm{E}\left[\sum_{n=-\infty}^{\infty} \operatorname{sinc}(2Wt-n) \sum_{k=-\infty}^{\infty} X\left(\frac{n}{2W}\right) X\left(\frac{k}{2W}\right) \operatorname{sinc}(2Wt-k)\right]$$

- Interchanging the order of expectation and inner summation:

$$E[(X'(t))^2] = \sum_{n=-\infty}^{\infty} \operatorname{sinc}(2Wt-n) \sum_{k=-\infty}^{\infty} E\left[X\left(\frac{n}{2W}\right) X\left(\frac{k}{2W}\right)\right] \operatorname{sinc}(2Wt-k)$$

$$= \sum_{n=-\infty}^{\infty} \text{sinc}\,(2Wt-n) \sum_{k=-\infty}^{\infty} R_X\left[\frac{n-k}{2W}\right] \text{sinc}\,(2Wt-k) \qquad (09)$$

- However, in view of Eq. (07), the inner summation on the right side ofEq. (09)equals $R_X\left(t-\frac{n}{2W}\right)$.
- Hence, we may simplify Eq. (09) as follows

$$E[(X'(t))^2] = \sum_{n=-\infty}^{\infty} R_X\left[t-\frac{n}{2W}\right] \text{sinc}\,(2Wt-n) \qquad (10)$$

$$= R_X(0)$$

- Finally, substituting Eqs. (04) , (08), into (10), we get the result

$$\xi = 0$$

as should be expected.

- We may therefore state the sampling theorem for message processes as follows.
  - ✓ If a stationary message process contains no frequencies higher than *W* hertz, it may be reconstructed from its samples at a sequence of points spaced *1/2W*seconds apart with zero mean squared error (i.e., Zero error power).

5.7 Quantization

10. Explain in detail about the quantization process. [Apr 2010, Apr 2011]
(or)
Illustrate and describe the types of quantizer? Describe the midtread and midrise type characteristics of uniform quantizer with a suitable diagram. [Dec 2016]

- A continuous signal (i.e., voice) has a continuous range of amplitudes and therefore its samples also have a continuous amplitude range.
- In other words, within the finite amplituderange of the signal, there are infinite number of amplitude levels.
- It is not necessary in fact to transmit the exact amplitudes of the samples.

- Any human sense (the ear or the eye), can detect only finite intensity differences.
- So, the original continuous signal will be approximated by a signal constructed of discrete amplitudes.
- The existence of a finite number of discrete amplitude levels is a basic condition of pulse-code modulation.

- **Amplitude quantization** is defined *as the process of transforming the sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible amplitudes.*

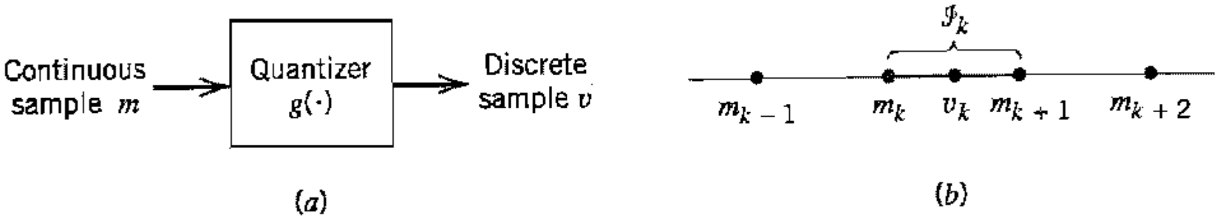

Fig: 10. Description of a memoryless quantizer

- Assume that the quantization process ismemoryless and instantaneous.
- It means, the transformation at time $t = nT_s$ is not affected by earlier or later samples of the message signal.
- This simple form of scalar quantization is commonly used in practice.

- When dealing with a memoryless quantizer, we may simplify the notation by dropping the time index.
- The symbol***m*** in place of ***m(nT$_s$)***as indicated in the block diagram of a quantizer shown in Figure 10a.
- Then, as shown in Figure. 10b, the signal amplitude ***m*** is specified by the index ***k*** if it lies inside the partition cell

$$\mathcal{J}_k: \{m_k < m \leq m_{k+1}\}, \qquad k = 1, 2, \ldots, L$$

where ***L*** is the total number of amplitude levels used in the quantizer.
- The discrete amplitudesm_k,$k = 1, 2, \ldots, L$, at the quantizer input are called ***decision levels*** or ***decision thresholds***.

- At the quantizer output, the index ***k*** is transformed into an amplitude v_k that represents all amplitudes of the cell $\mathcal{J}_k$.
- The discrete amplitudes v_k,$k = 1, 2, \ldots, L$, are called ***representation levels*** or ***reconstruction levels***,
- The spacing between two adjacent representation levels is called a ***quantum size or step-size***.
- Thus, the quantizer output v equals v_k if the input signal sample ***m*** belongs to the interval$\mathcal{J}_k$.
- The mapping,

$$v = g(m)$$

is the quantizer characteristic, which is a staircase function by definition.
- Types of quantizers:
  - ✓ Quantizers can be of a *uniform or nonuniform* type.
  - ✓ In a uniform quantizer, therepresentation levels are uniformly spaced; otherwise, the quantizer is nonuniform.

5.7.1 Uniform & non-uniform quantization:

11. Illustrate and describe the types of quantizer? Describe the midtread and midrise type characteristics of uniform quantizer with a suitable diagram. [Dec 2016]

- In a uniform quantizer, the representation levels are uniformly spaced; otherwise, the quantizer is nonuniform.

5.7.1.1 Uniform Quantization

- The quantizer characteristic can also be of a midtread or midrise type.
- Midread:
  - ✓ Figure 11(a) shows the input–output characteristic of a uniform quantizer of the midtread type
  - ✓ It is so called because ***the origin lies in the middle of a tread of the staircaselike graph***.

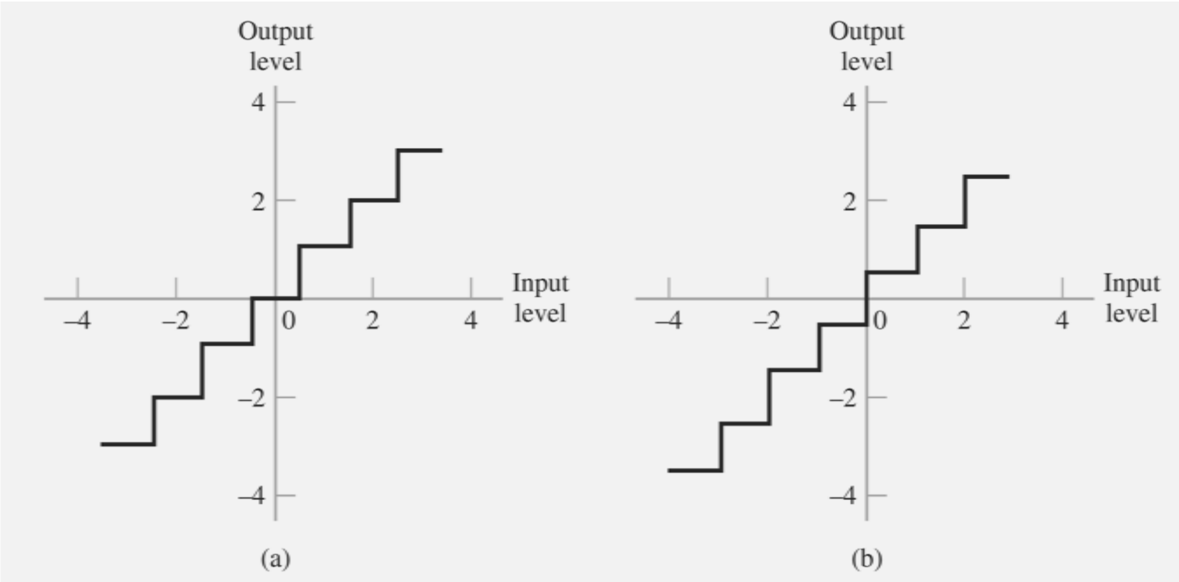

Figure 11. Two types of quantization: (a) midtread and (b) midrise.

- Midrise:
  - ✓ Figure 11(b) shows the corresponding input–output characteristic of a uniform quantizer of themidrise type.
  - ✓ It is so called because***the origin lies in the middle of a rising part of the staircaselike graph.***
- Note that both the midtread and midrise types of uniformquantizers aresymmetric about the origin.

5.7.1.2 Nonuniform Quantization

12. Explain non-uniform quantization. (Apr 2010, Apr 2011, May 2014)

- The sampled version of the message signal will be quantized.
- Quantization provides a newrepresentation of the signal that is ***discrete in both time and amplitude***.
- In some applications,it is preferred to use a variable separation between therepresentation levels.
- For example, the range of voltages covered by voice signals,from the peaks of loud talk to the weak passages of weak talk, is on the order of1000 to 1.

- By using *a nonuniformquantizer* with the feature that the step size increasesas the separation from the origin of the input–output amplitude characteristic isincreased
- The ***large end-step of the quantizer*** can take care of possible excursions ofthe voice signal into the large amplitude ranges that occur in rare.

- The use of a nonuniformquantizer is equivalent to *passing the message signalthrough a **compressor** and then applying the compressed signal to a **uniform quantizer***.

$\mu - law$

- A particular form of compression law that is used in practice is the so called $\mu - law$ defined by

$$|v| = \frac{\log(1 + \mu|m|)}{\log(1 + \mu)} \tag{01}$$

where the logarithm is the natural logarithm; **m** and***v*** are respectively the normalized input and output voltages, and μ is a positive constant.

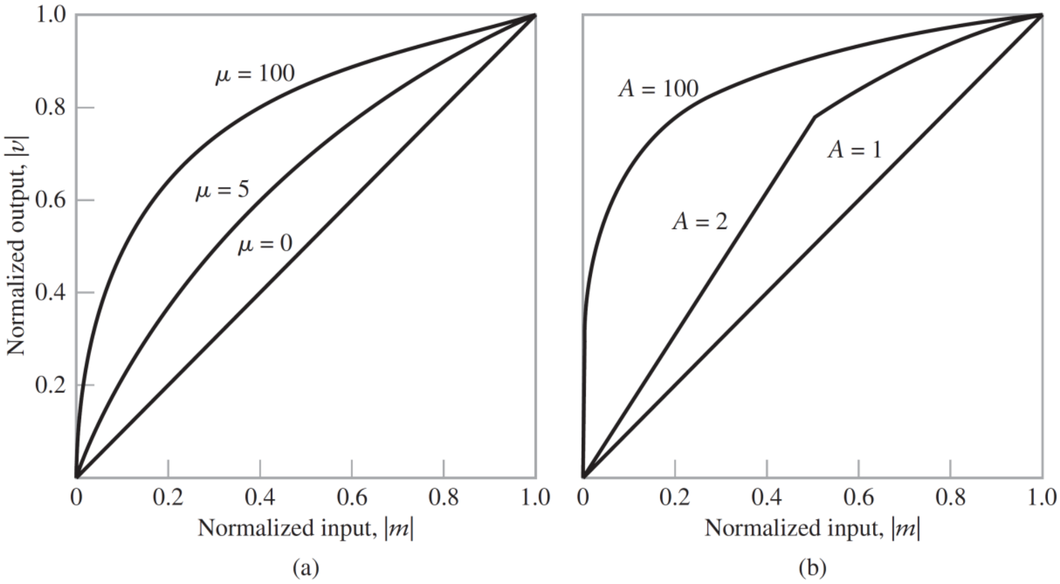

Figure 12. Compression laws. (a) m-law. (b) A-law.

- For convenience of presentation, the input to the quantizer and its output are both normalized to occupy ***a dimensionless range of values*** from zero to one, as shown in Figure12(a); here $\mu - law$is plotted for varying μ .
- Practical values of μtend to be approximately 255. The case of uniform quantization corresponds to $\mu = 0$.

- For a given value of μ,the reciprocal slope of the compression curve, which defines the quantum steps,is given by the derivative of $|m|$ with respect to $|v|$ that is,

$$\frac{d|m|}{d|v|} = \frac{\log(1 + \mu)}{\mu}(1 + \mu|m|) \qquad (02)$$

- The $\mu - law$ is neither strictly linear nor strictly logarithmic
- But it is approximately linear at low input levels corresponding to $\mu|m| << 1$ and approximately logarithmic at high input levels corresponding to $\mu|m| >> 1$.

A-Law:

- Another compression law that is used in practice is the so-called A-law, defined by

$$|v| = \begin{cases} \dfrac{A|m|}{1 + \log A}, & 0 \leq |m| \leq \dfrac{1}{A} \\ \dfrac{1 + \log(A|m|)}{1 + \log A}, & \dfrac{1}{A} \leq |m| \leq 1 \end{cases} \qquad (03)$$

which is shown plotted in Figure 12(b). Typical values of A used in practice tend to be in the vicinity of 100. The case of uniform quantization corresponds to $A = 1$.

- The reciprocal slope of this second compression curve is given by the derivative of $|m|$ with respect to $|v|$ as shown by

$$\frac{d|m|}{d|v|} = \begin{cases} \dfrac{1 + \log A}{A}, & 0 \leq |m| \leq \dfrac{1}{A} \\ (1 + \log A)|m|, & \dfrac{1}{A} \leq |m| \leq 1 \end{cases} \qquad (04)$$

- From the first line of Eq. (04), the quantum steps over the central linear segment, which have the dominant effect on small signals, are diminished by the factor $A/(1+\log A)$.
- This is typically about 25 dB in practice, as comparedwith uniform quantization.

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5.8 Quantization noise:

13. With proper diagram explain the noise due to quantization in digitalization process.
[May 2006, 2013], [Dec 2005, 2008, 2012, 2013, 2014]
Derive the expression for signal to noise ratio of uniform quantizer.
[April 2018, Nov 2017]

5.8.1 Illustration of *Quantization noise:*

Quantization introduces an error, defined as, the difference between the input signal *m* and the output signal *v*. The error is called ***quantization noise***.

- Figure 13 shows a typical variation of the quantization noise as a function of time, *assuming the use of a uniform quantizer of the midtread type.*
- Let the quantizer input *m* be the sample value of a ***zero-mean random variable****M*.(If the input has a nonzero mean, it can be always removed by subtracting the mean from the input and then adding it back after quantization.)

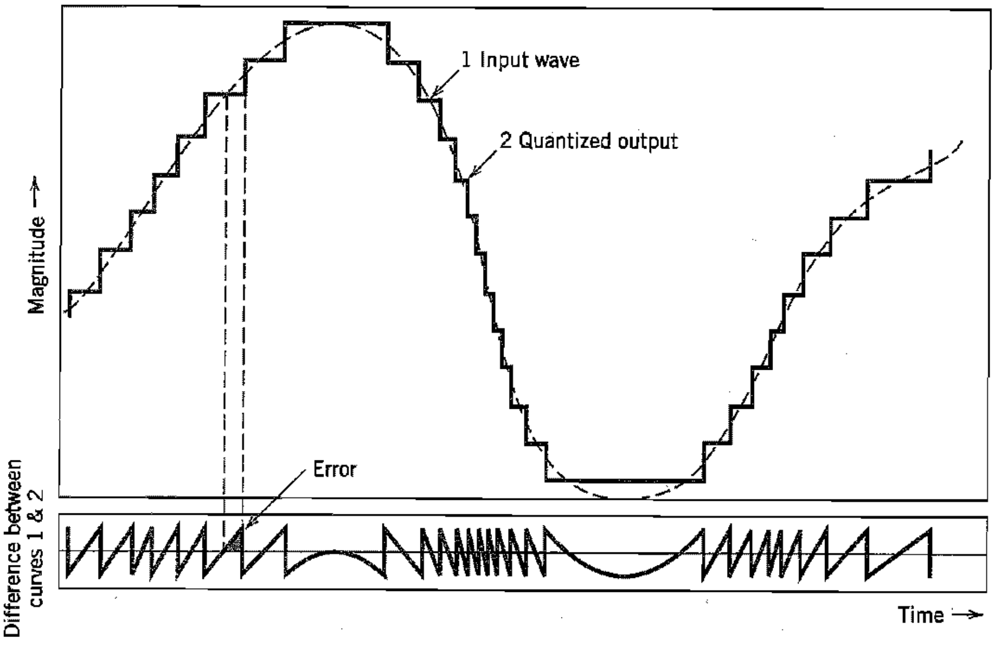


Figure13. Illustration of Quantization process

- A quantizer $g(\bullet)$ maps the inputrandom variable*M*of continuous amplitude into a discrete random variable *V*; their respective sample values *m* and *v* are related by Equation

$$v=g(m) \qquad (01)$$

- Let the quantization errorbe denoted by the random variable Q of sample value q.

We may thus write

$$q = m - v \qquad (02)$$

or, correspondingly,

$$Q = M - V \qquad (03)$$

- With the input *M* having zero mean, and the quantizer assumed to be symmetric as in *Figure 5.10*, it follows that the quantizer output *V*and therefore the quantization error *Q*, will also have zero mean.
- So, for the characterization of the quantizer in terms of output signal-to- quantization noise ratio, find the mean-square value of the quantization error Q.
- Consider then an input ***m*** of continuous amplitude in the range $(-m_{max}, m_{max})$.

5.8.2 Signal to noise ratio of uniform quantizer:

- Assuming a uniform quantizer of the midrise type illustrated in Figure 3.10b, we find that the step-size of the quantizer is given by

$$\Delta = \frac{2m_{\max}}{L} \qquad (04)$$

where L is the total number of representation levels.

- For a uniform quantizer, the quantization error Q will have its sample values bounded by $-\Delta/2 \le q \le \Delta/2$.

- If thestep-size Δ is sufficiently small (i.e., the number of representation levels L is sufficiently large)
- It isreasonable to assume that the ***quantization error Q is a uniformly distributed random variable***, and the effect of the ***quantization noise on the quantizer input is similar to that of thermal noise***.
- Thus the probability density function of the quantization error Q is expressed as follows:

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \le \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (05)$$

- For this to be true, the incoming signal does not overload the quantizer.
- Then, with the mean of the quantization error being zero, its variance σ_Q^2 is the same as the mean-square value:

$$\begin{aligned} \sigma_Q^2 &= E[Q^2] \\ &= \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq \end{aligned} \qquad (06)$$

- Substituting Equation (5) into (6), we get

$$\sigma_Q^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq$$

$$= \frac{\Delta^2}{12} \qquad (07)$$

- Typically, the *L-ary*number *k*, denoting the K[th] representation level of the quantizer, istransmitted to the receiver in binary form.
- Let *R* denote the number of *bits per sample*used in the construction of the binary code.
- We may then write

$$L = 2^R \qquad (08)$$

or, equivalently,

$$R = \log_2^L \qquad (09)$$

- Hence, substituting Equation (8) into (4), we get the step size

$$\Delta = \frac{2m_{\max}}{2^R} \qquad (10)$$

- Thus the use of Equation (10) in (7) yields

$$\sigma_Q^2 = \frac{1}{3} m_{\max}^2\, 2^{-2R} \qquad (11)$$

- Let *P* denote the average power of the message signal *m(t)*. We may then express the output signal-to-noise ratio of a uniform quantizer as

$$(SNR)_O = \frac{P}{\sigma_Q^2}$$

$$= \frac{3P}{m_{\max}^2} 2^{2R} \qquad (12)$$

- Equation (12) shows that the output signal-to-noise ratio of the quantizer increases exponentially with increasing number of bits per sample, *R*.

- An increase in *R* requires a proportionate increase in the channel (transmission) bandwidth B_T.
- The use of a binary code for the representation of a message signal (as in pulse- code modulation) provides a more efficient method than either frequency modulation (FM) or pulse-position modulation (PPM) for the trade-off of increased channel bandwidth for improved noise performance.

- In making this statement, we assume that the FM and PPM systems are limited by receiver noise, whereas the binary-coded modulation system is limited by quantization noise.

5.9 Logarithmic Companding of speech signal

14. Explain in detail about Logarithmic Companding of speech signal. (or)
In detail explain lograthmic companding of speech signals (4m) [Nov 2017]

- Introduction Pulse code modulation (PCM) is a common method of digitizing or quantizing an analog waveform.
- For any analog-to-digital conversion process, the quantization step produces an estimate of the waveform sample using a digital codeword.
- This digital estimate inherently contains some level of error due to the finite number of bits available.
- In practical terms, there is always tradeoff between the amount of error and the size of the digital data samples.
- The goal in any system design is quantizing the data in smallest number of bits that results in a tolerable level of error.
- In the case of speech coding, linear quantization with 13 bits sampled at 8 KHz is the minimum required to accurately produce a digital representation of the full range of speech signals.
- For many transmission systems, wired or wireless, 13 bits sampled at 8 KHz is an expensive proposition as far as bandwidth is concerned. To address this constraint, a compandingsystem is often employed.

- Companding is simply a system in which information is first compressed, transmitted through a bandwidthlimited channel, and expanded at the receiving end.
- It is frequently used to reduce the bandwidth requirements for transmitting telephone quality speech, by reducing the 13-bit codewords to 8-bit codewords.
- Two international standards for encoding signal data to 8-bit codes are A-law and μ-law. A-lawis the accepted European standard, while μ-law is the accepted standard in the United States and Japan.

5.9.1 Speech Companding

- The human auditory system is believed to be a logarithmic process in which high amplitude sounds do not require the same resolution as low amplitude sounds.
- The human ear is more sensitive to quantization noise in small signals than large signals.
- A-law and μ-law coding apply a logarithmic quantization function to adjust the data resolution in proportion to the level of the input signal. Smaller signals are represented with greater precision – more data bits – than larger signals.

- The result is fewer bits per sample to maintainan audible signal-to-noise ratio (SNR).

- Rather than taking the logarithm of the linear input data directly, which can be computationally difficult, A-law/ μ -law PCM matches the logarithmic curve with a piece-wise linear approximation.

- Eight straight-line segments along the curve produce a close approximation to the logarithm function. Each segment is known as a chord.
- Within each chord, the piece-wise linear approximation is divided into equally size quantization intervals called steps.
- The step size between adjacent codewords is doubled in each succeeding chord.
- Also encoded is the sign bit for the original integer.
- The result is an 8-bit logarithmic code composed of a 1-bit sign bit, a 3-bit chord, and a 4-bit step.

5.9.2 A-Law Compander

- A-law is the CCITT recommended companding standard used across Europe.
- Limiting the linearsample values to 12 magnitude bits, the A-law compression is defined by Equation 1, where A is the compression parameter (A=87.7 in Europe), and x is the normalized integer to be compressed.

$$F(x) = \begin{cases} \dfrac{A * |x|}{1+\ln(A)} & 0 \le |x| < \dfrac{1}{A} \\ \dfrac{\operatorname{sgn}(x) * (1+\ln(A|x|))}{1+\ln(A)} & \dfrac{1}{A} \le |x| \le 1 \end{cases}$$

(Eq. 1), A-law definiton

- Table 1 illustrates an A-law encoding table. The sign bit of the linear input data is omitted fromthe table.
- The sign bit (S) for the 8-bit code is set to 1 if the input sample is negative, and is set to0 if the input sample is positive.

Linear Input Data												A-law Encoded Output							
0	0	0	0	0	0	0	A	B	C	D	X	S	0	0	0	A	B	C	D
0	0	0	0	0	0	1	A	B	C	D	X	S	0	0	1	A	B	C	D
0	0	0	0	0	1	A	B	C	D	X	X	S	0	1	0	A	B	C	D
0	0	0	0	1	A	B	C	D	X	X	X	S	0	1	1	A	B	C	D
0	0	0	1	A	B	C	D	X	X	X	X	S	1	0	0	A	B	C	D
0	0	1	A	B	C	D	X	X	X	X	X	S	1	0	1	A	B	C	D
0	1	A	B	C	D	X	X	X	X	X	X	S	1	1	0	A	B	C	D
1	A	B	C	D	X	X	X	X	X	X	X	S	1	1	1	A	B	C	D

Table 1, A-Law Encoding

- After the input data is encoded through the logic defined in the table, an inversion pattern isapplied to the 8-bit code to increase the density of transitions on the transmission line, a benefit tohardware performance.
- The inversion pattern is applied by XOR'ing the 8-bit code with 0x55.
- Decoding the A-law encoded data is essentially a matter of reversing the steps in the encoding.
- Table 2 illustrates the A-law decoding table, applied after reversing the inversion pattern.
- Theleast significant bits discarded in the encoding process are approximated by the median value ofthe interval. This is shown in the output section by the trailing 1..0 pattern after the D bit.

A-law Encoded Input								Linear Output Data											
S	0	0	0	A	B	C	D	0	0	0	0	0	0	0	A	B	C	D	1
S	0	0	1	A	B	C	D	0	0	0	0	0	0	1	A	B	C	D	1
S	0	1	0	A	B	C	D	0	0	0	0	0	1	A	B	C	D	1	0
S	0	1	1	A	B	C	D	0	0	0	0	1	A	B	C	D	1	0	0
S	1	0	0	A	B	C	D	0	0	0	1	A	B	C	D	1	0	0	0
S	1	0	1	A	B	C	D	0	0	1	A	B	C	D	1	0	0	0	0
S	1	1	0	A	B	C	D	0	1	A	B	C	D	1	0	0	0	0	0
S	1	1	1	A	B	C	D	1	A	B	C	D	1	0	0	0	0	0	0

Table 2, A-Law Decoding

5.9.3 μ-Law Compander

- The United States and Japan use μ-law companding. Limiting the linear sample values to 13magnitude bits, the μ-law compression is defined by Equation 2, where m is the compressionparameter (m =255 in the U.S. and Japan) and x is the normalized integer to be compressed.

$$F(x) = \frac{\operatorname{sgn}(x) * \ln(1+\mu|x|)}{\ln(1+\mu)} \quad 0 \leq |x| \leq 1$$

Equation 2, μ-Law Definition

- The encoding and decoding process for μ-law is similar to that of A-law. There are, however, afew notable differences:

  1) μ-law encoders typically operate on linear 13-bit magnitude data, asopposed to 12-bit magnitude data with A-law,

  2) before chord determination a bias value of 33 isadded to the absolute value of the linear input data to simplify the chord and step calculations,

  3)the definition of the sign bit is reversed, and 4) the inversion pattern is applied to all bits in the 8 bit code.

- Table 3 illustrates a μ-law encoding table. The sign bit of the linear input data is omitted from thetable.
- The sign bit (S) for the 8-bit code is set to 1 if the input sample is positive, and is set to 0 ifthe input sample is negative.

Linear Input Data													µ-law Encoded Output							
0	0	0	0	0	0	0	1	A	B	C	D	X	S	0	0	0	A	B	C	D
0	0	0	0	0	0	1	A	B	C	D	X	X	S	0	0	1	A	B	C	D
0	0	0	0	0	1	A	B	C	D	X	X	X	S	0	1	0	A	B	C	D
0	0	0	0	1	A	B	C	D	X	X	X	X	S	0	1	1	A	B	C	D
0	0	0	1	A	B	C	D	X	X	X	X	X	S	1	0	0	A	B	C	D
0	0	1	A	B	C	D	X	X	X	X	X	X	S	1	0	1	A	B	C	D
0	1	A	B	C	D	X	X	X	X	X	X	X	S	1	1	0	A	B	C	D
1	A	B	C	D	X	X	X	X	X	X	X	X	S	1	1	1	A	B	C	D

Table 3, µ-Law Encoding

- After the input data is encoded through the logic defined in the table, an inversion pattern isapplied to the 8-bit code to increase the density of transitions on the transmission line, a benefit tohardware performance. The inversion pattern is applied by XOR'ing the 8-bit code with 0xFF.
- Decoding the μ-law encoded data is essentially a matter of reversing the steps in the encoding.Table 4 illustrates the μ-law decoding table, applied after reversing the inversion pattern.
- Theleast significant bits discarded in the encoding process are approximated by the median value ofthe interval. This is shown in the output section by the trailing 1..0 pattern after the D bit.

µ-law Encoded Input								Linear Output Data												
S	0	0	0	A	B	C	D	0	0	0	0	0	0	0	1	A	B	C	D	1
S	0	0	1	A	B	C	D	0	0	0	0	0	0	1	A	B	C	D	1	0
S	0	1	0	A	B	C	D	0	0	0	0	0	1	A	B	C	D	1	0	0
S	0	1	1	A	B	C	D	0	0	0	0	1	A	B	C	D	1	0	0	0
S	1	0	0	A	B	C	D	0	0	0	1	A	B	C	D	1	0	0	0	0
S	1	0	1	A	B	C	D	0	0	1	A	B	C	D	1	0	0	0	0	0
S	1	1	0	A	B	C	D	0	1	A	B	C	D	1	0	0	0	0	0	0
S	1	1	1	A	B	C	D	1	A	B	C	D	1	0	0	0	0	0	0	0

Table 4, µ-Law Decoding

Summary

- There is a wide array of audio transmission systems that employ A-law and/or μ-law companding for data rate reduction with good audio quality.
- The compression achieved by both A-law and μ-law coding is the result of utilizing the logarithmic characteristics of the human auditory system, where fewer bits of precision are required for larger signals than smaller ones.
- The logarithmic transfer function is implemented with a piece-wise linear approximation composed of a sign bit, a 3-bit chord, and a 4-bit segment.
- The encoding and decoding process is presented in table format, well suited for hardware orsoftware implementation.

5.10 Pulse Amplitude Modulation (PAM)

Discuss about the generation of PAM and its demodulation.	***[Nov/Dec 2010]***

Introduction

- The amplitude of the pulse carrier is changed in proportion with the instantaneous amplitude of the modulating signal.

Types of PAM

Depending upon the shape of the PAM pulse, there are two types of PAM. They are:

(i) Natural PAM

(ii) Flat top PAM

The flat top pulses have constant amplitude within the pulse interval.

Why flat top PAM is widely used?

- During the transmission, the noise interferes with the flat top of the transmitted pulses and this noise can be easily removed.
- In natural samples PAM, the pulse has varying top in accordance with the signal variation.
- When such type of pulse is received by the receiver, it always seems to be contaminated by noise.
- Then it becomes quite difficult to determine the shape of the top of the pulse and therefore amplitude detection of those pulses is not exact.
- As a result of this, errors are introduced in the received signal.
- The electronic circuitry needed to perform natural sampling is somewhat complicated because the pulse top shape is to be maintained. These complications are reduced by flat-top PAM.

Natural PAM

Generation of natural PAM

- The modulating signal x (t) is passed through a low pass filter which will band limit this signal to f_m.
- That means all the frequency components higher than the frequency f_m are removed.
- Band limiting is necessary to avoid the "aliasing" effect in the sampling process.
- The pulse train generator generates a pulse train of frequency f_s, such that $f_s > 2\ f_m$. Thus the Nyquist criterion is satisfied. This is nothing but sampling signal.

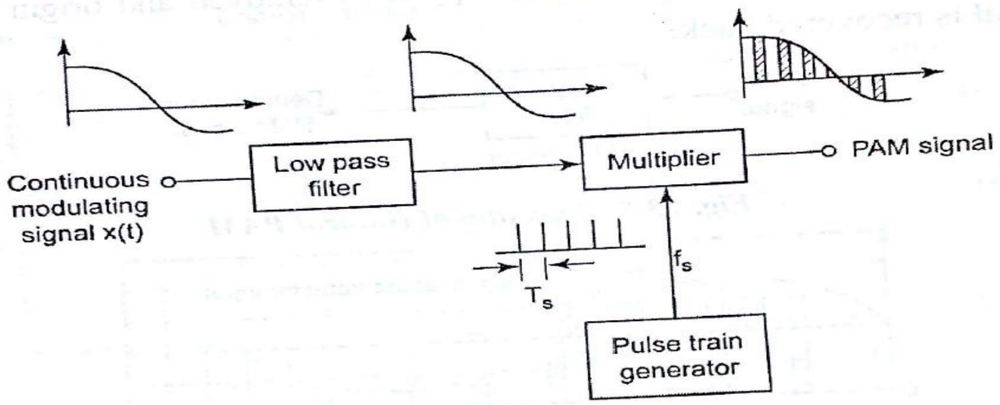


Fig : Generation of PAM

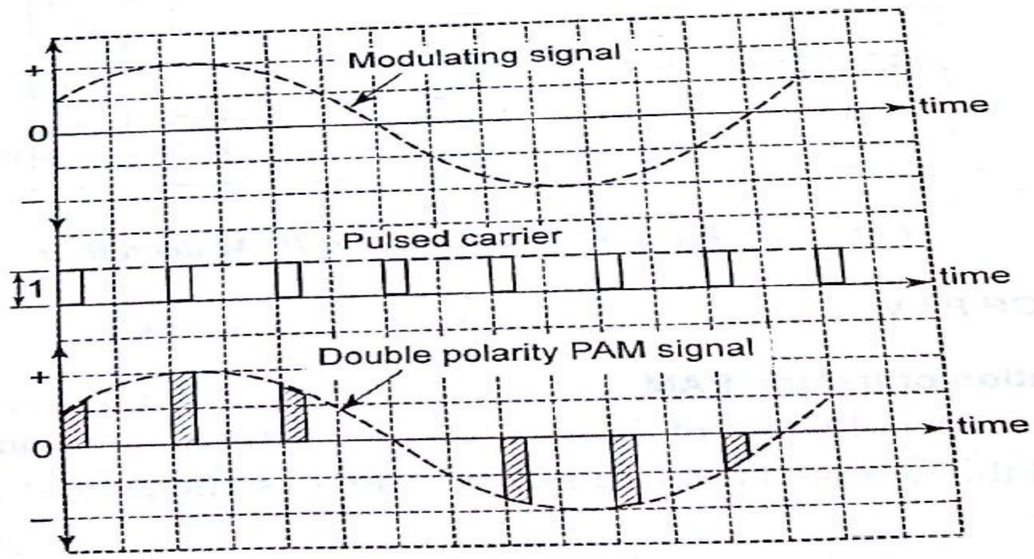


Fig : Waveforms of natural PAM generation

- The continuous time signal x (t) is applied at the input of a multiplier.
- The other input is a sampling signal s (t), which is a periodic train of pulses with unit amplitude and a period of "T_s" seconds.
- The uniform sampling takes place at the multiplier block to generate the PAM signal.
- The information in the modulating signal is contained in the "amplitude variations" of the pulsed carrier.

Detection of Natural PAM

- The PAM signal can be detected by passing it through a low pass filter, which is tuned to f_m.
- So all high frequency ripples is removed and original modulating signal is recovered back.

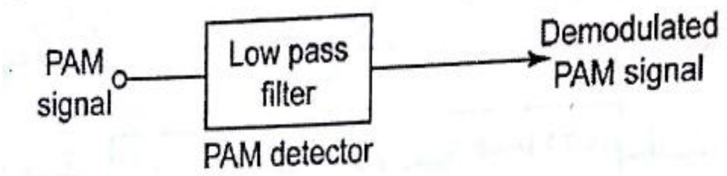


Fig : Detection of natural PAM

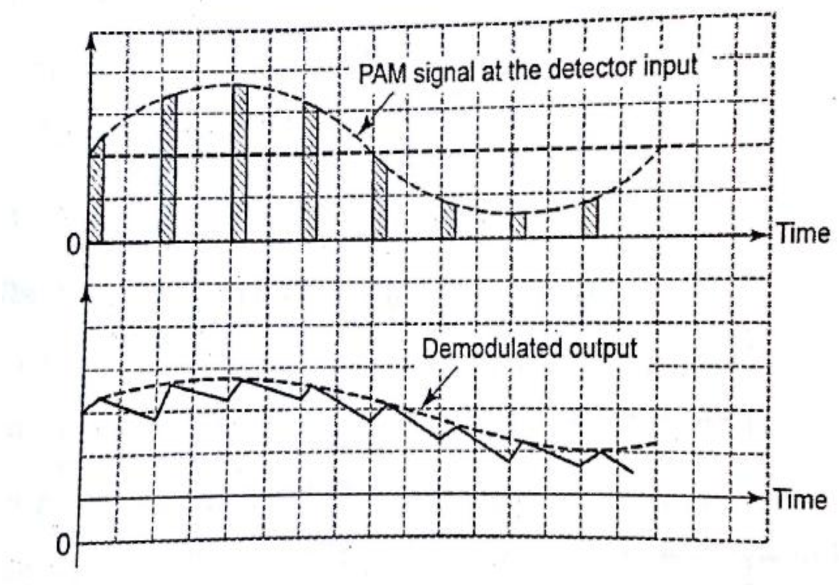


Fig : Waveforms of natural PAM detection

Flat top PAM

Generation of flat top PAM

- A sample and hold circuit is used to produce flat top sampled PAM. This consists of the two field effect transistors (FET) switches and a capacitor.
- Flat top PAM signals are generated by applying the input modulating signal x (t) to charging (sampling) switch.
- At the sampling instant, sampling switch is closed for a short duration by a short pulse applied to a gate G_1 of the transistor.
- During this period, the capacitor "C" quickly charged up to a voltage equal to the instantaneous sample value of the incoming signal x (t).
- Now, the sampling switch is opened and capacitor 'C' holds the charge.
- The discharge switch is then closed by a pulse applied to gate G2.
- Due to this, the capacitor "C" is discharged to zero volts.
- The discharges switch is then opened and thus capacitor has no voltage.

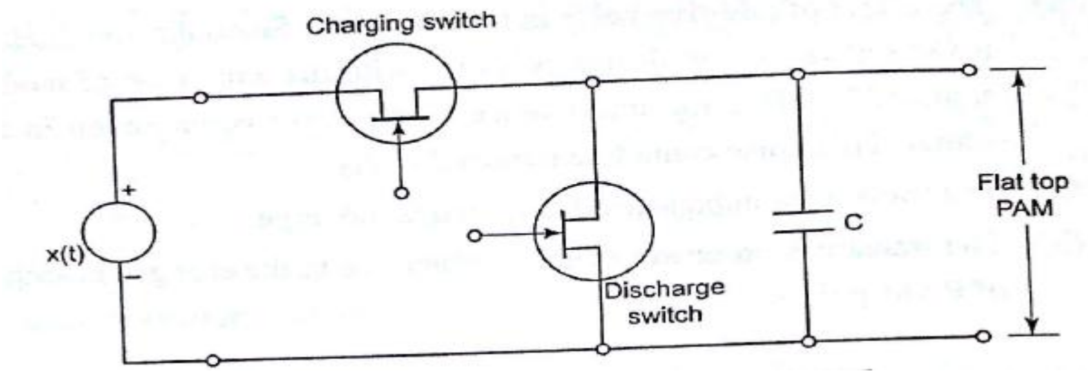


Fig (a): Circuit to generate flat top PAM

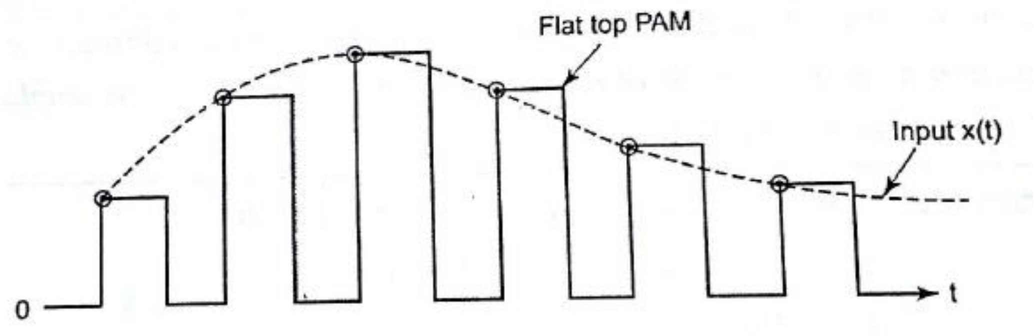


Fig (b): Flat top PAM signal

Fig : Generation of flat top PAM

Detection of flat top PAM

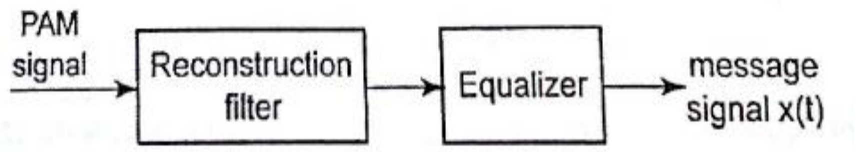


Fig : Detection of flat top PAM

- Detector contains a low-pass reconstruction filter with cut off frequency slightly higher than the maximum frequency present in the message signal x (t).
- The equalizer compensates for the aperture effect. It also compensates for the attenuation by a low pass filter.

Transmission Bandwidth of PAM Signal

✓ In a PAM signal, the pulse duration τ is considered to be very small in comparison to the time period (sampling period) T_S between any two samples.

$$\tau << T_S \qquad \text{……..(1)}$$

From sampling theorem,

$$f_S \geq 2\, f_m$$

$$\frac{1}{T_s} \geq 2\, f_m$$

$$T_s \leq \frac{1}{2\, f_m}$$

From (1),

$$\tau << T_s \leq \frac{1}{2\, f_m}$$

✓ If the ON and OFF time of PAM pulse is same, then maximum frequency of the PAM pulse will be,

$$f_{\max} = \frac{1}{\tau + \tau} = \frac{1}{2\tau}$$

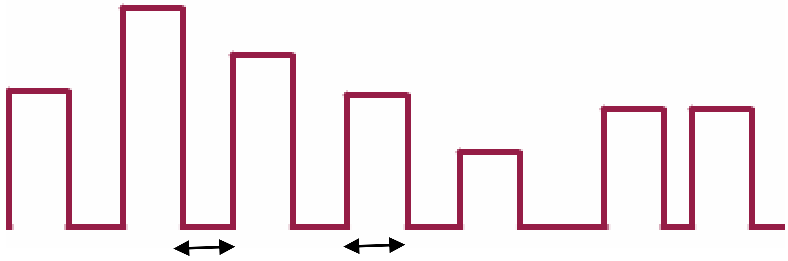


Fig: ON and OFF pulses of PAM

✓ Therefore, the bandwidth required for the transmission of a PAM signal would be equal to the maximum frequency f_{max}.

$$BW \geq f_{\max}$$

$$\geq \frac{1}{2\tau}$$

$$\text{But, } \tau << \frac{1}{2 f_m}$$

$$BW \geq \frac{1}{2\tau} >> f_m$$

$$\boxed{BW >> f_m}$$

Advantage: Simple generation and detection

Disadvantages:

- Effect of additive noise is high in PAM.
- Transmission bandwidth required is too large.
- The transmission power is not constant due to the changes in amplitudes of PAM pulses.

****

5.11 Pulse Time Modulation (PTM)

Explain the concept of pulse time modulation in detail.

Explain the concept and method of generating of PWM. What are the advantages and application of PTM? (May – 2013) [Nov/Dec 2013]

- In **pulse time modulation**, amplitude of pulse is held constant, whereas position of pulse is made proportional to the amplitude of signal at the sampling instant.
- There are two types of pulse time modulation. They are:
  - Pulse width modulation
  - Pulse position modulation **[Apr - 2019]**

Explain the generation and detection of PWM with neat diagram. (April / May – 2011)
With neat diagram, explain the generation and detection of PPM.

5.11.1 Pulse Width Modulation (PWM)

Introduction

- The width of the carrier pulses varies in proportion with the amplitude of modulating signal.
- The amplitude and frequency of the PWM wave remains constant.
- Only the width changes.
- The information is contained in the width variation.
- The additive noise, changes the amplitude of the PWM signal.
- Using the limiter circuit at the receiver, unwanted amplitude variations are easily removed.

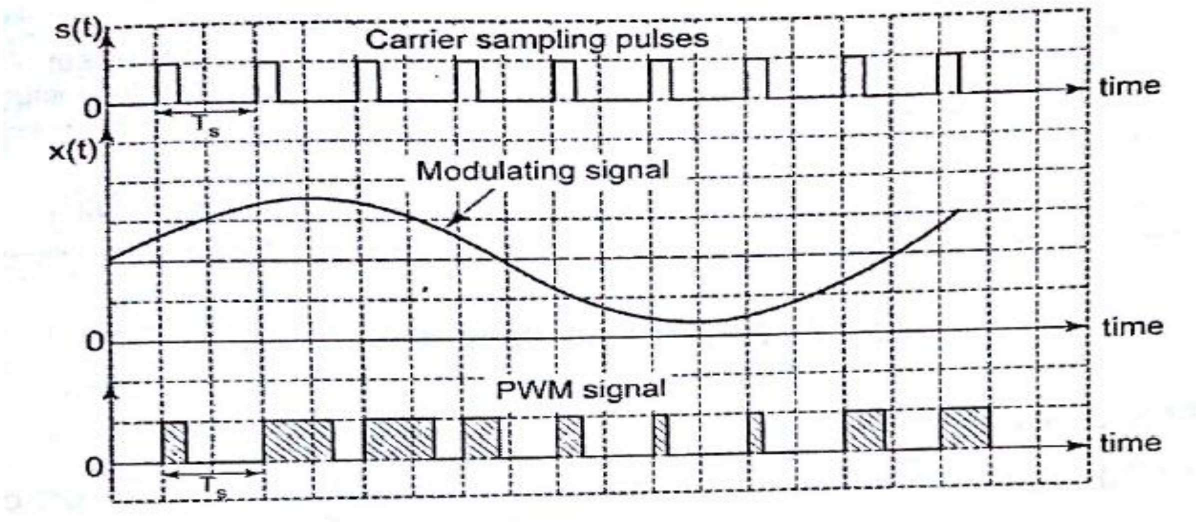


Fig: PWM signal

- Amplitude variations due to noise do not affect the performance. Thus PWM is more immune to noise than PAM.

PWM signal generation

- A saw tooth signal acts as a sampling signal which is applied to inverting terminal of a comparator.
- The modulating signal x (t) is higher than that of the saw tooth signal. This gives to a PWM signal.

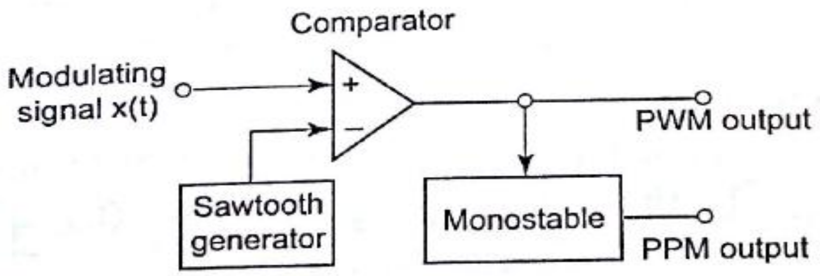


Fig : Block diagram of PWM and PPM generation

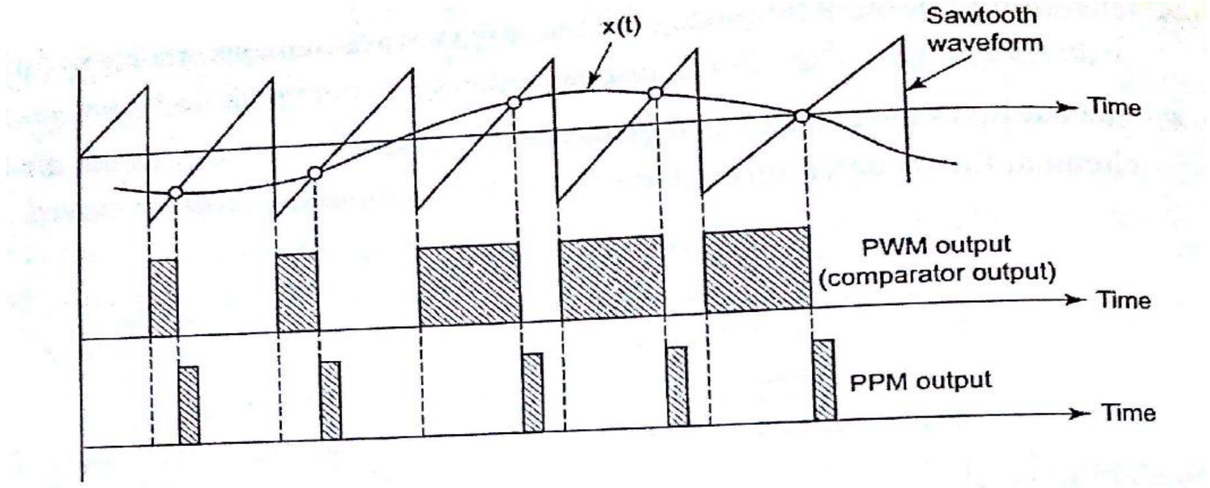


Fig : PWM and PPM waveforms

PWM signal detection

- The PWM signal received at the input of the detection circuit contains noise.
- It is applied to pulse generator which regenerates the PWM signal and remove noises.
- The regenerated pulses are applied to a reference pulse generator.
- The reference pulse generator produces reference pulses with constant amplitude and pulse width.
- These pulses are delayed by specific amount of delay.

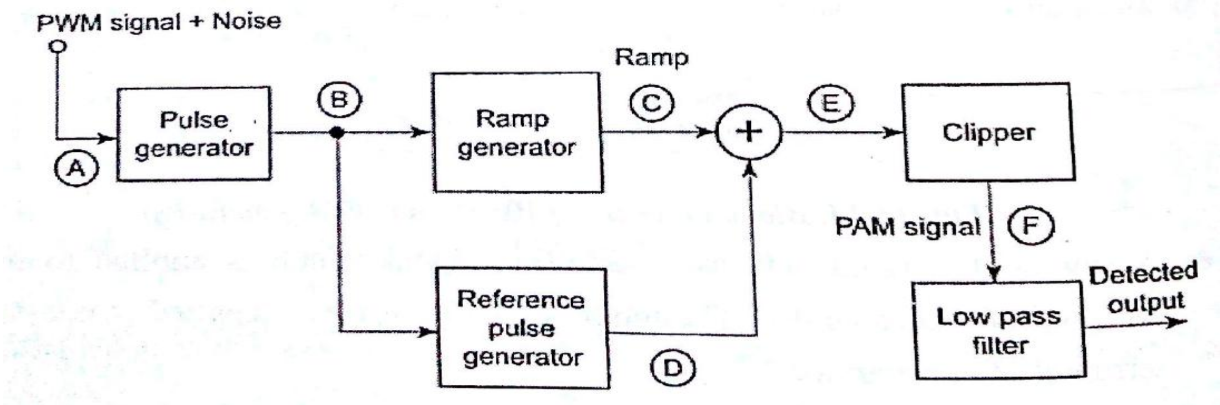


Fig: Block diagram of PWM detection circuit

- The regenerated PWM pulses are also applied to a ramp generator.
- The ramp generator produces ramps for the duration of pulses such that height of ramp is proportional to the widths of PWM pulses.
- The maximum ramp voltage is retained till the next pulse.
- The delayed reference pulses and the output of ramp generator is added with the help of adder.

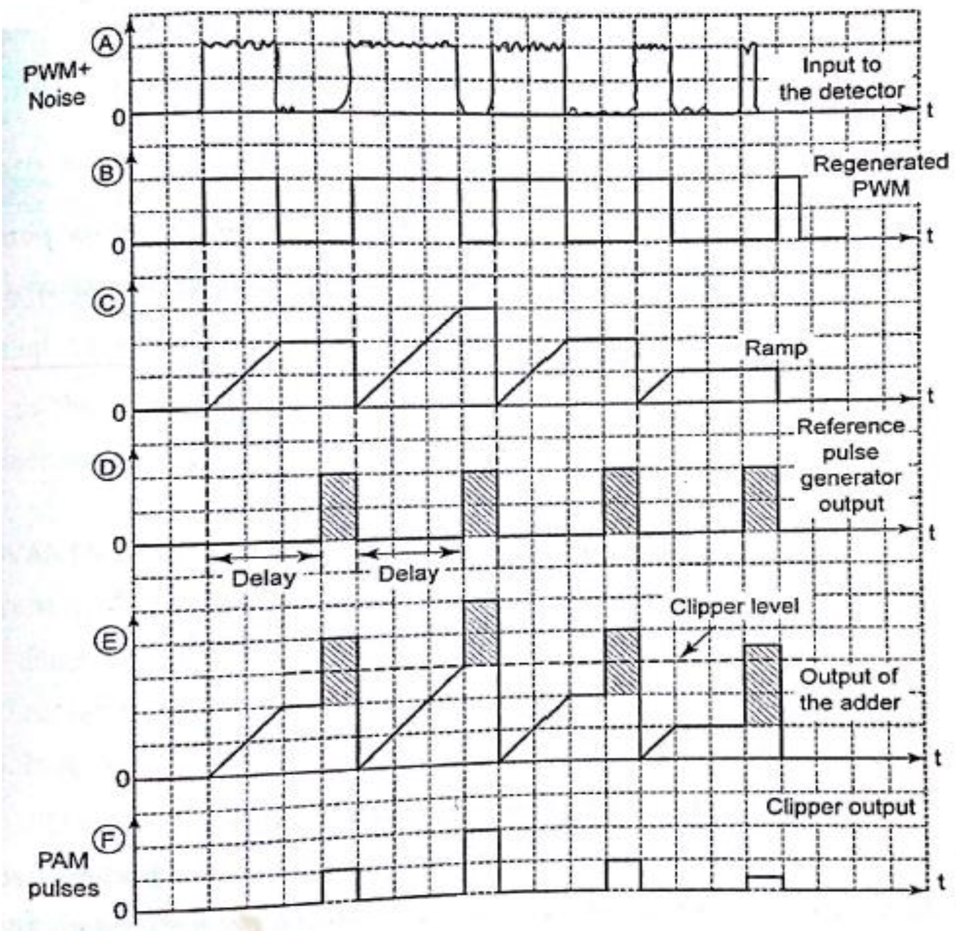


Fig : Waveform for PWM detection circuit

- The output of the adder is then clipped off at a threshold level to generate PAM signals at the output of the clipper.
- A low pass filter is used to recover the original modulating signal from PAM signal.

Advantages

- In PWM noise is less because here amplitude is constant.
- No synchronization required between transmitter and receiver.
- It is easy to separate the signal from noise.

Disadvantages

- Variable pulse width causes variable power contents. So, transmission must be powerful enough to handle the maximum width.
- Bandwidth requirement is higher than PAM.

5.11.2 Pulse Position Modulation (PPM)

- The amplitude and width of the pulses are kept constant but the position of each pulse is varied in accordance with the amplitude of the sampled values of the modulating signal.

PPM signal generation

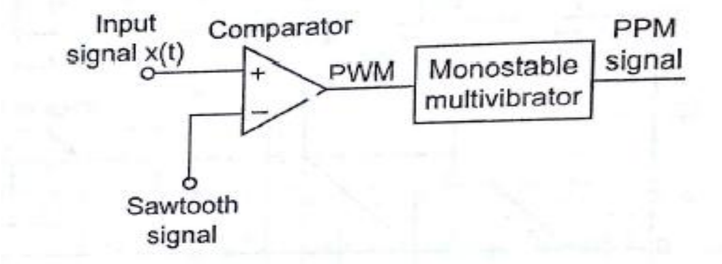


Fig : Generation of PPM signal

- To generate pulse position modulation, the PWM pulses obtained at the output of the comparator are used as the trigger input to a monostable multivibrator.
- The monostable is triggered on negative (falling) edge of PWM.
- The output of monostable goes high. This voltage remains high for the fixed period then goes low.
- As a result of shifting the trailing edges of PWM signal in proportion with the modulating signal x (t), the PPM pulses also results in keep shifting.

PPM signal demodulation

- The received PPM signal is noise corrupted.
- The pulse generator develops a pulsed waveform at its output of fixed duration and applies these pulses to reset pin (R) of a SR flip flop.

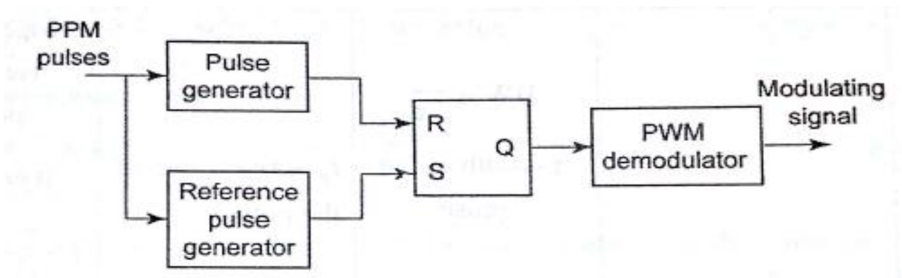


Fig : PPM demodulator circuit

- A fixed period reference pulse is generated from the incoming PPM.
- The SR flip flop is set by the reference pulses.
- Due to the set and reset signals applied to the flip-flop, a PWM signal is obtained in the output which can be demodulated with a PWM demodulator.

Advantages

- Due to constant amplitude of pulses, the transmitted power always remains constant.
- It is easy to reconstruct PPM signal from the noise contaminated PPM signal.

Disadvantages

- Synchronization required between the transmitter and receiver.
- Large bandwidth requirement.

Difference Between PAM, PWM, and PPM

Difference Between PAM, PWM, and PPM

The below table gives the detailed difference between PWM, PAM, and PPM.

Sr. No.	Parameter	PAM	PWM	PPM
1	**Type of Carrier**	Train of Pulses	Train of Pulses	Train of Pulses
2	**Variable Characteristic of the Pulsed Carrier**	Amplitude	Width	Position
3	**Bandwidth Requirement**	Low	High	High
4	**Noise Immunity**	Low	High	High
5	**Information Contained in**	Amplitude Variations	Width Variations	Position Variations
6	**Power efficiency (SNR)**	Low	Moderate	High
7	**Transmitted Power**	Varies with amplitude of pulses	Varies with variation in width	Remains Constant
8	**Need to transmit synchronizing pulses**	Not needed	Not needed	Necessary
9	**Bandwidth depends on**	Bandwidth depends on the width of the pulse	Bandwidth depends on the rise time of the pulse	Bandwidth depends on the rise time of the pulse
10	**Transmitter power**	Instantaneous transmitter power varies with the amplitude of the pulses	Instantaneous transmitter power varies with the amplitude and width of the pulses	Instantaneous transmitter power remains constant with the width of the pulses
11	**Complexity of generation and detection**	Complex	Easy	Complex
12	**Similarity with other Modulation Systems**	Similar to AM	Similar to FM	Similar to PM

****

5.12 Pulse-Code Modulation

15. Explain the operation of PCM in detail with proper block diagrams. (May 2013, Nov 2013)(or)
Describe PCM waveform coder and decoder with neat sketch and list the merits compared with analog coders. [Dec 2015] (or)
Explain in detail about temporal waveform encoding scheme. (or)
Explain pulse code modulation system with neat block diagram. [May 2016] [Apr - 2019]

- PCM is the most basic form of digital pulse modulation.
- In pulse-code modulation (PCM), a message signal is ***represented by a sequence of coded pulses***, this is accomplished by ***representing the signal in discrete form in both time and amplitude***.
- The basic operations performed in the transmitter of a PCM system are
  - ✓ Sampling
  - ✓ quantization, and
  - ✓ encoding, as shown in Fig.;

Operations in the transmitter

- ***The low-pass filter***, prior to sampling, is included just to prevent aliasing of the message signal.
- In practice, an anti-alias (low-pass) filter is used at the front end of the sampler to reject frequencies greater than Wbefore sampling,Figure14(a).

- The ***quantizing and encoding operations*** are usually performed in the same circuit, which is called an ***analog-to-digital converter***.

(i) ***Sampling***

- The incoming message (baseband) signal is sampled with a train of rectangular pulses, narrow enough to closely approximate the instantaneous sampling process.
- For perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency component Wof the message signal (***in accordance with the sampling theorem***).
- ***Function of sampling:*** Sampling permits the reduction of the continuously varying message signal (of some finite duration) to a limitednumber of discrete values per second.

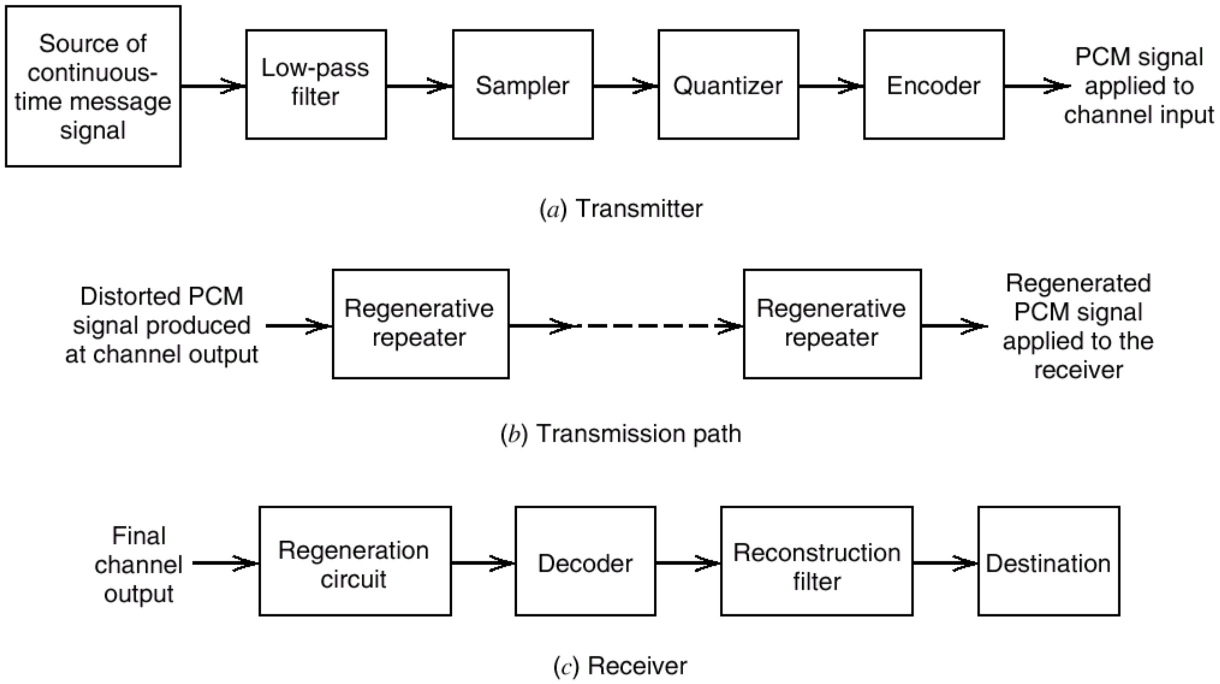


Figure14. The basic elements of a PCM system

(a) Transmitter, (b) transmission path,connecting the transmitter to the receiver, and (c) receiver.

(ii) Nonuniform Quantization

- The sampled version of the message signal is then quantized.
- It provides a newrepresentation of the signal that is ***discrete in both time and amplitude***.

(iii) Encoding

- **The use of an encoding process to ***convert the discrete set of sample values to a more suitable form of signal***.
- ****Code:** Plan for representing this discrete set of values as a particular arrangement of discrete events is called a code. One of the discrete events in a code is called a code element or symbol.
- ****Code word:** A particular arrangement of symbols to represent a single value of the discrete set is called a code word or character.
- In a binary code, each symbol may be either of two distinct values, such as a negative pulse or positive pulse.

The two symbols of the binary code are customarily denoted as 0 and 1. In practice, a binary code is preferred over other codes (e.g., ternary code) for two reasons:

1. ***The maximum advantage over the effects of noise in a transmission medium is obtained by using a binary code, because a binary symbol withstands a relatively high level of noise.***
2. ***The binary code is easy to generate and regenerate.***

Regeneration along the Transmission Path

- This capability is attained by reconstructing the PCM signal by means of a chain of regenerative repeaters located at sufficiently close spacing along the transmission route.
- As illustrated in Figure15, *three basic functions are performed by a regenerative repeater*:
  - Equalization, Timing and Decision making.
- ****Equalizer:** The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the transmission characteristics of the channel.
- ****Timing circuitry:**The timing circuitry provides a periodic pulse train, derived from the received pulses; this is done for renewed sampling of the equalized pulses at the instants of time where the signal-to-noise ratio is a maximum.
- ****Decision-making device:**The sample so extracted is compared to a predetermined threshold in the decision-making device. In each bit interval, a decision is then made on whether the received symbol is a 1 or 0 on the basis of whether the threshold is exceeded or not.
- If the threshold is exceeded, a clean new pulse representing symbol 1 is transmitted to the next repeater.

  Otherwise, another clean new pulse representing symbol 0 is transmitted.

- In this way, the accumulation of distortion and noise in a repeater span is removed.
- In practice, however, the ***regenerated signal departs from the original signal*** for two main reasons:
  1. The unavoidable presence of channel noise and interference causes the repeater to make wrong decisions occasionally, thereby introducing bit errors into the regeneratedsignal.
  2. If the spacing between received pulses deviates from its assigned value, a jitter is introduced into the regenerated pulse position, thereby causing distortion.

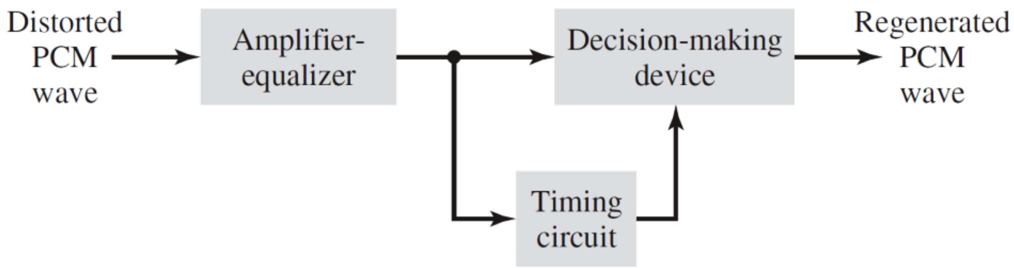


Figure15. Block diagram of Regenerative repeater

Operations in the Receiver

(i) Decoding and Expanding

- The first operation in the receiver is to regenerate (i.e., reshape and clean up) the received pulses.
- These clean ***pulses are then regrouped into code words and decoded*** (i.e., mapped back) into a quantized PAM signal.

- ***Decoding:*** The decoding process involves generating a pulse whose amplitude is the linear sum of all the pulses in the code word; each pulse is weighted by its place value $(2^0, 2^1, 2^2, ..., 2^{R-1})$ in the code, where R is the number of bits per sample.

- The sequence of decoded samples represents an estimate of the sequence of compressed samples produced by the quantizer in the transmitter.
- In order to restore the sequence of decoded samples to their correct relative level, a subsystem is used in the receiver called an expander (complementary to the compressor, used in the transmitter).
- The combination of a compressorand an expander is referred to as a compander.

(ii) Reconstruction

- The final operation in the receiver is to recover the message signal.
- This operation isachieved by passing the expander output through a low-pass reconstruction filterwhose cutoff frequency is equal to the message bandwidth.
- Recovery of the messagesignal is intended to signify estimation rather than exact reconstruction.

5.13 Time Division Multiplexing:

16. Explain in detail about the process of Time division multiplexing. [May 2010, Nov 2011] (or) What is TDM? Explain the difference between analog TDM and digital TDM. [May 2016]

- ***Concept of TDM:***The transmission of the message samples engages the communication channel for only a fraction of the sampling interval on a periodic basis, and in this way some of the time interval between adjacent samples is cleared for use by other independent message sources on a time-shared basis.

➢ ***The*time-division multiplex* (TDM) *system*, enables the joint utilization of a common communication channel by a plurality of independent message sources ***without mutual interferenceamong them***.

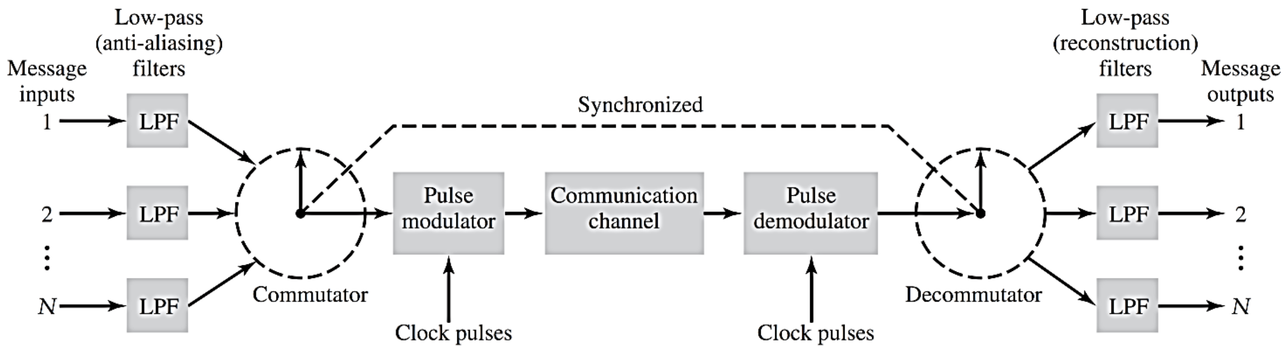


FIGURE 16 Block diagram of TDM system.

➢ The concept of TDM is illustrated by the block diagram shown in Fig. 16.

Transmitting system:

Low-pass (anti-aliasing) filter:

➢ Each input message signal is first restricted in bandwidth by a low-pass anti-aliasing filter.

➢ It removes the frequencies that are non-essential to a satisfactory signal representation.

Commutator:

➢ The low-pass filter outputs are then applied to a *commutator*.

➢ It is usually implemented using electronic switching circuitry.

➢ The function of the commutator is twofold*(dual)*:

(1) to take a narrow sample of each of the N input messages at a rate that is slightly higher than $2W$, where W is the cutoff frequency of the anti-aliasing filter, and

(2) to sequentially interleave these N samples inside the sampling interval Indeed, this latter function is the essence of thetime-division multiplexing operation.

Pulse modulator:

➢ Next to the commutation process, the multiplexed signal is applied to a *pulse modulator*.

➢ Pulse modulator transforms the multiplexed signal into a form suitable for transmission over the common channel.

➢ The use of time-division multiplexing introduces a bandwidth expansion factor N, because the scheme must squeeze N samples derived from N independent message sources into a time slot equal to one sampling interval.

Receiving System

Pulse Demodulator:

- At the receiving end of the system, the received signal is applied to a *pulse demodulator*, which performs the reverse operation of the pulse modulator.

Decommutator:

- The narrow samples produced at the pulse demodulator output are distributed to the appropriate low-pass reconstruction filters through a *decommutator*.
- *Decommutator* operates in *synchronism* with the commutator in the transmitter.
- This synchronization is essential for a satisfactory operation of the system.
- Synchronization depends on the method of pulse modulation used to transmit the multiplexed sequence of samples.

Equalization:

- The TDM system is highly sensitive to dispersion in the common channel.
- A non-constant magnitude response of the channel and a nonlinear phase response, both being measured with respect to frequency.
- Accordingly, *equalization* of both magnitude and phase responses of the channel is necessary to ensure a satisfactory operation of the system; in effect, equalization compensates for dispersion in the channel.
  - However, unlike frequency-division multiplexing (FDM), to a first-order approximation TDM is immune to nonlinearities in the channel as a source of cross-talk.
  - The reason for this behavior is that different message signals are not simultaneously applied to the channel.

Synchronization

- For a PCM system with time-division multiplexing to operate satisfactorily, it is necessary that the timing operations at the receiver, except for the time lost in transmission and regenerative repeating, follow closely the corresponding operations at the transmitter.
- In a general way, this amounts to requiring a local clock at the receiver to keep the same time as a distant standard clock at the transmitter, except that the local clock is delayed by an amount equal to the time required to transport the message signals from the transmitter to the receiver.

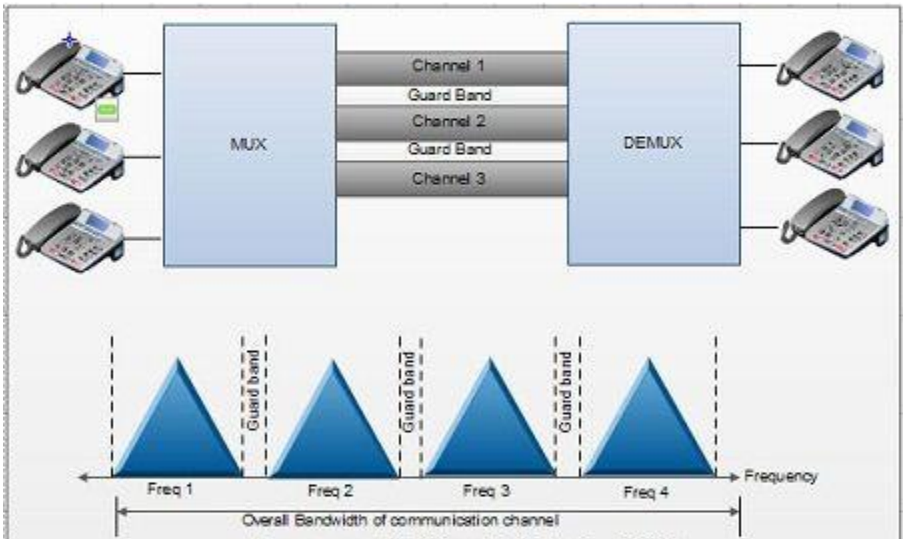
5.14 Frequency-Division Multiplexing (FDM)

Explain in detail about Frequency-Division Multiplexing (FDM) . **[Apr - 2019]**

- Frequency-Division Multiplexing (FDM) is a scheme in which numerous signals are combined for transmission on a single communications line or channel.
- It is analog multiplexing technique. Each signal is assigned a different frequency (sub channel) within the main channel. its requires channel synchronization.
- FDM multiplexing technique is based on orthogonality of sinusoids.
- FDM requires that the bandwidth of a link should be greater than the combined bandwidths of the various signals to be transmitted.
- Thus each signal having different frequency forms a particular logical channel on the link and follows this channel only.
- These channels are then separated by the strips of unused bandwidth called guard bands.

  These guard bands prevent the signals from overlapping as shown in Fig.

In FDM, signals to be transmitted must be analog signals. Thus digital signals need to be converted to analog form, if they are to use FDM.



Frequency Division Multiplexing FDM

A typical analog Internet connection via a twisted pair telephone line requires approximately three kilohertz (3 kHz) of bandwidth for accurate and reliable data transfer.

Twisted-pair lines are common in households and small businesses. But major telephone cables, operating between large businesses, government agencies, and municipalities, are capable of much larger bandwidths.

Advantages of FDM:
1. A large number of signals (channels) can be transmitted simultaneously.
2. FDM does not need synchronization between its transmitter and receiver for proper operation.
3. Demodulation of FDM is easy.
4. Due to slow narrow band fading only a single channel gets affected.

Disadvantages of FDM:
1. The communication channel must have a very large bandwidth.
2. Intermodulation distortion takes place.
3. Large number of modulators and filters are required.

4. FDM suffers from the problem of crosstalk.
5. All the FDM channels get affected due to wideband fading.

Applications of FDM

1. FDM is used for FM & AM radio broadcasting. Each AM and FM radio station uses a different carrier frequency. In AM broadcasting, these frequencies use a special band from 530 to 1700 KHz. All these signals/frequencies are multiplexed and are transmitted in air. A receiver receives all these signals but tunes only one which is required. Similarly FM broadcasting uses a bandwidth of 88 to 108 MHz

2. FDM is used in television broadcasting.

3. First generation cellular telephone also uses FDM.

PROBLEMS

17. A PCM sinusoidal has a uniform quantizer followed by a '*v*' bit encoder. Show that the rms signal to noise ratio is approximately given by 1.8 + 6 *v* dB, assuming a sinusoidal input. [April/May 2018]

Solution:

Assume that the modulating signal be a sinusoidal voltage, having peak amplitude Am. Let the signal cover the complete excursion os representation levels.

The power of the signal will be,

$$P = \frac{v^2}{r}$$

$$= \left(\frac{A_m}{\sqrt{2}}\right)^2$$

When R=1, the power P is normalized, i.e.,

Normalized power:

$$P = \frac{A_m^2}{2}, \quad \text{with R=1 in the above equation}$$

Therefore, The signal to quantization noise ratio is given by

$$(SNR)_O = \frac{P}{\sigma_Q^2}$$

$$= \frac{3P}{m_{\max}^2} 2^{2R}$$

Substitute:

$$P = \frac{A_m^2}{2}, \quad m_{\max} = A_m$$

$$(SNR)_O = \frac{3P}{m_{\max}^2} 2^{2R} = \frac{3\frac{A_m^2}{2}}{A_m^2} 2^{2R} = \frac{3}{2} 2^{2R} = 1.5 \times 2^{2R}$$

Expressing signal to noise ratio in dB,

$$(SNR)dB = 10\log_{10}^{1.5} + 10\log_{10}^{2^{2R}} = 1.76 + (2v \times 10 \times 0.3)$$

$$(SNR)dB\ in\ PCM = 1.8 + 6v;$$ for sinusoidal signal

18. Show that the signal to noise power ratio of a uniform quantizer is PCM system increases significantly with increase in number of bits per sample. Also determine the signal to quantization noise ratio of an audio signal $S(t) = 4\sin(2\pi 500t)$, which is quantized using a 10 bit PCM. [April/May 2018, Nov 2017]

Given:

$$S(t) = 4\sin(2\pi 500t)$$

Solution:

For 10 bit PCM

$$L = 2^n$$
$$n = 10$$

$\therefore$ Number of levels = 1024

The amplitude$A_m$of sinusoidal waveform means that $m_p = 4$ volts.

The total signal swing possible ($-m_p$to $+m_p$)will be $2m_p = 8$ volts.

The average signal power is

$$Pave = \left[\frac{(Am)^2}{2}\right] = \left[\frac{(4)^2}{2}\right] = 8\ watts$$

The interval,

$$\Delta V = \frac{2mp}{L}$$
$$= \frac{8}{1024\ levels}$$
$$= 7.81 \times 10^{-3}\ volt$$

Quantization noise,

$$N_q = \frac{(\Delta V)^2}{12}$$

SNR:

$$SNR = \left(\frac{S}{N_q}\right) = \left(\frac{P_{ave}}{N_q}\right) = \frac{8}{(\Delta V)^2} \times 12$$

$$= \frac{96}{6.10 \times 10^{-5}}$$

$$= 15{,}73{,}770$$

$$SNR_{dB} = 10\log_{10}^{1573770} = 61.96dB$$

UNIT V - SAMPLING & QUANTIZATION

TWO MARKS

1. **What is Communication system?**

   The Communication System is the system which is used to transport an information bearing signal from a source to a user destination via a communication channel.

2. **What are different categories of Communication Systems?**
   - Analog Communication Systems are designed to transmit analog information using analog modulation methods.
   - Digital Communication Systems are designed for transmitting digital information using digital modulation schemes, and
   - Hybrid Systems that use digital modulation schemes for transmitting sampled and quantized values of an analog message signal.

3. **How can BER of an system be improved? [NOV/DEC2012]**

   Increasing the transmitted signal power Employing modulation and demodulation technique Employing suitable coding and decoding methods Reducing noise interference with help of improved filtering.

4. **Which parameter is called figure of merit of a digital communication systemand why? [NOV/DEC 2010]**

   The ratio Eb/No or bit energy to noise power spectral density is called figure of merit of a digital communication system

5. **Define half power bandwidth. [NOV/DEC2011]**

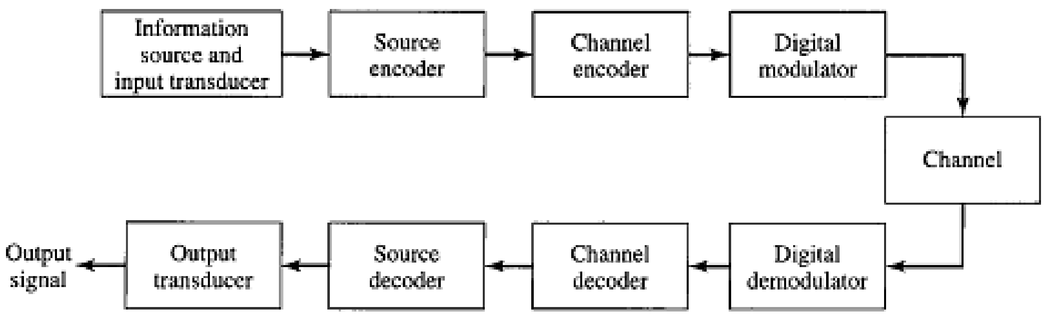
   Half power bandwidth is the bandwidth where PSD of the signal drops to half (3dB) of its maximum value.It is called 3dB bandwidth.

6. **What is channel? Give examples. [Nov/Dec 2013]**

   A channel is used to convey an information signal, for example a digital bit stream, from one or several senders (or transmitters) to one or several receivers. A channel has a certain capacity for transmitting information, often measured by its bandwidth in Hz or its data rate in bits per second.

   Ex: Physical transmission medium such as a wire, logical connections over multiplexed medium such as a radio channel.

7. **Draw a typical digital communication system. [Nov/Dec 2012], [Nov/Dec 2011]**



8. **What are the Advantages of Digital Communication? [Nov/Dec 2013]**
   - The effect of distortion, noise and interference is less in a digital communication system.
   - Regenerative repeaters can be used at fixed distance along the link, to identify and regenerate a pulse before it is degraded to an ambiguous state.
   - Digitalcircuits are more reliableand cheaper compared to analog circuits.
   - Signal processing functions like encryption, compression can be employed to maintain the secrecy of the information.
   - Error detecting and Error correcting codes improve the system performance by reducing the probability of error.
9. **What are Disadvantages of Digital Communication? (or)**
   State the demerits of digital communication. [May/June 2014]
   - Large System Bandwidth:- Digital transmission requires a large system bandwidth to communicate the same information in a digital format as compared to analog format.
   - System Synchronization:- Digital detection requires system synchronization whereas the analog signals generally have no such requirement.
10. **What is sampling process?**
    - **SAMPLING:** A message signal may originate from a digital or analog source. If the message signal is analog in nature, then it has to be converted into digital form before it can transmit by digital means.
    - The process by which the continuous-time signal is converted into a discrete–time signal is called Sampling.

SAMPLING THEOREM FOR LOW-PASS SIGNALS:-

11. **Define sampling rate. [Apr - 2019]**

    The bandpass signal $g(t)$ whose maximum bandwidth is $2W$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate of twice the bandwidth.

12. **Why prefilterring done before sampling? [AUC NOV/DEC 2010]**

    The signal must be limited to some highest frequency W Hz before sampling. Then the signal is sampled at the frequency of fs=2W of higher. Hence the single should be prefiltered at higher that W Hz.If the signal is not prefiltered, then frequency component higher that W Hz will generate aliasing in the sampled signal spectrum.

13. Draw the circuit theoretic representation of ideal sampling process.

This circuit-theoretic interpretation of $g_\delta(t)$ is depicted in Fig. (2)

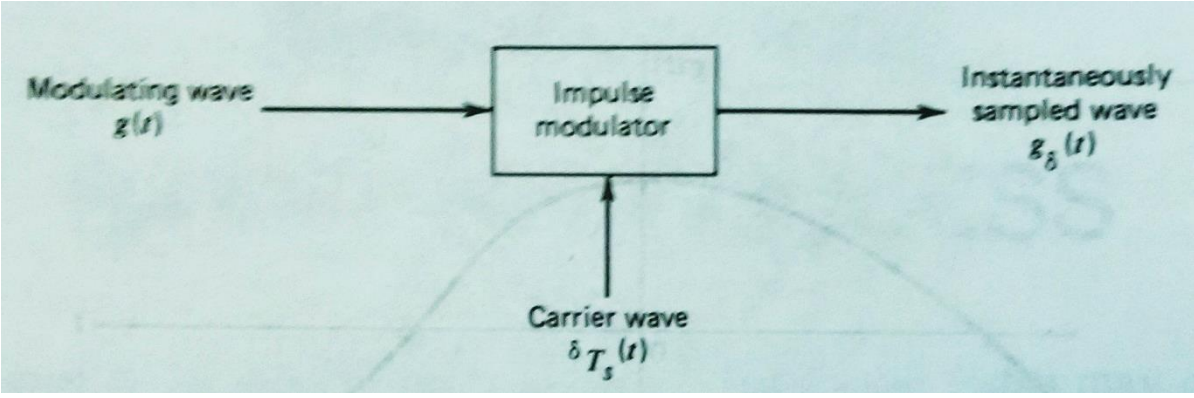


Figure: Circuit-theoretic interpretation of the ideal sampling process as impulse modulation

where, $g(t)$- Modulatiing wave, δ_{T_s} -Carrier wave and $g_\delta(t)$-Instantaneously sampled value

14. Draw the spectrum of (a) analog signal $g(t)$ (b) Spectrum of sampled signal $g_\delta(t)$ for a sampling rate $f_s = 2W$. (c) Ideal amplitude response of reconstruction filter.

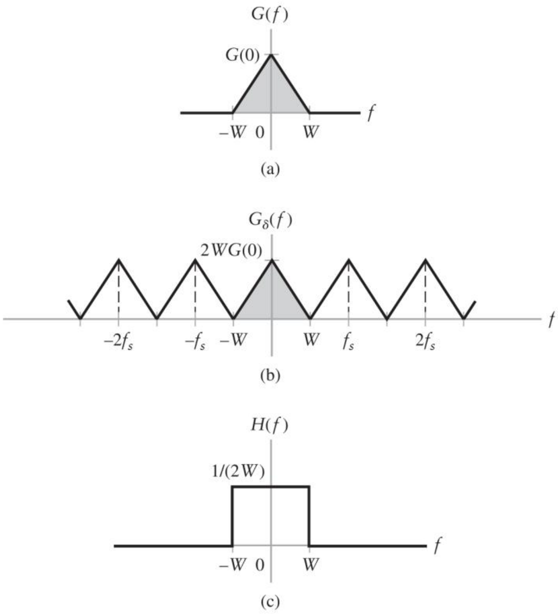


Figure: (a) Spectrum of signal $g(t)$. (b) Spectrum of sampled signal $g_\delta(t)$ for a sampling rate $f_s = 2W$. (c) Ideal amplitude response of reconstruction filter.

15. Write about sinc function.

The *sinc function* exhibits an important property known as the *interpolatory property*, which is describes as follows:

$$\text{sinc}x = \begin{cases} 1, x = 0 \\ 0, x = \pm 1, \pm 2, ... \end{cases}$$

16. Draw the block diagram of Reconstruction filter.

Reconstruction filter.

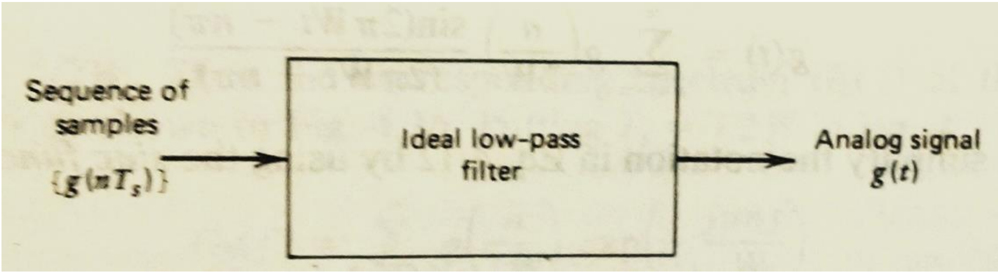


Figure: Reconstruction filter

17. Give the complete statement of sampling theorem. (or)
State sampling theorem for lowpass signals. [May/June 2009] (or)
State sampling theorem. [May/June 2014], [May/June 2012](or)
Define Band pass sampling. [April/May 2018]

Statement of the Sampling Theorem:

1. If a finite-energy signal contains no frequencies higher than W hertz, it is completely determined by specifying its ordinates at a sequence of points spaced 1/2W seconds apart.
2. If a finite-energy signal contains no frequencies higher than W hertz, it may be completely recovered from its ordinates at a sequence of points spaced 1/2W seconds apart.

18. Define Nyquist rate and Nyquist interval.

The minimum sampling rate of *2W* samples per second, for a signal bandwidth of *W* hertz, is called the *Nyquist rate*.Correspondingly, the reciprocal, *1/2W*, is called the *Nyquist interval*.

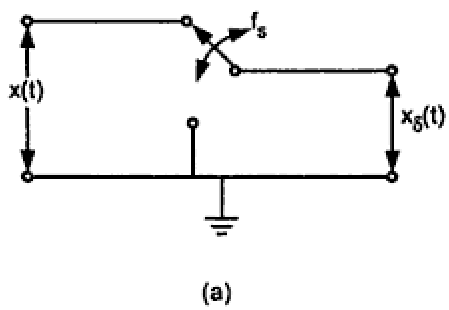
19. What are the different types of sampling?

Types of sampling (Practical Sampling):

1. Ideal sampling
2. Natural sampling
3. Flat top sampling

20. Wirte in short about Ideal Sampling (or)Instantaneous sampling (or) Impulse sampling.

Ideal Sampling (or)Instantaneous sampling (or) Impulse sampling:



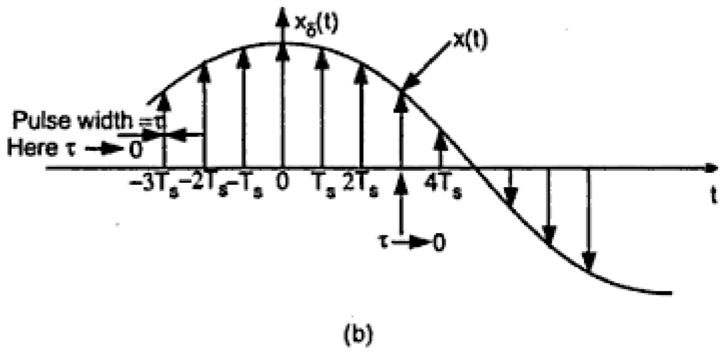


***Fig (a)** Functional diagram of a switching sampler*

***Fig (b)** Message $x(t)$ and sampled $x_\delta(t)$ signals*

- Ideal sampling is same as instantaneous sampling.
- Fig. (a)shows the switching sampler.
- If closing time 't' of the switch approaches zero the output $x_\delta(t)$ gives only instantaneous value. The waveforms are shown in Fig. (b).
- Since the width of the pulse approaches zero, the instantaneous sampling gives train of impulses in $x_\delta(t)$. The area of each impulse in the sampled version is equal to instantaneous value of input signal $x(t)$.

21. Write about Natural Sampling (or) Chopper Sampling.

Natural Sampling (or) Chopper Sampling:

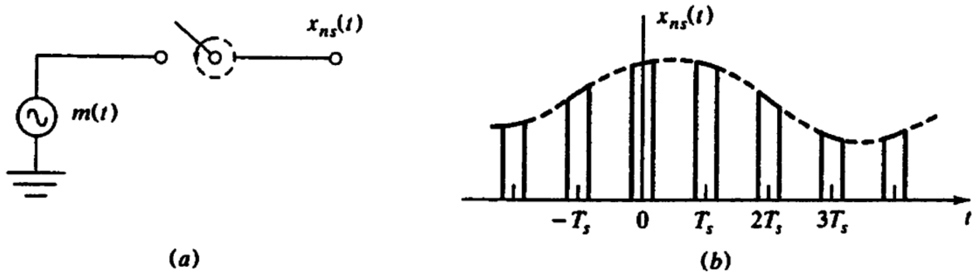


Fig. 5.3. Natural sampling

- Although instantaneous sampling is a convenient model, a more practical way of sampling a band-limited analog signal *m(t)* is performed by high-speed switching circuits.
- An equivalent circuit employing a mechanical switch and the resulting sampled signal are shown in Fig. 5-3(a) and (b),respectively.
- The sampled signal $x_{ns}(t)$ can be written as

$$x_{ns}(t) = m(t)x_p(t) \qquad (5.4)$$

Where $x_p(t)$ is the periodic train of rectangular pulses with period T_s, and each rectangular pulse in $x_p(t)$ has width *d* and unit amplitude.

- The sampling here is termed natural sampling, since the top of each pulse in $x_{ns}(t)$ retains the shape of its corresponding analog segment during the pulse interval.

22. Write about Flat top sampling (or) rectangular pulse shaping.

Flat-Top Sampling (or) Rectangular Pulse Shaping:

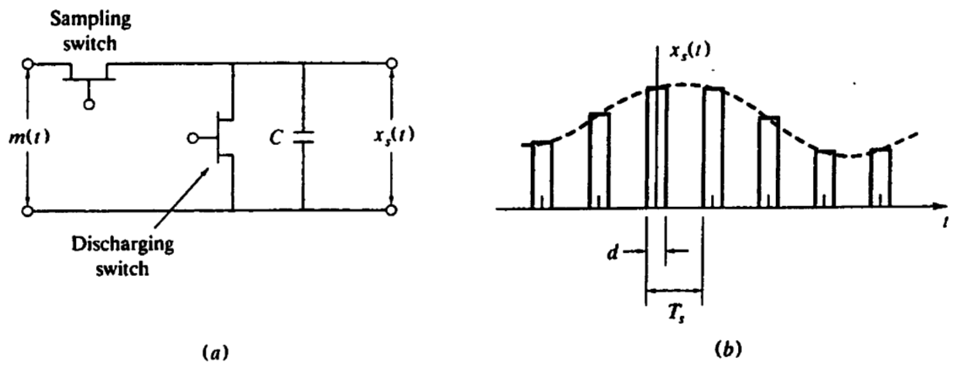
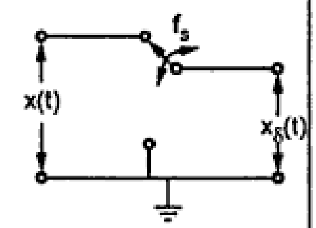
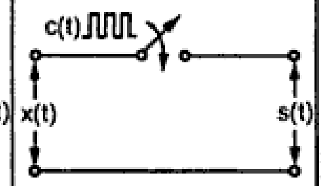
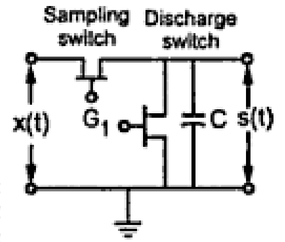
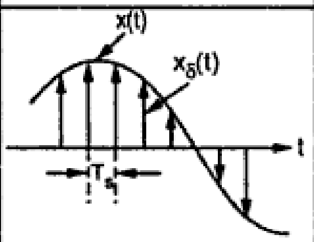
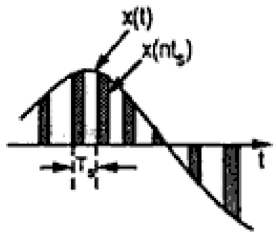
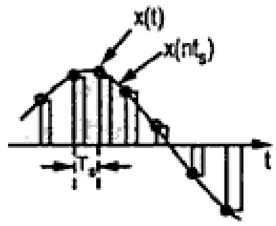


Fig. 5.4. Flat-top Sampling

- The simplest and thus most popular practical sampling method is actually performed by a functional block termed the sample-and-hold (S/H) circuit [Fig. 5-4(a)].
- This circuit produces a flat-top sampled signal $x_s(t)$ [Fig. 5-4(b)].

23. Compare Instantaneous, Natural and flat top sampling techniques.

Comparison of Various Sampling Techniques:

Sr. No.	Parameter of comparison	Ideal or instantaneous sampling	Natural sampling	Flat top sampling
1	Principle of sampling	It uses multiplication by an impulse function	It uses chopping principle	It uses sample and hold circuit
2	Circuit of sampler			
3	Waveforms			
4	Realizability	This is not practically possible method	This method is used practically	This method is used practically

24. What is aliasing in sampling process? [May/June 2016], [Nov/Dec 2012]

Aliasing Phenomenon

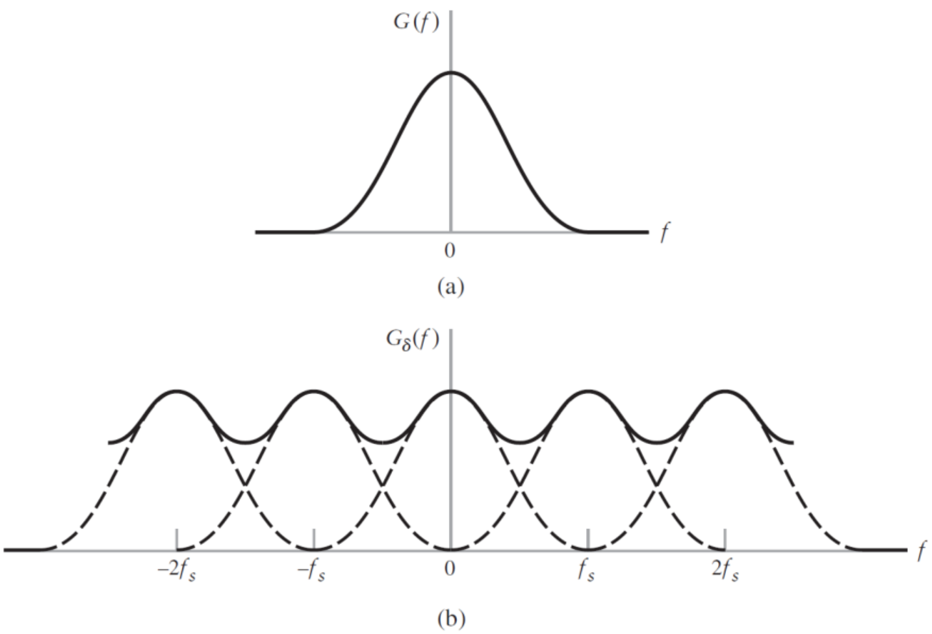


Fig. (a) Spectrum of a signal. (b) Spectrum of an undersampled version of the signal, exhibiting the aliasing phenomenon.

- Aliasing refers to the phenomenon of a high-frequency component in the spectrum of the signal seemingly taking on the identity of a lower frequency in the spectrum of its sampled version, as illustrated in Fig.

25. What are the corrective measures of aliasing effects?[Nov/Dec 2012]

To combat the effects of aliasing in practice, we may use two corrective measures:

1. Prior to sampling, a low-pass anti-alias filter is used to attenuate the high-frequencycomponents of a message signal that are not essential to the information..
2. The filtered signal is sampled at a rate slightly higher than the Nyquist rate.

26. Draw the spectrum of (a) Anti-alias filtered spectrum of an information-bearing signal. (b) Spectrum of instantaneously sampled version of the signal, assuming the use of a sampling rate greater than the Nyquist rate. (c) Idealized amplitude response of the reconstruction filter.

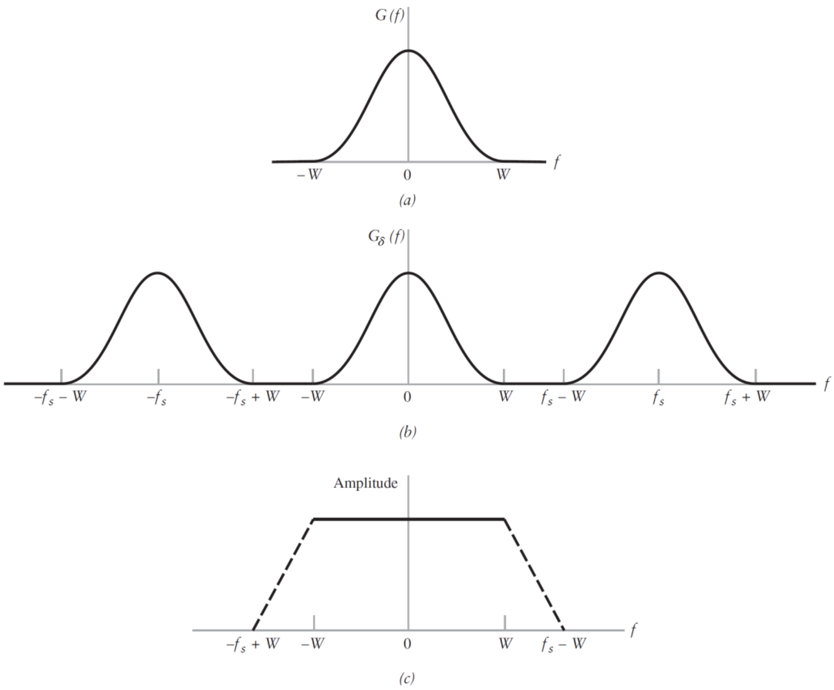


Fig 5.4 (a) Anti-alias filtered spectrum of an information-bearing signal. (b) Spectrumof instantaneously sampled version of the signal, assuming the use of a sampling rate greaterthan the Nyquist rate. (c)Idealized amplitude response of the reconstruction filter.

Reconstruction of a message process from its samples:

27. Write about the nature of reconstruction of samples.

If a stationary message process contains no frequencies higher than *W* hertz, it may be reconstructed from its samples at a sequence of points spaced *1/2W*seconds apart with zero mean squared error (i.e., Zero error power).

Quantization

28. What is meant by amplitude quantization?

Amplitude quantization is defined *as the process of transforming the sample amplitude $m(nT_s)$ of a message signal $m(t)$ at time $t = nT_s$ into a discrete amplitude $v(nT_s)$ taken from a finite set of possible amplitudes.*

The discrete amplitudes $m_k, k = 1, 2, \ldots, L$, at the quantizer input are called *decision levels* or *decision thresholds*.

29. Compare uniform and non uniform quantization. [AUC NOV/DEC 2011]

S.NO	UNIFORM QUANTIZATION	NON QUANTIZATION
1	The quantization step size remains samethroughout the dynamic range of the signal	The quantization step size varies with the amplitude of the input signal
2	SNR ratio varies with input signal amplitude	SNR ratio can be maintained constant

30. What are the types of quantizers?

Quantizers can be of a *uniform or nonuniform* type. In a uniform quantizer, therepresentation levels are uniformly spaced; otherwise, the quantizer is nonuniform.

Uniform & non-uniform quantization:

31. Write about uniform quantizer.

a. Uniform quantizer
b. Nonuniform quantizer

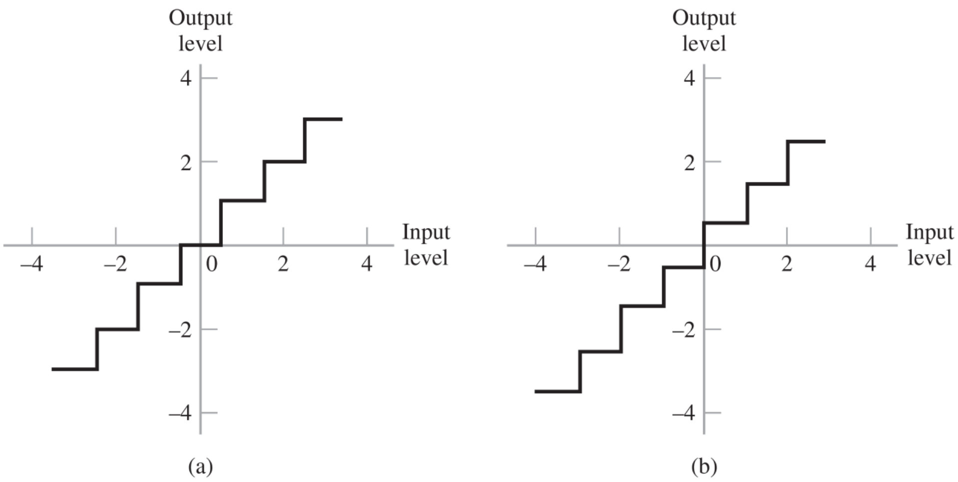


FIGURE. Two types of quantization: (a) midtread and (b) midrise.

- In a uniform quantizer, the representation levels are uniformly spaced; otherwise, the quantizer is nonuniform.

32. Write a note on uniform quantizer.

Uniform Quantization

- The quantizer characteristic can also be of a midtread or midrise type.

- Figure (above) shows the input–output characteristic of a uniform quantizer of the midtread type,which is called as uniform, because the origin lies in the middle of a tread of the staircase-like graph.

33. Write about Nonuniformquantizer.

Nonuniform Quantization

- The *nonuniformquantizer* with the feature that the step size increasesas the separation from the origin of the input–output amplitude characteristic isincreased, the large end-step of the quantizer can take care of possible excursions ofthe voice signal into the large amplitude ranges that occur relatively infrequently.

34. Write about $\mu - law$.

A particular form of compression law that is used in practice is the so called $\mu - law$ defined by

$$|v| = \frac{\log(1 + \mu|m|)}{\log(1 + \mu)} \qquad (01)$$

where the logarithm is the natural logarithm; m and are respectively the normalized input and output voltages, and μ is a positive constant.

35. Write about $A - law$.

- One of the compression law that is used in practice is the A-law, defined by

$$|v| = \begin{cases} \dfrac{A|m|}{1 + \log A}, & 0 \le |m| \le \dfrac{1}{A} \\ \dfrac{1 + \log(A|m|)}{1 + \log A}, & \dfrac{1}{A} \le |m| \le 1 \end{cases} \qquad (03)$$

which is shown plotted in Fig. 5.12(b). Typical values of A used in pratice tend to be in the vicinity of 100. The case of uniform quantization corresponds to $A=1$.

- The reciprocal slope of this second compression curve is given by the derivative of $|m|$ with respect to $|v|$ as shown by

$$\frac{d|m|}{d|v|} = \begin{cases} \dfrac{1 + \log A}{A}, & 0 \le |m| \le \dfrac{1}{A} \\ (1 + \log A)|m|, & \dfrac{1}{A} \le |m| \le 1 \end{cases} \qquad (04)$$

Quantization noise

36. Draw the diagram to illustrate Quantization process.

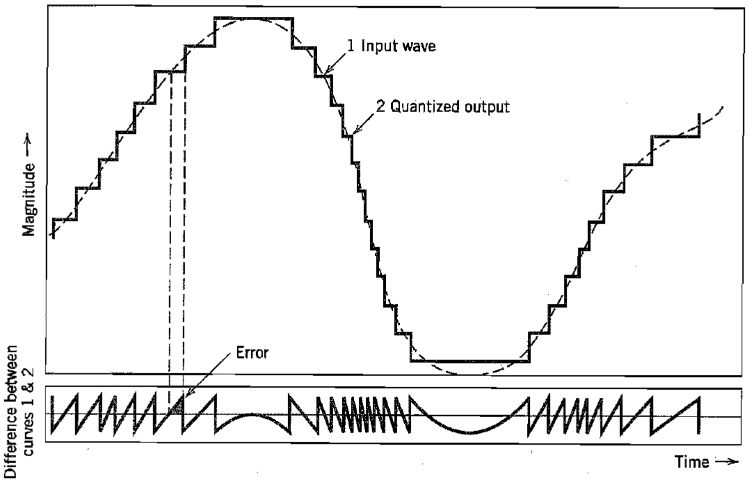


Fig. 3.11. Illustration of Quantization process

37. Write about Quantization noise.

- The use of quantization introduces an error defined as the difference between the input signal *m* and the output signal *v*. The error is called ***quantization noise***.
- Figure 3.11 illustrates a typical variation of the quantization noise as a function of time, assuming the use of a uniform quantizer of the midtread type.

38. Write about the quantizer step-size.

- Consider then an input m of continuous amplitude in the range $(-m_{max}, m_{max})$.
- Assuming a uniform quantizer of the midrise type, we find that the step-size of the quantizer is given by

$$\Delta = \frac{2m_{\max}}{L} \qquad (04)$$

where L is the total number of representation levels.

- For a uniform quantizer, the quantization error Q will have its sample values bounded by $-\Delta/2 \le q \le \Delta/2$.

39. What will happen if the quantization step size is smaller?

If the step-size Δ is sufficiently small (i.e., the number of representation levels L is sufficiently large), it isreasonable to assume that the quantization error *Q* is a uniformly distributed random variable, and the interfering effect of the quantization noise on the quantizer input is similar to that of thermal noise.

40. Write the expression for probability density function. (or)

Derive the expression for quantization noise of a PCM system. [Nov 2017]

The expression for the probability density function of the quantization error Q as follows:

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \leq \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases}$$

41. Write the expression for the output SNR of a uniform quantizer.

Let *P* denote the average power of the message signal *m(t)*. We may then express the output signal-to-noise ratio of a uniform quantizer as

$$(SNR)_O = \frac{P}{\sigma_Q^2}$$

$$= \frac{3P}{m_{\max}^2} 2^{2R}$$

Logarithmic Companding of speech signal

42. What is companding? [May/June 2016]

- Companding is simply a system in which information is first compressed, transmitted through a *bandwidthlimited channel*, and expanded at the receiving end.
- It is frequently used to reduce the bandwidth requirements for transmitting telephone quality speech, by reducing the 13-bit codewords to 8-bit codewords.

Pulse Amplitude Modulation

43. What is PAM?write its types.

- The amplitude of the pulse carrier is changed in proportion with the instantaneous amplitude of the modulating signal.

Types of PAM

Depending upon the shape of the PAM pulse, there are two types of PAM. They are:

(iii) Natural PAM

(iv) Flat top PAM

44. Why flat top PAM is widely used? [Dec – 2016]

- During the transmission, the noise interferes with the flat top of the transmitted pulses and this noise can be easily removed.
- In natural samples PAM, the pulse has varying top in accordance with the signal variation.
- When such type of pulse is received by the receiver, it always seems to be contaminated by noise.
- Then it becomes quite difficult to determine the shape of the top of the pulse and therefore amplitude detection of those pulses is not exact.
- As a result of this, errors are introduced in the received signal.
- The electronic circuitry needed to perform natural sampling is somewhat complicated because the pulse top shape is to be maintained. These complications are reduced by flat-top PAM.

45. What are the advantages and disadvantages of PAM?

Advantage: Simple generation and detection

Disadvantages:

- Effect of additive noise is high in PAM.
- Transmission bandwidth required is too large.
- The transmission power is not constant due to the changes in amplitudes of PAM pulses.

Pulse-Time Modulation

46. What is Pulse-Time Modulation and its types?

- In **pulse time modulation**, amplitude of pulse is held constant, whereas position of pulse is made proportional to the amplitude of signal at the sampling instant.

There are two types of pulse time modulation. They are:

- Pulse width modulation
- Pulse position modulation

47. Define Pulse Width Modulation (PWM)

- The width of the carrier pulses varies in proportion with the amplitude of modulating signal.
- The amplitude and frequency of the PWM wave remains constant.
- Only the width changes.
- The information is contained in the width variation.
- The additive noise, changes the amplitude of the PWM signal.
- Using the limiter circuit at the receiver, unwanted amplitude variations are easily removed.

48. Draw the waveform of PWM.

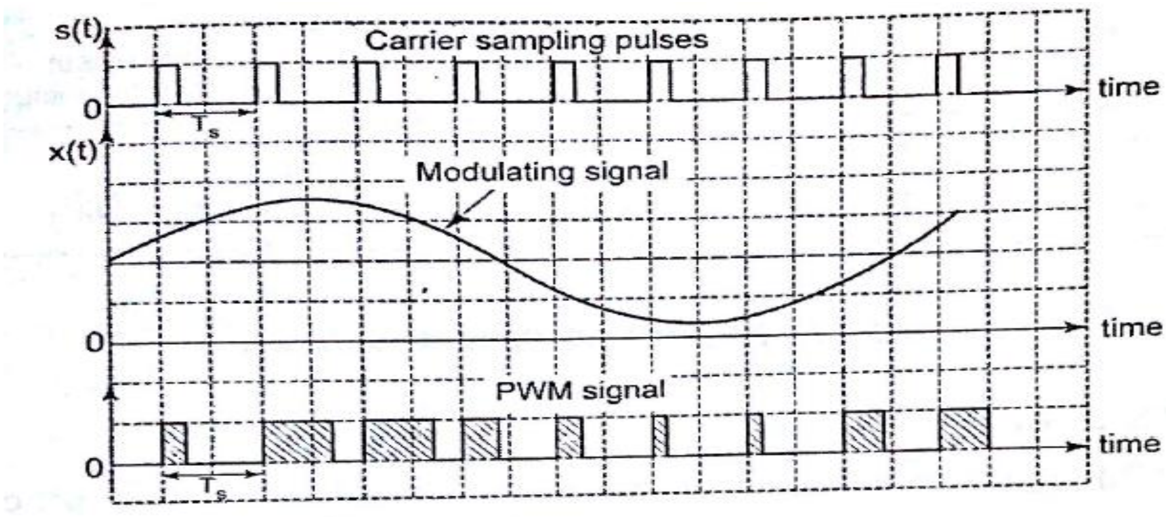


Fig: PWM signal

- Amplitude variations due to noise do not affect the performance. Thus PWM is more immune to noise than PAM.

49. **Draw the block diagram and waveform of PWM and PPM.**

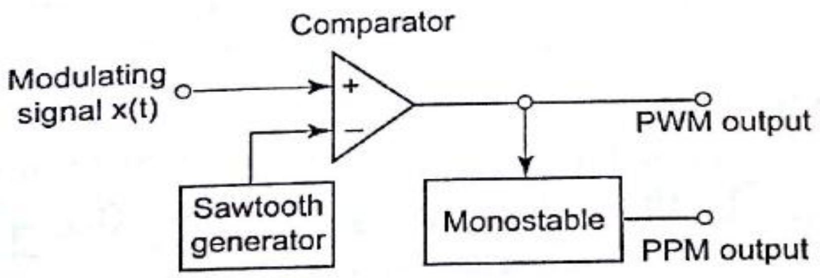


Fig : Block diagram of PWM and PPM generation

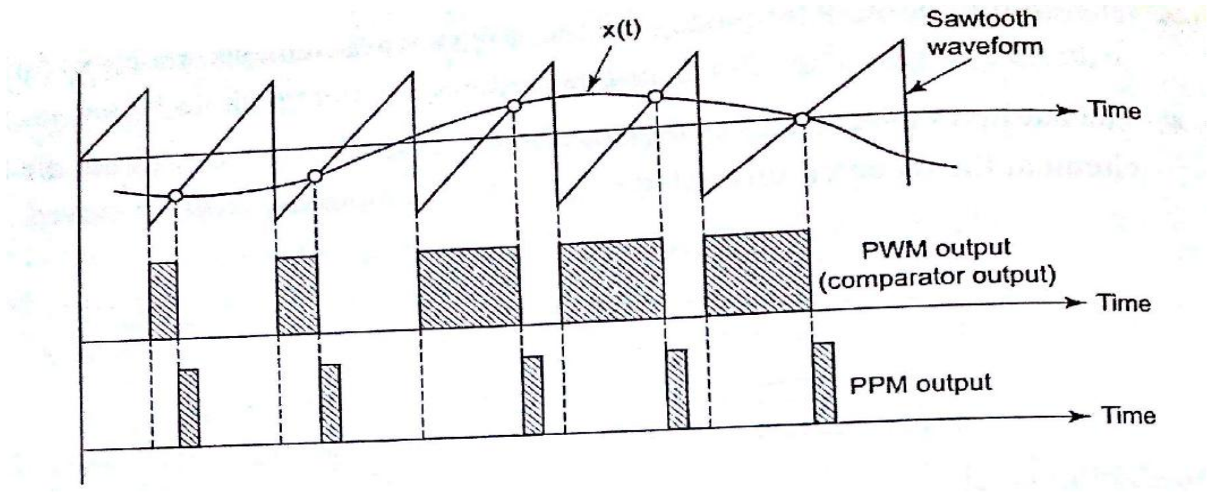


Fig : PWM and PPM waveforms

50. **Draw the Block diagram of PWM detection circuit.**

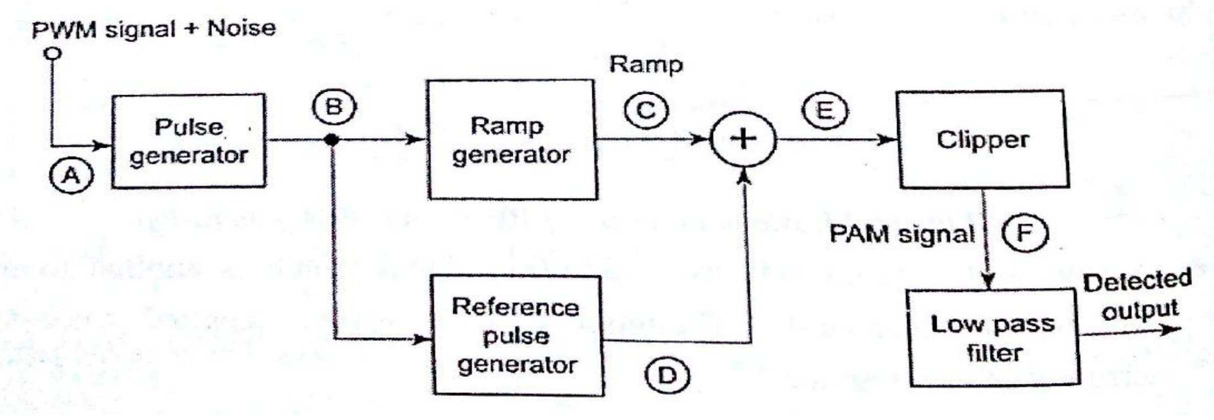


Fig: Block diagram of PWM detection circuit

51. **List the advantages and disadvantages of PWM.**

Advantages

- In PWM noise is less because here amplitude is constant.
- No synchronization required between transmitter and receiver.
- It is easy to separate the signal from noise.

Disadvantages

- Variable pulse width causes variable power contents. So, transmission must be powerful enough to handle the maximum width.
- Bandwidth requirement is higher than PAM.

52. Define Pulse Position Modulation (PPM).

The amplitude and width of the pulses are kept constant but the position of each pulse is varied in accordance with the amplitude of the sampled values of the modulating signal.

53. Draw PPM demodulator circuit.

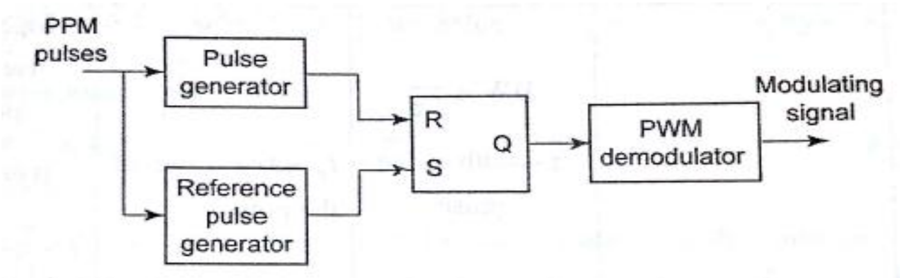


Fig : PPM demodulator circuit

54. List the advantages and disadvantages of PPM.

Advantages

- Due to constant amplitude of pulses, the transmitted power always remains constant.
- It is easy to reconstruct PPM signal from the noise contaminated PPM signal.

Disadvantages

- Synchronization required between the transmitter and receiver.
- Large bandwidth requirement.

55. Difference Between PAM, PWM, and PPM.

The below table gives the detailed difference between PWM, PAM, and PPM.

Sr. No.	Parameter	PAM	PWM	PPM
1	**Type of Carrier**	Train of Pulses	Train of Pulses	Train of Pulses
2	**Variable Characteristic of the Pulsed Carrier**	Amplitude	Width	Position
3	**Bandwidth Requirement**	Low	High	High
4	**Noise Immunity**	Low	High	High
5	**Information Contained in**	Amplitude Variations	Width Variations	Position Variations
6	**Power efficiency (SNR)**	Low	Moderate	High
7	**Transmitted Power**	Varies with amplitude of pulses	Varies with variation in width	Remains Constant

Pulse-Code Modulation

56. What is Pulse code modulation?

Pulse code modulation:

In pulse-code modulation (PCM), a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.

57. What are the basic operations performed in PCM?

The basic operations performed in the transmitter of a PCM system are sampling, quantization, and encoding; the low-pass filter prior to sampling is included merely to prevent aliasing of the message signal.

58. Write about quantization process in PCM.

The quantizing and encoding operations are usually performed in the same circuit, which is called an analog-to-digital converter.

59. What are the operations performed in PCM receiver?

- The basic operations in the receiver are regeneration of impaired signals, decoding, and reconstruction of the train of quantized samples.
- Regeneration also occurs at intermediate points along the transmission path as necessary.

60. Write about sampling in PCM.

Sampling in PCM

- The incoming message (baseband) signal is sampled with a train of rectangular pulses, narrow enough to closely approximate the instantaneous sampling process.
- Thus the application of sampling permits the reduction of the continuously varying message signal (of some finite duration) to a limitednumber of discrete values per second.

61. What is the purpose of ternary code used in PCM?

The two symbols of the binary code are customarily denoted as 0 and 1. In practice, a binary code is preferred over other codes (e.g.,ternary code) for two reasons:

1. Binary symbol withstands a relatively high level of noise.
2. The binary code is easy to generate and regenerate.

62. Draw the block diagram of Pulse code modulation.

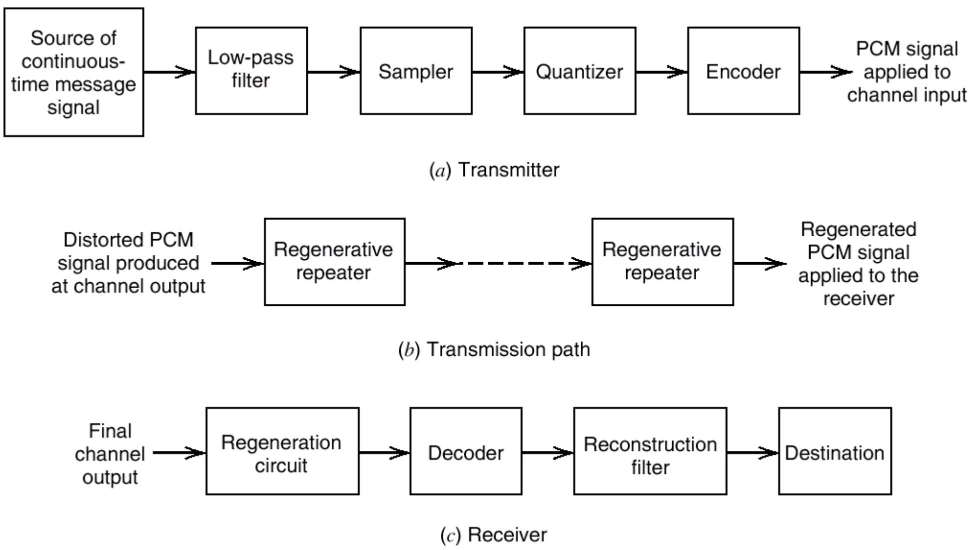


FIGURE. The basic elements of a PCM system

(a) Transmitter, (b) transmission path,connecting the transmitter to the receiver, and (c) receiver.

63. In a PCM system, the output of the transmitting quantizer is digital. Then why is it further encoded. [Nov 2017, May 2018]

In a PCM system, the output of the transmitting quantizer is digital. It is required to translate the discrete set of sample values to a more appropriate form of the signal. So it is further encoded.

64. Write about the regeneration in the transmission path of PCM.

Regeneration along the Transmission Path

Three basic functions are performed by a regenerative repeater: equalization, timing, and decision making.

- The equalizer shapes the received pulses so as to compensate for the effects of amplitude and phase distortions produced by the transmission characteristics of the channel.
- The timing circuitry provides a periodic pulse train, derived from the received pulses.

65. What are reasons for the regenerated signal departs from the original signal?

- In practice, however, the regenerated signal departs from the original signal for two main reasons:
  1. The unavoidable presence of channel noise and interference causes the repeater to make wrong decisions, thereby introducing bit errors into the regeneratedsignal.
  2. If the spacing between received pulses deviates from its assigned value, a jitter is introduced into the regenerated pulse position, thereby causing distortion.

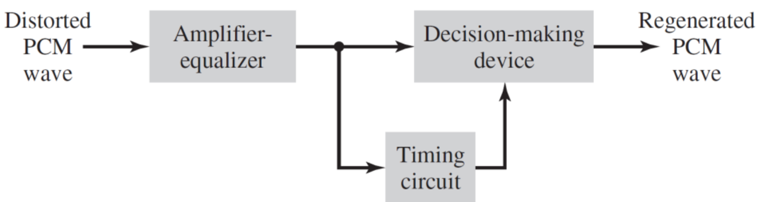


Fig. 5.13. Block diagram of Regenerative repeater

66. Explain the reconstruction process in PCM.

Reconstruction

- The final operation in the receiver is to recover the message signal.
- This operation isachieved by passing the expander output through a low-pass reconstruction filterwhose cutoff frequency is equal to the message bandwidth.
- Recovery of the messagesignal is intended to signify estimation rather than exact reconstruction.

Time Division Multiplexing:

67. What is the need for TDM system? **[Apr - 2019]**

A *time-division multiplex* (TDM) *system*, which enables the joint utilization of a common communication channel by a plurality of independent message sources without mutual interference among them.

68. Draw the block diagram of TDM system.

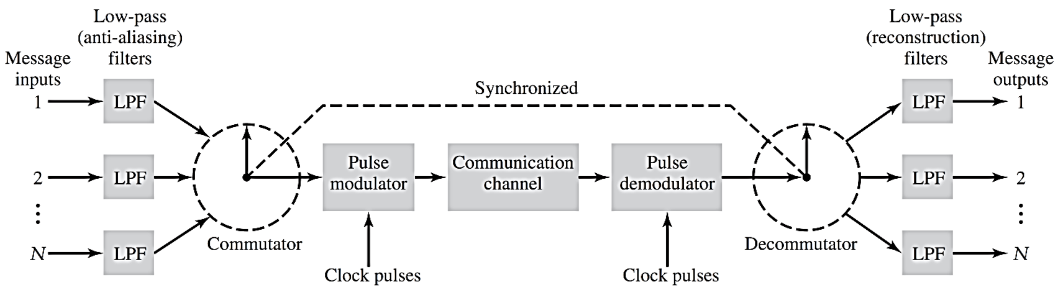


FIGURE 5.21 Block diagram of TDM system.

69. What is the function of commutator?

The function of the commutator is twofold:

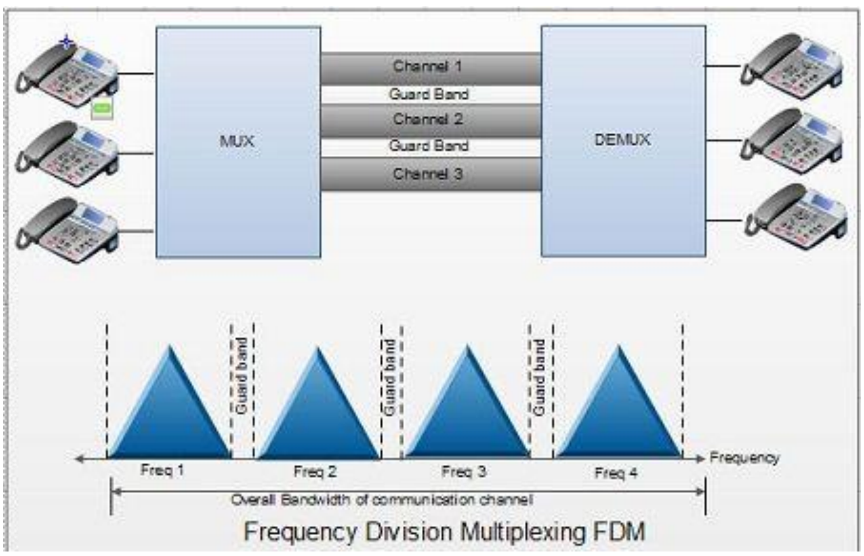
(1) to take a narrow sample of each of the *N* input messages at a rate that is slightly higher than 2*W*, where *W* is the cutoff frequency of the anti-aliasing filter, and

(2) to sequentially interleave these *N* samples inside the sampling interval Indeed, this latter function is the essence of thetime-division multiplexing operation.

70. Define Frequency-Division Multiplexing (FDM) .

- Frequency-Division Multiplexing (FDM) is a scheme in which numerous signals are combined for transmission on a single communications line or channel.
- It is analog multiplexing technique. Each signal is assigned a different frequency (sub channel) within the main channel. its requires channel synchronization.

71. Draw the block diagram of FDM.



72. List the advantages and disadvantages of FDM.

Advantages of FDM:

1. A large number of signals (channels) can be transmitted simultaneously.
2. FDM does not need synchronization between its transmitter and receiver for proper operation.
3. Demodulation of FDM is easy.
4. Due to slow narrow band fading only a single channel gets affected.

Disadvantages of FDM:

1. The communication channel must have a very large bandwidth.
2. Intermodulation distortion takes place.
3. Large number of modulators and filters are required.
4. FDM suffers from the problem of crosstalk.
5. All the FDM channels get affected due to wideband fading.

73. Mention the applications of FDM.

Applications of FDM

1. FDM is used for FM & AM radio broadcasting.
2. FDM is used in television broadcasting.
3. First generation cellular telephone also uses FDM.

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